Readings about Strings and Tries

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I. Manacher's Algorithm

Statement: Find all pairs (i, j) which make a palyndrome substring.

A. Detailed Statement

Worst case we would have $O(n^2)$ palyndromes.

Compact way of keeping palyndromes: $i = 0 \dots n-1$, $d_1[i], d_2[i]$ represent the number of palyndromes with odd and even length with their center in i.

Ex. *abababc*, has $d_1 = 3$ (odd length), and *cbaabd* has $d_2[3] = 2$ (even length).

Sub-palindrome with l size with center in i, we also have with $l-2, l-4, \dots$.

Both palyndrom arrays can be calculated in linear time.

Solution: Can be done with string hashing and suffix trees, but this has a smaller constant and memory complexity.

B. Trivial Algorithm

Tries to increase the answer by one until it's possible for each center i. It is $O(n^2)$ in time. Implementation being:

```
vector<int> d1(n),  d2(n);
for (int i = 0; i < n; i++) {
    d1[i] = 1; // Pair with itself
    while (0 <= i - d1[i] && i + d1[i] < n &&
        s[i - d1[i]] == s[i + d1[i]]) {
        d1[i]++;
    }

    d2[i] = 0;
    while (0 <= i - d2[i] - 1 && i + d2[i] < n
         && s[i - d2[i] - 1] == s[i + d2[i]])
        {
        d2[i]++;
    }
}
```

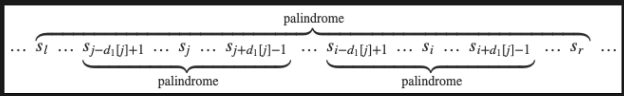
C. Manacher's Algorithm

Allows to find all the sub-palyndromes with odd length (even length is just a small mod).

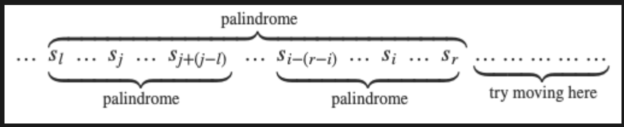
Borders: (l, r) of the palyndrome with maximal r. Initially we assume $l = 0, r = -1$

We want to calculate $d_1[i]$ with all the previous i already calculated.

- If it is ourside of the current rightmost sub-palyndrome $i > r$, we launch the trivial algorithm, allowing us to calculate $d_i[i]$ and updating (l, r).
- $i \leq r$. We can flip i into $j = l + (r - i)$ and since they are symmetrical, we can **almost always** do $d_1[i] = d_1[j]$. Like so:



- **Trick case:** When the inner palyndrome reaches the border of the outer one. $j - d_1[j] + 1 \leq l$. Since the symmetry of the outer palyndrome is not guaranteed, we assign $d_1[i] = r - i + 1$, and then run the trivial algorithm to increase the value if necessary, updating in the end. Like so:



Note we only use the part of the palyndrome with guaranteed symmetry before moving to the *try moving here part*.

A similar algorithm is then used for the even part.

D. Complexity

Not as intuitive, but if we look at *Z-function building algorithm* we can check it's similar and linear.

We can also notice that r can only increase by one ine very iteration of the trivial algorithm, and that it can't decrease. $O(n)$ iterations. The rest is linear.

E. Implementation

Here is the implementation, which is fairly similar for both d_1, d_2.

```
vector<int> d1(n), d_2(n);
for (int i = 0, l = 0, r = -1; i < n; i++) {
    int k = (i > r) ? 1 : min(d1[l + r - i], r
         - i + 1);
    while (0 <= i - k && i + k < n && s[i - k]
         == s[i + k]) {
        k++;
    }
    d1[i] = k--;
    if (i + k > r) {
        l = i - k;
        r = i + k;
    }
}
for (int i = 0, l = 0, r = -1; i < n; i++) {
    int k = (i > r) ? 0 : min(d2[l + r - i +
        1], r - i + 1);
    while (0 <= i - k - 1 && i + k < n && s[i
        - k - 1] == s[i + k]) {
        k++;
    }
    d2[i] = k--;
    if (i + k > r) {
```

```
        l = i - k - 1;
        r = i + k ;
    }
}
```

REFERENCES

[1] *Manacher's Algorithm - Finding all sub-palindromes in* $O(N)$, jakobkogler for E-maxx. From: https://cp-algorithms.com/string/manacher.html