1 Normalized Dispersion Response

Linear Approximation:

$$\begin{aligned}
\Delta D_N &= D_N - D_{N_{model}} \\
&= \frac{D}{\sqrt{\beta}} - \frac{D_m}{\sqrt{\beta_m}} \\
&= \frac{\Delta D + D_m}{\sqrt{\Delta\beta + \beta_m}} - \frac{D_m}{\sqrt{\beta_m}} \\
&= \frac{\Delta D}{\sqrt{\Delta\beta + \beta_m}} + \frac{D_m}{\sqrt{\Delta\beta + \beta_m}} - \frac{D_m}{\sqrt{\beta_m}} && \Rightarrow \text{First order linearisation} \\
&\approx \frac{\Delta D}{\sqrt{\beta_m}} + \frac{D_m}{\sqrt{\Delta\beta + \beta_m}} - \frac{D_m}{\sqrt{\beta_m}} && \Rightarrow \text{Taylor } \tfrac{1}{\sqrt{\Delta\beta+\beta_m}} \text{ to first order} \\
&\approx \frac{\Delta D}{\sqrt{\beta_m}} + \left[\left(\frac{1}{\sqrt{\beta_m}} - \frac{1}{2}\frac{\Delta\beta}{(\beta_m + \Delta\beta)^{3/2}}\right) - \frac{1}{\sqrt{\beta_m}}\right] D_m && \Rightarrow (\beta_m + \Delta\beta)^3 \approx \beta_m^3 \\
&\approx \frac{1}{\sqrt{\beta_m}}\Delta D - \frac{1}{2}\frac{D_m}{\sqrt{\beta_m^3}}\Delta\beta && \text{both linear } \Delta D = \Delta D(\delta K_0) \text{ and } \Delta\beta = \Delta\beta(\delta K_1) \\
&= \frac{\Delta D}{\sqrt{\beta_m}} - \frac{1}{2} D_{N_{model}} \Delta BetaBeat
\end{aligned}$$