# Item Characteristic Curves generated from common CTT Item Statistics

Diego Figueiras[1] & John T. Kulas[1]

[1] Montclair State University

Author Note

Add complete departmental affiliations for each author here. Each new line herein must be indented, like this line.

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The authors made the following contributions. John T. Kulas: .

Correspondence concerning this article should be addressed to Diego Figueiras, Dickson Hall 226. E-mail: figueirasd1@montclair.edu

# Item Characteristic Curves generated from common CTT Item Statistics

Item characteristic curves are referenced by psychometricians as visual indicators of important attributes of assessment items - most frequently *difficulty* and *discrimination*. Within these visual presentations the x-axis ranges along "trait" levels (by convention annotated with the greek $\theta$), whereas the y-axis displays probabilities of responding to the item within a given response category. In the context of true tests, the response categories are binary, and the y-axis probability refers to the likelihood of a "correct" response. From this visualization, the observer extracts the likelihood that respondents of any trait level would answer a focal item correctly. If the curve transitions from low to high likelihood at a location toward the lower end of the trait (e.g., "left" on the plotting surface), this indicates that it is relatively easy to answer the item correctly. Stated differently, it does not take much $\theta$ to have a high likelihood of answering correctly. On the contrary, if the growth in the curve occurs primarily at higher trait levels, this indicates that the item is relatively more difficult. If the curve is sharp (e.g., strongly vertical), this indicates high discrimination; if it is flatter, that is an indication of poorer discrimination.

Assessment specialists who examine ICCs usually do so from within the psychometric framework of either Item Response Theory or Rasch modeling. These frameworks provide the parameters necessary to plot the curves. For the IRT 2 parameter logistic model (2PL), these parameters represent item difficulty and item discrimination. Item difficulty (the $b$-parameter) is scaled as the trait level associated with a 50% likelihood of correct response (e.g., it is scaled to $\theta$). Item discrimination ($a$-parameter) is the degree to which an item differentiates across individuals who are characterized as being relatively lower or higher on the trait. From a Classical Test Theory (CTT) orientation, item difficulty is most commonly represented by the percent of individuals answering the item correctly (also referred to as a *p-value*). Item discrimination can be conveyed via a few CTT indices, but the most commonly calculated and consulted index is the corrected item total correlation.

Assessment specialists who consult these CTT parameters don't typically (to our knowledge!) attempt to represent them visually, as is common in IRT and Rasch applications. However, there is perhaps little reason for them not to, as ICCs based on CTT parameters could provide snapshot psychometric information as valuable as those gained from IRT- or Rasch-derived ICCs. The largest obstacle to psychometricians is the property of invariance, which refers to IRT parameter independence across item and person estimates. However, this property is often overstated, as invariance is only attained with perfect model-data fit (which never occurs), and is also only true when subjected to linear transformation (commonly across samples). Additionally, several comparative investigations have noted commonality between IRT and CTT difficulty and discrimination estimates as well as relative stability of CTT estimates when samples are large and/or judiciously constructed (Fan, 1998). Fan in fact summarizes that IRT and CTT "... framework produce very similar item and person statistics" (p.379). Hambleton and Jones (1993) concluded that "no study provides enough empirical evidence on the extent of disparity between the two frameworks and the superiority of IRT over CTT despite the theoretical differences."

Fan (1998) looked at the correlations between ability estimates and item difficulty in CTT and all three IRT models. These correlations were very high, generally between .80 and .90. As of item discrimination, correlations were moderate to high, with only a few being very low.[ˆfootie] [ˆ]:...and in fact, as is presented below, the relationship between the IRT and CTT discrimination indices is non-linear - the correlation is an inappropriate index to capture the magnitude of this relationship.

He also looked at the item invariance for all models. In theory, the major advantage of IRT models over CTT is that the latter has a circular dependency between the item and person statistics, while IRT has no such dependency, which means that the item parameters don't depend on the sample and the person parameters don't depend on the set of items. This property of invariance is very important, since item estimates can be used regardless of the sample you are giving the test or assessment to. An item will always have the same level of difficulty regardless of who is responding, for example.

What Fan (1998) got on his study, however, shows empirical evidence against this supposed advantage of IRT against CTT. The CTT item difficulty and discrimination degrees of invariance were highly correlated with those of IRT, indicating that they were highly comparable.

Lord (2012) described a function that approximates the relationship between IRT parameters and the CTT discrimination index of an item-test biserial correlation:

$$a_i \cong \frac{r_i}{\sqrt{1 - r_i^2}}$$

This formula wasn't intended for practical purposes but rather to assist in the conceptual comprehension of the discrimination parameter in IRT for people who were more familiar with CTT procedures. In an effort to move from the conceptual to a practical application, Kulas et al. (2017) proposed a modification that minimized the average residual (either $a_i$ or $r_i$, where $r_i$ is the *corrected* item-total *point-biserial* correlation).

Simulations identified systematic slope and inflection differences across item with differing item difficulty values, so the formula was further changed to include the following modifiers This revised formula is used in the current presentation:

$$\hat{a_i} \cong [(.51 + .02z_g + .3z_g^2)r] + [(.57 - .009z_g + .19z_g^2)\frac{e^r - e^{-r}}{e - e^r}]$$

Where $g$ is the absolute deviation from 50% responding an item correctly and 50% responding incorrectly (e.g., a "p-value" of .5). $Z_g$ is the standard normal deviation associated with $g$. The transformation of the standard p-value was recommended in order to scale this index along an interval-level metric more directly anaologous to the IRT $b$

parameter. Figure XX visualizes the re-specifications of Lord's formula at p-values (difficulty) of .5, .3 (or .7), and .1 (or .9).

As we can see, the higher the corrected item-total correlations, the higher the estimated IRT a-parameter (discrimination). Also, as the p-values (difficulty) deviates from 0, the relationship between the estimated IRT a-parameter and the corrected item-total correlations becomes stronger.

Practitioners and researchers that don't use IRT or Rasch models and instead opt to follow a CTT philosophy would benefit from having ICCs that use CTT statistics. This study intends to show evidence of the overlapping nature of CTT and IRT parameters when it comes to plotting ICCs.

## Study 1 - Visual of discrimination relationship

The purpose of study 1 is to look at the visualizations resulting from Kulas et al. (2017) formula on simulated data. We hypothesize that the relationship between the estimated IRT a-parameter and the corrected item-total correlations will be stronger as the later deviates from 0, which would mean that the item has more discrimination.

### Procedure and methods

We simulated data using Han (2007) software. Our sample was 10,000 observations, with a mean of 0 and a standard deviation of 1. The number of items were 100, with response categories of either correct or incorrect (1 and 0).The mean for the a parameter was 2, and the standard deviation 0.8. The mean for parameter b was 0 and the standard deviation 0.5.

## Results

### Study 2 - Item Characteristic Curves comparisons.

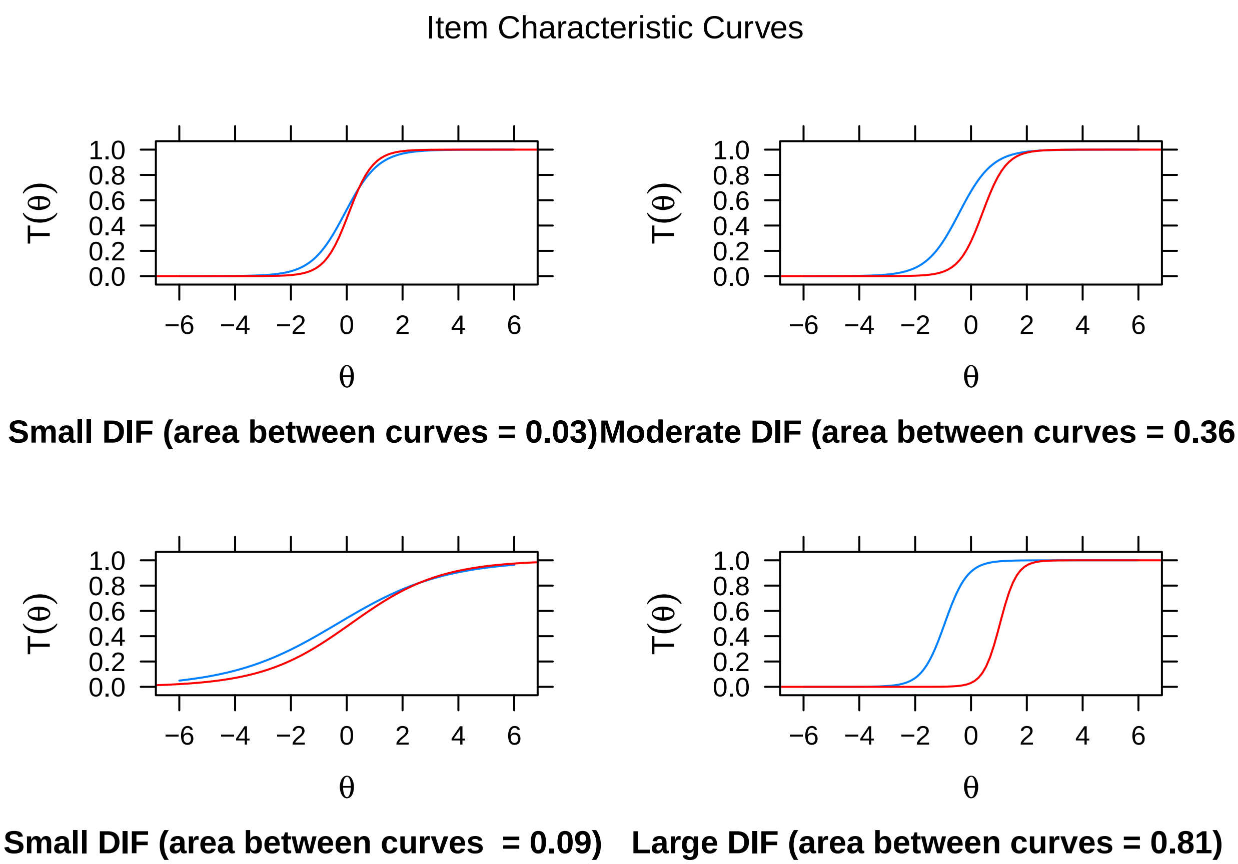
The purpose of study 2 is to simulates a lot of test data and then generate ICCs based on the IRT model and then we compare that to our CTT estimates.

## Procedure and materials

The same simulated data as in study 1 was used. The mirt package was used to compute the IRT statistics. The blue curves were plotted using 2PL IRT parameters (a and b), while the red curves were plotted using CTT parameters (p-values and corrected item-total correlations, modifying them with Kulas et al. (2017) formulas).

## Results

We used R [Version 4.1.1; R Core Team (2020)] and the R-packages *}ape* [@}R-ape], *dplyr* [Version 1.0.7; Wickham et al. (2021)], *DT* [Version 0.19; Xie et al. (2021)], *forcats* [Version 0.5.1; Wickham (2021a)], *formattable* (Ren & Russell, 2021), *geiger* [Version 2.0.7; Alfaro et al. (2009); Eastman et al. (2011); Slater et al. (2012); Harmon et al. (2008); Pennell et al. (2014)], *ggplot2* [Version 3.3.5; Wickham (2016)], *gridExtra* [Version 2.3; Auguie (2017)], *irtplay* [Version 1.6.2; Lim (2020)], *jpeg* [Version 0.1.9; Urbanek (2021)], *knitr* [Version 1.34; Xie (2015)], *lattice* [Version 0.20.44; Sarkar (2008); Sarkar and Andrews (2019)], *latticeExtra* [Version 0.6.29; Sarkar and Andrews (2019)], *markdown* [Version 1.1; Allaire et al. (2019); Xie et al. (2018); Xie et al. (2020)], *mirt* [Version 1.34; Chalmers (2012)], *officer* (Gohel, 2021), *papaja* [Version 0.1.0.9997; Aust and Barth (2020)], *pdftools* [Version 3.0.1; Ooms (2021)], *psych* [Version 2.1.9; Revelle (2021)], *purrr* [Version 0.3.4; Henry and Wickham (2020)], *readr* [Version 2.0.1; Wickham and Hester (2021)], *readxl* [Version 1.3.1; Wickham and Bryan (2019)], *reticulate* [Version 1.22; Ushey et al. (2021)], *rmarkdown* [Version 2.11; Xie et al. (2018); Xie et al. (2020)], *shiny* [Version 1.7.0; Chang et al. (2021)], *stringr* [Version 1.4.0; Wickham (2019)], *tibble* [Version 3.1.4; Müller and Wickham (2021)], *tidyr* [Version 1.1.3; Wickham (2021b)], *tidyverse* [Version 1.3.1; Wickham et al. (2019)], and *tinytex* [Version 0.33; Xie (2019)] for all our analyses. The area between ICC's was calculated between CTT-derived and IRT-derived ICCs. The average difference for all 100 curves was 0.35.




## Results

## Discussion

If this general idea is well-recieved (SIOP members would seem to represent a great barometer!) we would like to stress the CTT ICCs via further and more extensive conditions. That is, are there patterns that help explain CTT ICCs that diverge from their

IRT counterparts? Although our simulations did generate a range of item difficulties and discriminations, we have not yet fully explored systematic patterns of extremly difficult/easy items as well as very poorly discriminating items. If patterns emerge, we would like to model predicted discrepancies via incorporating error bars within our visualizations.

Additionally, if there is interest in this general idea we would likely publish our function as a small `R` package, perhaps to supplement the `psych` package's "alpha" function, which produces corrected item-total correlations as well as p-values within the same output table (e.g., the "input" data is already available in tabular format).

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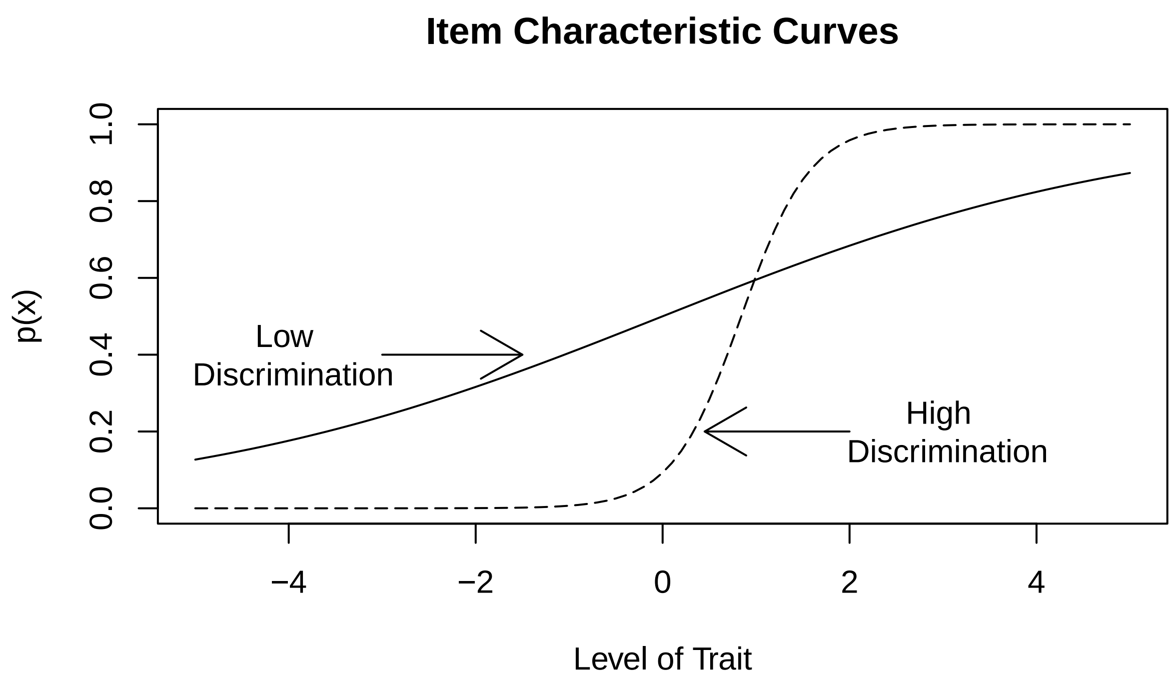


*Figure 1*. Item characteristic curves primarily reflecting differences in discrimination.

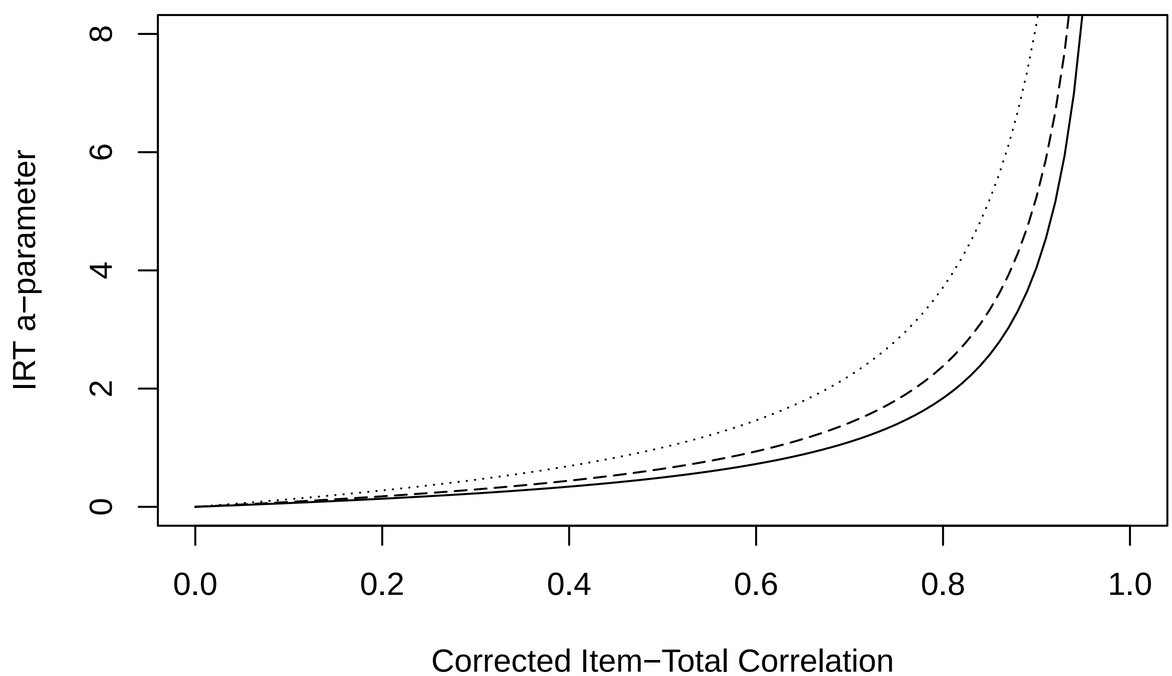


*Figure 2*. Empricially-derived functional relationship between the IRT $a$ parameter and the CTT corrected-item total correlation as a function of item difficulty (p-value; solid = .5, dashed = .3/.7, dotted = .1/.9).

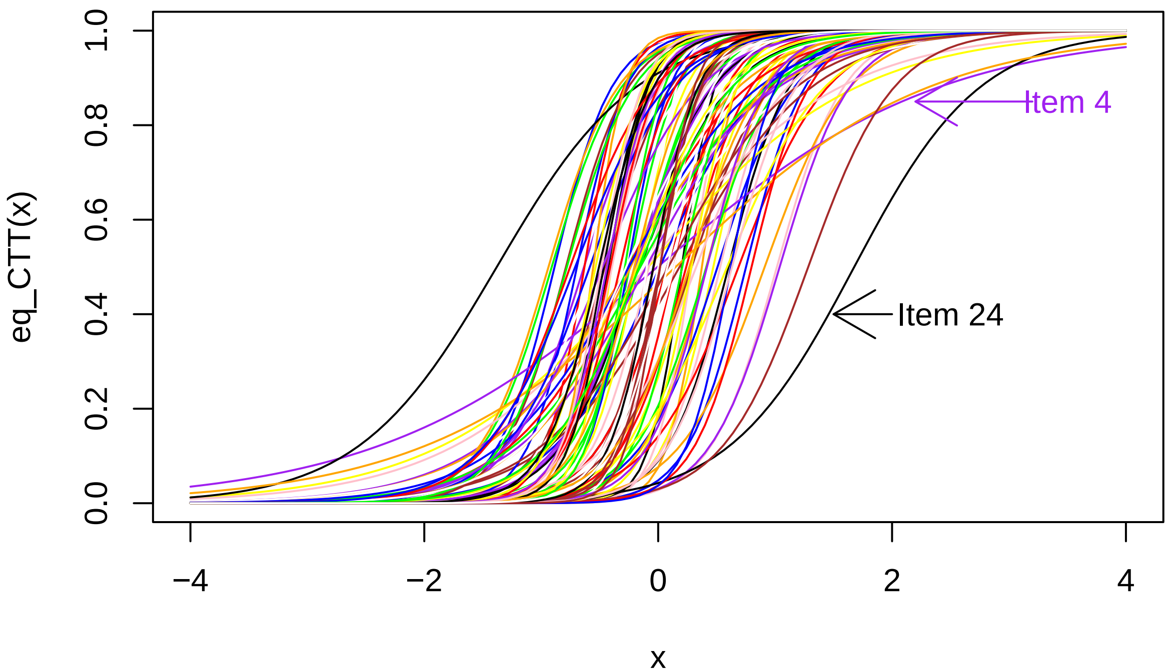


*Figure 3*. ICCs derived from only CTT parameters (with two noteworthy ICCs annotated).

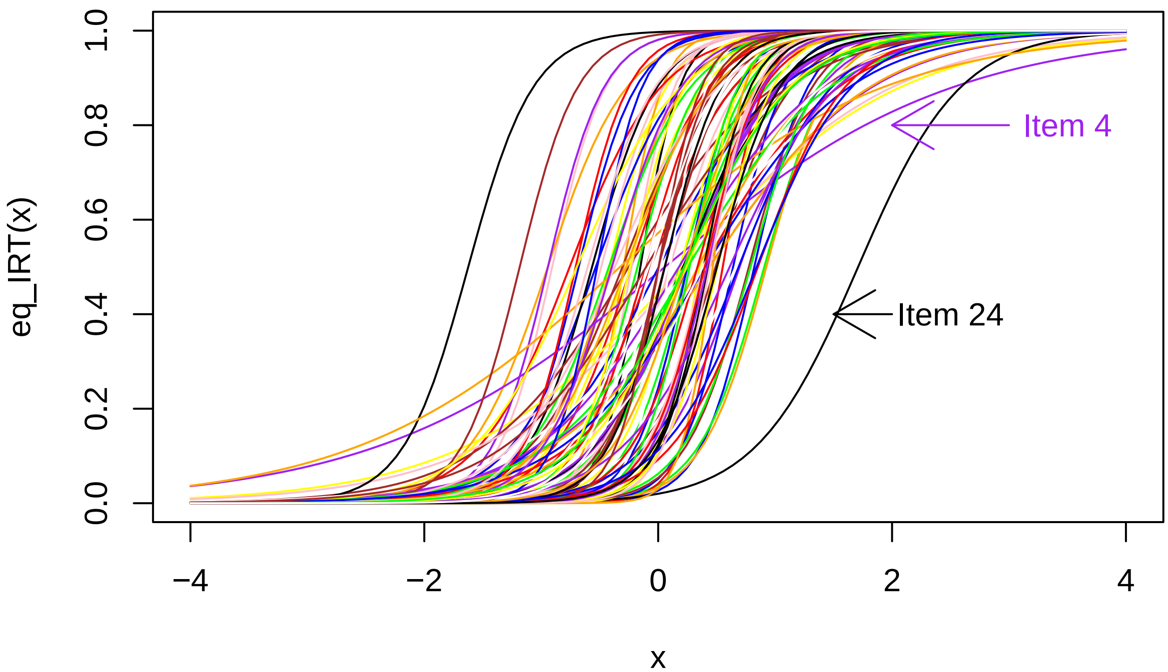


*Figure 4*. Typical ICCs derived from IRT parameters (same noteworthy items annotated).