MATH 315, Fall 2020
Homework 7

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Homework Team *(team number)*: *(names of team members)*

NOTE: We are returning to team submissions for this assignment. Submit only one set of solutions for all team members.

This assignment is associated with Chapter 5, Differentiation and Integration. It illustrates numerical methods for differentiating a function at a point and across an interval, and also some of the resulting errors including those caused by implementing these methods on a computer.

1. Use Taylor series to show that the one-sided difference approximation

$$f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$$

   is second order in h.

2. (a) Write a short MATLAB script to "reproduce" the data in Table 5.1 of your text for the derivative of $f(x) = \frac{1}{1+25x^2}$ at $x = 1$. Create a similar table that includes a third column that implements the second order one-sided difference approximation given in Problem 1. Compute an additional table showing the absolute value of the error. Discuss your results. When does round off error begin to dominate for each approximation?

   (b) Use `polyfit` to determine the unique 10th order polynomial that interpolates $\frac{1}{1+25x^2}$ at equally spaced points on the interval $[-1, 1]$. Differentiate and evaluate this polynomial using `syms`, `diff`, and `subs` to approximate the derivative at $x = 1$. Create a plot that shows the analytical derivative of $\frac{1}{1+25x^2}$ on the interval $[0.8, 1]$ and the result obtained using this approximation (i.e. the derivative of the function on the interval $[0.8, 1]$). *Hint: you can use* `double(subs(f,x))` *to evaluate the symbolic function for the vector of values,* `x`.

   (c) Use `spline` and `unmkpp` to determine the spline that interpolates $\frac{1}{1+25x^2}$ on the interval $[-1, 1]$ (use the same interpolation points you used in Problem 2b). Differentiate and evaluate this appropriate piecewise polynomial using `syms`, `diff`, and `subs` to approximate the derivative at $x = 1$. Create a plot that shows the analytical derivative and the result obtained using this approximation.

   (d) Compare your results for the approximation of $f'(1)$ from Problems 2b and 2c to that obtained using `gradient`. (For the spacing, use the interpolation points you used in Problem 2b and 2c)