MATH 315, Fall 2020
Homework 6

NAME:

Important Information: This will **not** be a team assignment. You must work on this on your own, and turn in your own solutions.

You should not ask friends or anyone but me for help before Friday's lab. You will have an opportunity to work with a small group in the lab on Friday, and you can then ask that small group questions if you are having trouble. Your lab group can provide help and guidance to each other, but they **should not give** solutions to each other.

I am happy to provide individual guidance during office hours, so please do not hesitate to ask me for help.

This assignment is associated with Chapter 4, Curve Fitting – we experiment with MATLAB's `polyfit, spline` and `pchip` functions. You should write one script m-file to perform all of the experiments and produces all of the plots outlined in the assignment.

1. **Math Problem:** Suppose $f(x)$ is a function that is n times continuously differentiable, and let $p(x)$ be the polynomial of degree $n-1$ that interpolates $f(x)$ at the points $\{x_i, f(x_i)\}$, $i = 1, 2, 3, \cdots, n$. Then for any $t \in [x_1, x_n]$

$$|f(t) - p(t)| = \left| \frac{f^{(n)}(c)}{n!} \right| |w(t)| \,,$$

   where $c \in [x_1, x_n]$ and $w(t) = (t - x_1)(t - x_2) \cdots (t - x_n)$.

   Show that the maximum error associated with **linear** interpolation using equally spaced points is bounded by $\dfrac{M}{8}(x_2 - x_1)^2$, where $M = \max\limits_{x_1 \le x \le x_2} |f''(x)|$.

   (Hint: Find the expression for $w(t)$ for linear interpolation, and then find the maximum value of $|w(t)|$ on the interval $[x_1, x_2]$.)

2. **MATLAB Problem:** The following is weekly measured laboratory data (the application is not important):

Week:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Measurement:	2	7	5	5	8	9	10	9	15	14	25	45	50	35	48	74	102

Week:	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Measurement:	147	171	199	226	208	200	150	121	54	27	30	12	10	10	4	5

(a) Use `polyfit` and `polyval` to construct an interpolating polynomial for *all* of the given data. Create a plot that contains the data points as red circles and the interpolating polynomial as a blue curve. Is the interpolating polynomial a good representation of the data? Explain.

(b) You get a warning about that suggests trying "centering and scaling". This only requires a small modification of calls to `polyfit` and `polyval`. Try, and comment about whether this improves your result.

(c) Use `polyfit` and `polyval` to construct an interpolating polynomial using data for weeks $1, 3, 5, \ldots, 33$. Create a plot that contains *all* data points in the above table as red circles, and the interpolating polynomial through data for weeks $1, 3, 5, \ldots, 33$. Is the interpolating polynomial a good representation of the data? Explain.

(d) Use `polyfit` and `polyval` to construct an interpolating polynomial using data for weeks

$$1, 2, 3, 4, 6, 9, 11, 14, 17, 20, 23, 25, 28, 30, 31, 32, 33$$

Create a plot that contains *all* data points in the above table as red circles, and the interpolating polynomial through data for the weeks specified in this problem. Is the interpolating polynomial a good representation of the data? Why do we get a better interpolating polynomial when using the data for this problem than when using the data in the previous problem?

(e) Repeat the above experiments, but this time use MATLAB's built-in `spline` function. (You will not need centering and scaling for this). Does the use of `spline` change the results?

(f) Repeat the above experiments, but this time use MATLAB's built-in `pchip` function. (You will not need centering and scaling for this).

Does the use of `pchip` change the results?

Turn in all codes, as well as any plots and tables that you need to clearly explain your results. Your work should be neatly organized.