MATH 315, Fall 2020
Homework 0

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This homework is intended for practice, and to remind you of some important concepts from Linear Algebra. This assignment also introduces you to the LaTeX typesetting system and the MATLAB computing language. This will not count toward your grade, but I encourage you to work through it, and please do not be shy about asking for help.

If you want to learn LaTeX to prepare your homework assignments for this course, then you can use this document as a template, and make the following modfications:

- Edit the title, author name, and team letter to which you were assigned.
- Include a description of the problem you are solving.
- Provide information about the approach you used.
- Include any figures and results from your computations.
- Provide a summary of what you learned.

1 LaTeX

LaTeX is like a programming language, so you need to learn a little about how to use it. You can download the software for free to your computer; see, for example, https://www.latex-project.org/get/. There are also online services that can be good for collaborative group work; one that is used a lot at Emory is Overleaf.

1.1 Math Expressions

LaTeX is much better at creating and displaying mathematical formulas than MS Word. For example, if you want to create some displayed equations, you can do it as follows:

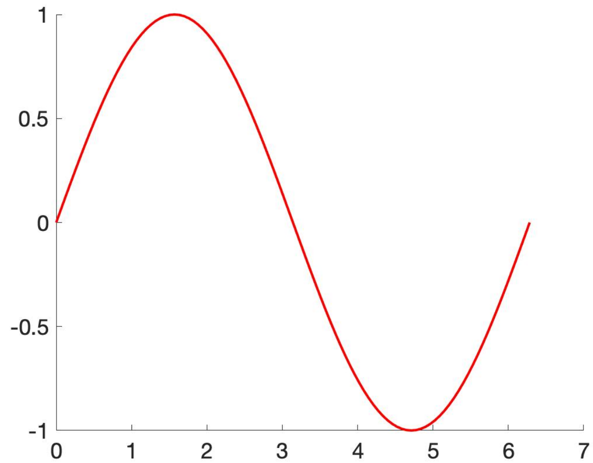
$$x(t) = S_x(t)\,, \quad y(t) = S_y(t),$$

or include some more complicated math expressions:

$$L = \int_{t_1}^{t_n} \sqrt{[S_x'(t)]^2 + [S_y'(t)]^2}\,dt = \sum_{i=1}^{n-1} \int_{t_i}^{t_{i+1}} \sqrt{[S_x'(t)]^2 + [S_y'(t)]^2}\,dt$$

1.2 Including Figures

In MATLAB, when you create a figure, you can use the `File --> Save as` menu option to save the figure in a variety of formats. In this illustration I'll use `JPEG`. For example, if you save the figure into a file with the name `MySinPlot.jpg`, then you can put it into the LaTeX document as follows:



See Section 3 for more information on how this plot was created in MATLAB.

1.3 Including MATLAB code in a document

There are many ways to include your code in the document. One easy way is to cut-and-past it into the *verbatim* environment, such as:

```
function f = NormalPDF(x, mu, sigma)
%
% Compute f(x) for normal PDF, with mean mu and standard deviation sigma.
%
c = 1 / (sigma*sqrt(2*pi));
d = (x - mu).^2 / (2*sigma^2);
f = c * exp( -d );
```

2 Linear Algebra Review

A prerequisite of this course is MATH 221, Linear Algebra. We will review some things as we go through the course, but it will also be assumed that you have a good Linear Algebra background. Some things you should recall are discussed in this section.

2.1 Invertible Matrices

If a matrix A has m rows and n columns with real entries, then we usually write $A \in \Re^{m\times n}$. If $m = n$, so that the matrix has the same number of rows and columns, we say $A \in \Re^{n\times n}$

is a square matrix. If $A \in \Re^{n \times n}$ is invertible, then there is another $n \times n$ matrix, denoted as A^{-1}, such that

$$AA^{-1} = A^{-1}A = I,$$

where I is the $n \times n$ identity matrix.

Here are some fill-in-the-blank exercises to remind you of some of the important things you learned about invertible matrices:

1. $A \in \Re^{n \times n}$ is invertible if and only if the columns of A are _______________.
2. $A \in \Re^{n \times n}$ is invertible if and only if $Ax = 0$ has only the _______________ solution, $x =$_____.
3. $A \in \Re^{n \times n}$ is invertible if and only if the determinant, $\det(A)$ is _______________.
4. $A \in \Re^{n \times n}$ is invertible if and only if all eigenvalues of A are _______________.
5. If A and B are $n \times n$ invertible matrices, then so is $C = AB$, and $C^{-1} =$_______________. Hint to remember this rule: If you put on your socks then your shoes, what is the inverse operation?

2.2 Orthogonality

In a basic linear algebra class you learn what it means for a set of vectors to be orthogonal and orthonormal. For example, the Gram-Schmidt process is used to transform a linearly independent set of vectors into and orthonormal set. These fill-in-the-blank exercises will remind you of some of the important things you learned:

1. If the set of vectors $\{u_1, u_2, \ldots, u_n\}$ is orthogonal, then
$$u_i^T u_j = \underline{\qquad} \text{ if } i \neq j.$$
2. If the set of vectors $\{u_1, u_2, \ldots, u_n\}$ is orthonormal, then
$$u_i^T u_j = \begin{cases} \underline{\qquad} & \text{if } i \neq j \\ \underline{\qquad} & \text{if } i = j \end{cases}$$
3. If $Q \in \Re^{n \times n}$ is an orthogonal matrix, then $Q^T Q = QQ^T =$_____. That is, $Q^T =$_____.

2.3 Eigenvalues and Eigenvectors

An important chapter in Linear Algebra covers eigenvalues and eigenvectors of $n \times n$ matrices. These fill-in-the-blank exercises will remind you of some of the important things you learned:

1. Definition of eigenvalue and eigenvector: If $A \in \Re^{n \times n}$, then λ is an eigenvalue of A if there is a vector $x \neq 0$ such that $Ax =$__________.
2. If $A \in \Re^{n \times n}$ is symmetric (that is, $A^T = A$) then all eigenvalues of are _______________.

3. If $A \in \Re^{n \times n}$ is symmetric then A is orthogonally diagonalizable. This means there is an orthogonal matrix $Q \in \Re^{n \times n}$ such that $A = QDQ^T$,where
   (a) The columns of Q are the ______________ of A.
   (b) D is a diagonal matrix, where the diagonal entries of D are equal to the ______________ of A.

3 MATLAB Exercises

There is a lot to discuss about MATLAB – each week you will learn more and more. Hopefully the process of installing and accessing MATLAB from Emory's Software Distribution will not be too difficult to understand.

Once you have MATLAB installed, or you access it via an online version, you should read the first chapter in the course reference book, and then download the script m-file `BasicPlot.m` from Canvas. To "execute the code" in this script, you can either:

- In the command window, type the name of the function at the `>>` prompt:

  ```
  >> BasicPlot
  ```

  and press the enter (or return) key.

- Or at the top of the MATLAB gui, you can click on the "Run" arrow: 

Now here are some exercises for you to try.

1. Add some additional commands to `BasicPlot.m` that will generate a plot of $y = \cos(4x)$ on the interval $0 \le x \le 2\pi$.

2. Write a new script m-file that will plot the polynomial:

$$p(x) = x^4 - 8x^3 + 17.75x^2 - 14.5x + 3.75$$

   on the interval $-1 \le x \le 6$.

3. Finding the roots of $p(x)$ looks complicated. But actually we can relate this to an eigenvalue problem. Here is how: Let A be the "companion" matrix associated with $p(x)$:

$$A = \begin{bmatrix} 8 & -17.75 & 14.5 & -3.75 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

   It can be shown (we skip the derivation) that the characteristic polynomial of A is $p(x)$, and so the eigenvalues of A are the roots of $p(x)$. Here is MATLAB code to setup the matrix and to compute the eigenvalues:

```
A = [8 -17.75 14.5 -3.75;
1 0 0 0;
0 1 0 0;
0 0 1 0];

r = eig(A);
```

Add the above commands to your script m-file, and also include additional plotting commands to include circles on the plot in locations indicating the roots of the polynomial. That is, your plot should look like: