Homework 4:

1) a.

$$A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & -2 & 3 \\ 3 & -7 & 4 \end{pmatrix} \qquad b = \begin{pmatrix} 8 \\ 1 \\ 10 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & -2 & 3 \\ 3 & -7 & 4 \end{pmatrix} \xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 + 1R_1} \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 5 \\ 0 & -10 & -2 \end{pmatrix} \xrightarrow[R_3 = R_3 - 10R_2]{} \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 5 \\ 0 & 0 & -52 \end{pmatrix}$$

The highlited numbers are the negative of the multipliers. We thus get:

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 10 & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 5 \\ 0 & 0 & -52 \end{pmatrix}$$

b.

multipliers

$$A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & -2 & 3 \\ 3 & -7 & 4 \end{pmatrix}, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$A = \begin{pmatrix} 3 & -7 & 4 \\ -1 & -2 & 3 \\ 1 & 1 & 2 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$A = \begin{pmatrix} 3 & -7 & 4 \\ 0 & -13/3 & 13/3 \\ 0 & -4/3 & 2/3 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad m_{21} = -\frac{1}{3} \quad m_{31} = \frac{1}{3}$$

$$A = \begin{pmatrix} 3 & -7 & 4 \\ 0 & -13/3 & 13/3 \\ 0 & 0 & -2/3 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad m_{23} = 4/13$$

Thus, we get

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad L = \begin{pmatrix} 1 & 0 & 0 \\ -1/3 & 1 & 0 \\ 1/3 & 4/13 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 3 & -7 & 4 \\ 0 & -13/3 & 13/3 \\ 0 & 0 & -2/3 \end{pmatrix}$$

c. Since $A = LU$, then $A^{-1} = (LU)^{-1} = U^{-1}L^{-1}$. Let's compute:

For L:

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 3 & 10 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 + R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 10 & 1 & -3 & 0 & 1 \end{array}\right) \xrightarrow[R_3 = R_3 - 10R_2]{}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -13 & -10 & 1 \end{array}\right), \text{ Thus } L^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -13 & -10 & 1 \end{pmatrix}$$

For U:

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 5 & 0 & 1 & 0 \\ 0 & 0 & -52 & 0 & 0 & 1 \end{array}\right) \xrightarrow{\frac{1}{-52}R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 5 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/52 \end{array}\right) \xrightarrow[R_2 = R_2 - 5R_3]{R_1 = R_1 - 2R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 1/26 \\ 0 & -1 & 0 & 0 & 1 & 5/52 \\ 0 & 0 & 1 & 0 & 0 & -1/52 \end{array}\right) \xrightarrow{\frac{1}{-1}R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 1/26 \\ 0 & 1 & 0 & 0 & 1 & -5/52 \\ 0 & 0 & 1 & 0 & 0 & -1/52 \end{array}\right) \xrightarrow{R_1 = R_1 - 1R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 7/52 \\ 0 & 1 & 0 & 0 & 1 & -5/52 \\ 0 & 0 & 1 & 0 & 0 & -1/52 \end{array}\right), \text{ Thus } U^{-1} = \begin{pmatrix} 1 & -1 & 7/52 \\ 0 & 1 & 5/52 \\ 0 & 0 & -1/52 \end{pmatrix}$$

Finally, we multiply

$$\begin{pmatrix} 1 & -1 & 7/52 \\ 0 & 1 & -5/52 \\ 0 & 0 & -1/52 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -13 & -10 & 1 \end{pmatrix} = \begin{pmatrix} -7/4 & -61/26 & 7/52 \\ 9/4 & 51/26 & -5/52 \\ 1/4 & 5/26 & -1/52 \end{pmatrix}$$

Thus:

$$A^{-1} = \frac{1}{52} \begin{pmatrix} -91 & -122 & 7 \\ 117 & 102 & -5 \\ 13 & 10 & -1 \end{pmatrix}$$