Solución de la pregunta #6

a.) $\begin{vmatrix} 1 & -2 & -3 \\ 3 & -1 & 5 \\ 4 & 1 & t^2-14 \end{vmatrix} = 5t^2 - 136 \implies$ hay solución única si $t \neq \pm\sqrt{\frac{136}{5}}$

Solución del sistema:

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & t^2-14 & t+2 \end{array}\right) \xrightarrow[4F_2 \; - \; F_3]{3F_1 \; - \; F_2} \left(\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & -5 & -14 & 10 \\ 0 & -9 & 2-t^2 & 14-t \end{array}\right)$$

$$\xrightarrow{9F_2 \; - \; 5F_3} \left(\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & -5 & -14 & 10 \\ 0 & 0 & 5t^2-136 & 5(t+4) \end{array}\right)$$

Ahora, si $t \neq \pm\sqrt{\frac{136}{5}}$,

$$\begin{cases} x \quad -2y \quad -3z & = & 4 & \implies & x = -\dfrac{13(t+4)}{5t^2-136} \\ \\ \quad -5y \quad -14z & = & 10 & \implies & y = -\dfrac{2\left(5t^2+7t-108\right)}{5t^2-136} \\ \\ \quad\quad (5t^2-136)z & = & 5(t+4) & \implies & z = \dfrac{5(t+4)}{5t^2-136} \end{cases}$$

b.) Si $t = +\sqrt{\frac{136}{5}}$

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & \frac{66}{5} & 2\sqrt{\frac{34}{5}}+2 \end{array}\right) \xrightarrow[4F_2 \; - \; F_3]{3F_1 \; - \; F_2} \left(\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & -5 & -14 & 10 \\ 0 & -9 & -\frac{126}{5} & 14-2\sqrt{\frac{34}{5}} \end{array}\right)$$

$$\xrightarrow{9F_2 \; - \; 5F_3} \left(\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & -5 & -14 & 10 \\ 0 & 0 & 0 & 2\left(\sqrt{170}+10\right) \end{array}\right)$$

$\therefore$ Por tanto el sistema es inconsistente. No hay solución

c.)

d.) Si $t = -\sqrt{\frac{136}{5}}$

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & \frac{66}{5} & 2-2\sqrt{\frac{34}{5}} \end{array}\right) \xrightarrow[4F_2 \;-\; F_3]{3F_1 \;-\; F_2} \left(\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & -5 & -14 & 10 \\ 0 & -9 & -\frac{126}{5} & 2\sqrt{\frac{34}{5}}+14 \end{array}\right)$$

$$\xrightarrow{9F_2 \;-\; 5F_3} \left(\begin{array}{ccc|c} 1 & -2 & -3 & 4 \\ 0 & -5 & -14 & 10 \\ 0 & 0 & 0 & -2\left(\sqrt{170}-10\right) \end{array}\right)$$

$\therefore$ Por tanto el sistema es inconsistente. No hay solución