feasible set is assumed convex; in [42] only the feasible set is approximated; in [43] it is assumed that the objective function approximation is concave; in [45] the surrogate objectives need not lower-bound the original objective, as in the general sequential method, but several additional assumptions are made: the original objective is required to be Lipschitz continuous and coercive; the surrogate objectives are required to be uniformly strongly concave.

- It is shown that several applications related to resource exploitation problems can be tackled by the newly proposed optimization framework. Specifically, as for communication systems, the problems of energy-efficient beamforming in MIMO interference networks, and of power control for weighted sum-energy-efficiency optimization are described. Moreover, for radar systems, an advanced interference covariance estimator is developed.

The remainder of this paper is organized as follows. Section II provides the necessary background on the MBI and sequential optimization methods, that are the fundamental building blocks to develop the novel optimization framework. Section III describes the general class of problems considered herein, and presents the solution algorithm and its convergence analysis. Several novel applications of the proposed framework are considered in Section IV. Finally, concluding remarks are provided in Section V.

II. Mathematical Preliminaries

This section provides the necessary background on the main optimization methods which will be used to develop the proposed optimization framework.

A. Alternating Optimization and MBI

Let us consider the non-concave optimization problem

$$\max_{\boldsymbol{x}} f(\boldsymbol{x}) \quad \text{s.t.} \;\; \boldsymbol{x} \in \mathcal{X} \tag{1}$$

where $f(\cdot)$ is a continuously differentiable function and $\mathcal{X}$ is a compact set, defined in terms of differentiable constraint functions. The idea of alternating optimization is to partition the variable $\boldsymbol{x}$ into the blocks $\boldsymbol{x} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_K)$ and then to alternately solve (1) with respect to one block while keeping the others fixed. The formal description of the alternating optimization procedure is reported in **Algorithm 1**.

Algorithm 1: Alternating optimization.

Initialization: $n = 0$ and $\boldsymbol{x}^{(n)} \in \mathcal{X}$;
repeat
  for $k = 1, \ldots, K$ **do**

$$\boldsymbol{x}_k^* = \arg\max_{\boldsymbol{y}} f\left(\boldsymbol{x}_1^{(n)}, \ldots, \boldsymbol{x}_{k-1}^{(n)}, \boldsymbol{y}, \ldots, \boldsymbol{x}_K^{(n)}\right) \quad \text{s.t.} \;\; \left(\boldsymbol{x}_1^{(n)}, \ldots, \boldsymbol{x}_{k-1}^{(n)}, \boldsymbol{y}, \ldots, \boldsymbol{x}_K^{(n)}\right) \in \mathcal{X} \tag{2}$$

  $\boldsymbol{x}^{(n+1)} = [\boldsymbol{x}_{1^{(n)}}, \ldots, \boldsymbol{x}_{k-1^{(n)}}, \boldsymbol{x}_k^\star, \ldots, \boldsymbol{x}_K^{(n)}]^T$;
  $n = n + 1$;
  end for
until Convergence

The alternating optimization algorithm enjoys the following result [9].

Proposition 1: The sequence $f(\boldsymbol{x}^{(n)})$ is monotonically increasing and converges. Now, assume that the feasible set $\mathcal{X}$ can be expressed as the Cartesian product of closed convex sets, i.e., $\mathcal{X} = \mathcal{X}_1 \times, \ldots, \times \mathcal{X}_K$, with $\boldsymbol{x}_k \in \mathcal{X}_k$ for all $k = 1, \ldots, K$. If the solution to the generic subproblem (2) is unique, then **Algorithm 1** converges to a stationary point for Problem (1).

The latter assumption in Proposition 1 can be relaxed by considering an improvement of the alternating optimization, which updates at each iteration only the block yielding the maximum increase of the objective function. This approach is called MBI [41], and can be formally stated in **Algorithm 2**.

Algorithm 2: MBI.

Initialization: $n = 0$ and $\boldsymbol{x}^{(n)} \in \mathcal{X}$;
repeat
  for $k = 1, \ldots, K$ **do**

$$\tilde{\boldsymbol{x}}_k^* = \arg\max_{\boldsymbol{y}} f\left(\boldsymbol{x}_1^{(n)}, \ldots, \boldsymbol{x}_{k-1}^{(n)}, \boldsymbol{y}, \ldots, \boldsymbol{x}_K^{(n)}\right) \quad \text{s.t.} \;\; \left(\boldsymbol{x}_1^{(n)}, \ldots, \boldsymbol{x}_{k-1}^{(n)}, \boldsymbol{y}, \ldots, \boldsymbol{x}_K^{(n)}\right) \in \mathcal{X} \tag{3}$$

  end for
  $\tilde{k} = \arg\max_{1 \leq k \leq K} f(\boldsymbol{x}_1^{(n)}, \ldots, \boldsymbol{x}_{k-1}^{(n)}, \tilde{\boldsymbol{x}}_k^\star, \ldots, \boldsymbol{x}_K^{(n)})$;
  $\boldsymbol{x}^{(n+1)} = [\boldsymbol{x}_1^{(n)}, \ldots, \boldsymbol{x}_{\tilde{k}-1}^{(n)}, \tilde{\boldsymbol{x}}_{\tilde{k}}^\star, \ldots, \boldsymbol{x}_K^{(n)}]^T$;
  $n = n + 1$;
until Convergence

A similar result as in Proposition 1 holds for **Algorithm 2** [28], but, as already mentioned, the assumption on the uniqueness of the solution of Problem (3) can be removed. Instead, the assumption on the Cartesian product structure of the feasible set is still required, which implies that the constraint functions are decoupled, or equivalently, that each constraint function depends only on one variable block.

B. Sequential Optimization

Consider again Problem (1), but now let us explicitly denote the constraints defining the feasible set $\mathcal{X}$ as the set of K inequalities $g_i(\boldsymbol{x}) \geq 0$, with $i = 1, \ldots, K_1$, where the function $g_i(\boldsymbol{x})$ is, in general, continuously differentiable for all $i = 1, \ldots, K_1$. Then, the basic idea of sequential optimization is to tackle the non-concave Problem (1) by solving a sequence of approximate maximization problems. More formally, consider the sequence of approximate problems $\mathcal{P}_n$ with objectives $\tilde{f}_n(\boldsymbol{x})$ and constraint functions $\tilde{g}_{i,n}(\boldsymbol{x})$, such that the following three properties are fulfilled:

(P1) $\tilde{f}_n(\boldsymbol{x}) \leq f(\boldsymbol{x})$, $\tilde{g}_{i,n}(\boldsymbol{x}) \leq g_i(\boldsymbol{x})$ for all $\boldsymbol{x}$ and $i = 1, \ldots, K_1$;

(P2) $\tilde{f}_n(\boldsymbol{x}^{(n-1)}) = f(\boldsymbol{x}^{(n-1)})$, $\tilde{g}_{i,n}(\boldsymbol{x}^{(n-1)}) = g_i(\boldsymbol{x}^{(n-1)})$ with $i = 1, \ldots, K_1$, with $\boldsymbol{x}^{(n-1)}$ the optimal solution of $\mathcal{P}_{n-1}$;

(P3) $\nabla \tilde{f}_n(\boldsymbol{x}^{(n-1)}) = \nabla f(\boldsymbol{x}^{(n-1)})$, $\nabla \tilde{g}_{i,n}(\boldsymbol{x}^{(n-1)}) = \nabla g_i(\boldsymbol{x}^{(n-1)})$ with $i = 1, \ldots, K_1$.

The following results are known about the sequential optimization method:

(R1) The sequence $f(\boldsymbol{x}^{(n)})$ is monotonically increasing and converges. Moreover, if $\boldsymbol{x}^{(n)} \to \boldsymbol{x}^\star$ the limit point $\boldsymbol{x}^\star$