CS 106B

Lecture 23: Depth First and Breadth First Searching

Wednesday, May 23, 2018

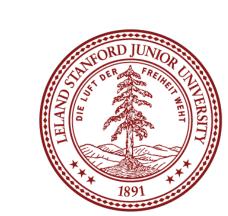
Programming Abstractions
Spring 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

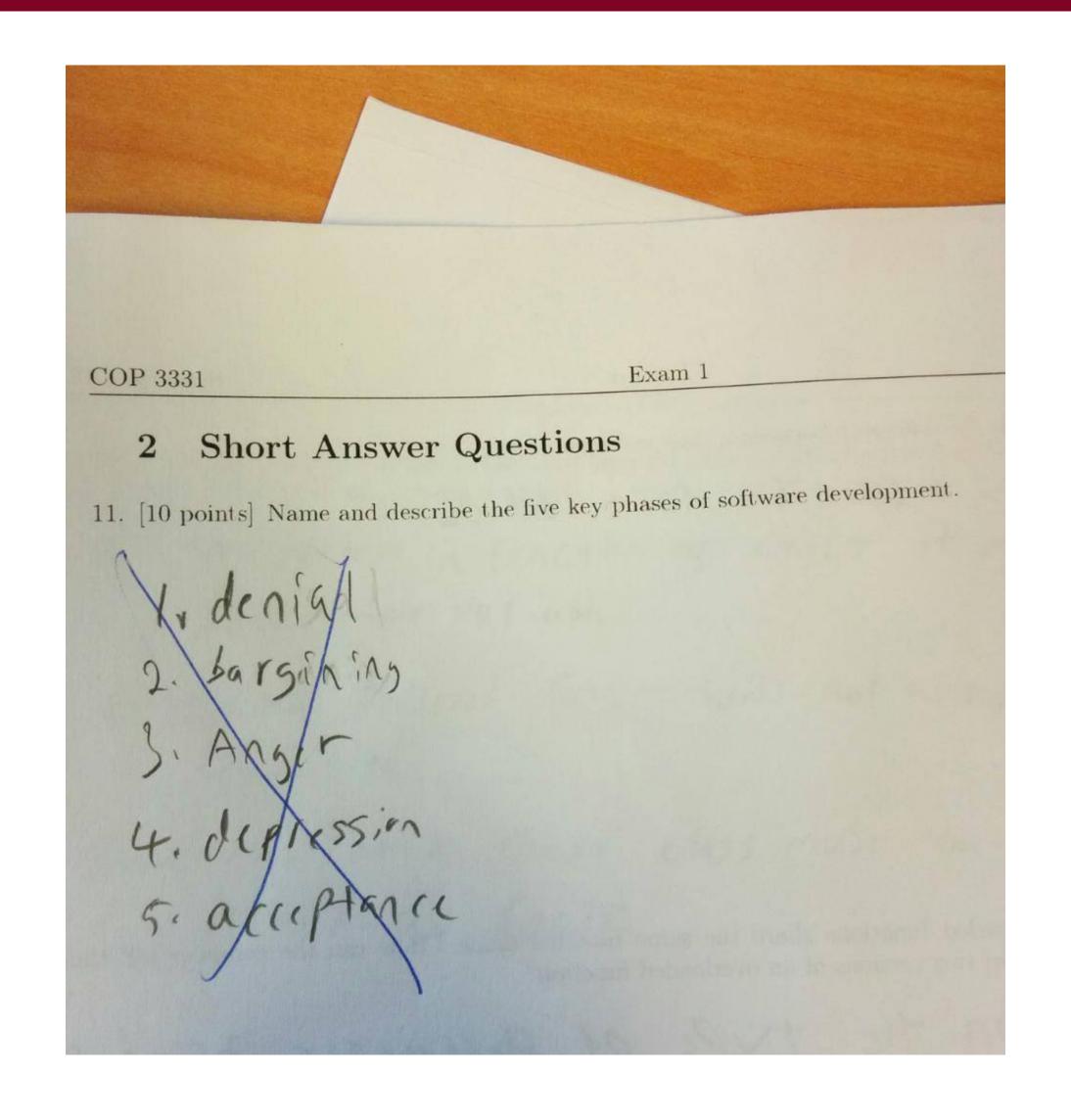
reading:

Programming Abstractions in C++, Chapter 18.6





At this point in the quarter...



https://i.redd.it/e5uylwsqzizx.jpg

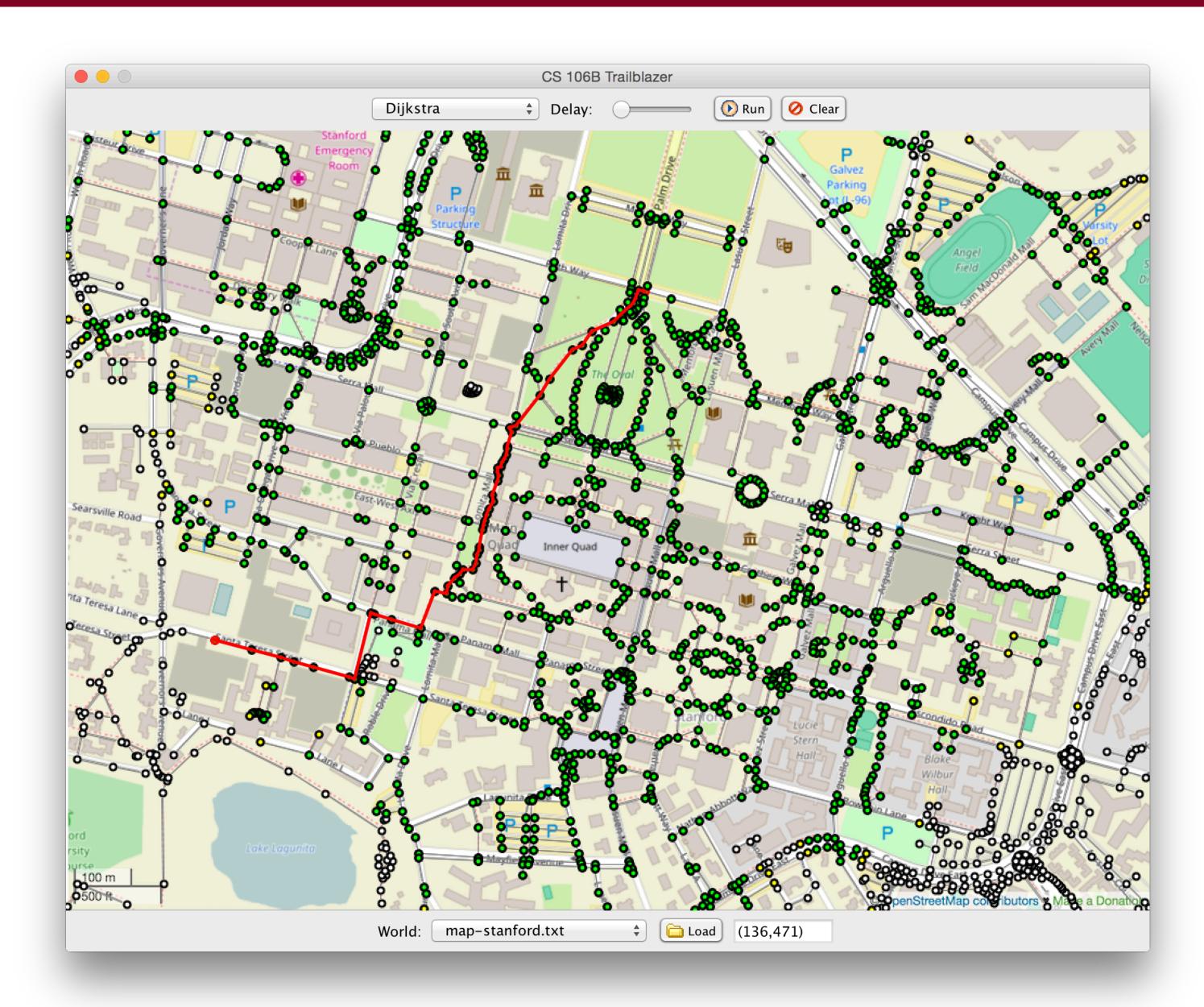


Today's Topics

- Logistics
- •Trailblazer: Final assignment! Out tomorrow.
- More on Graphs (and a bit on Trees)
- Depth First Search
- Breadth First Search



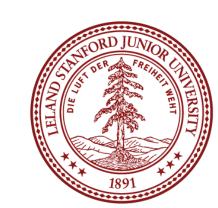
Trailblazer



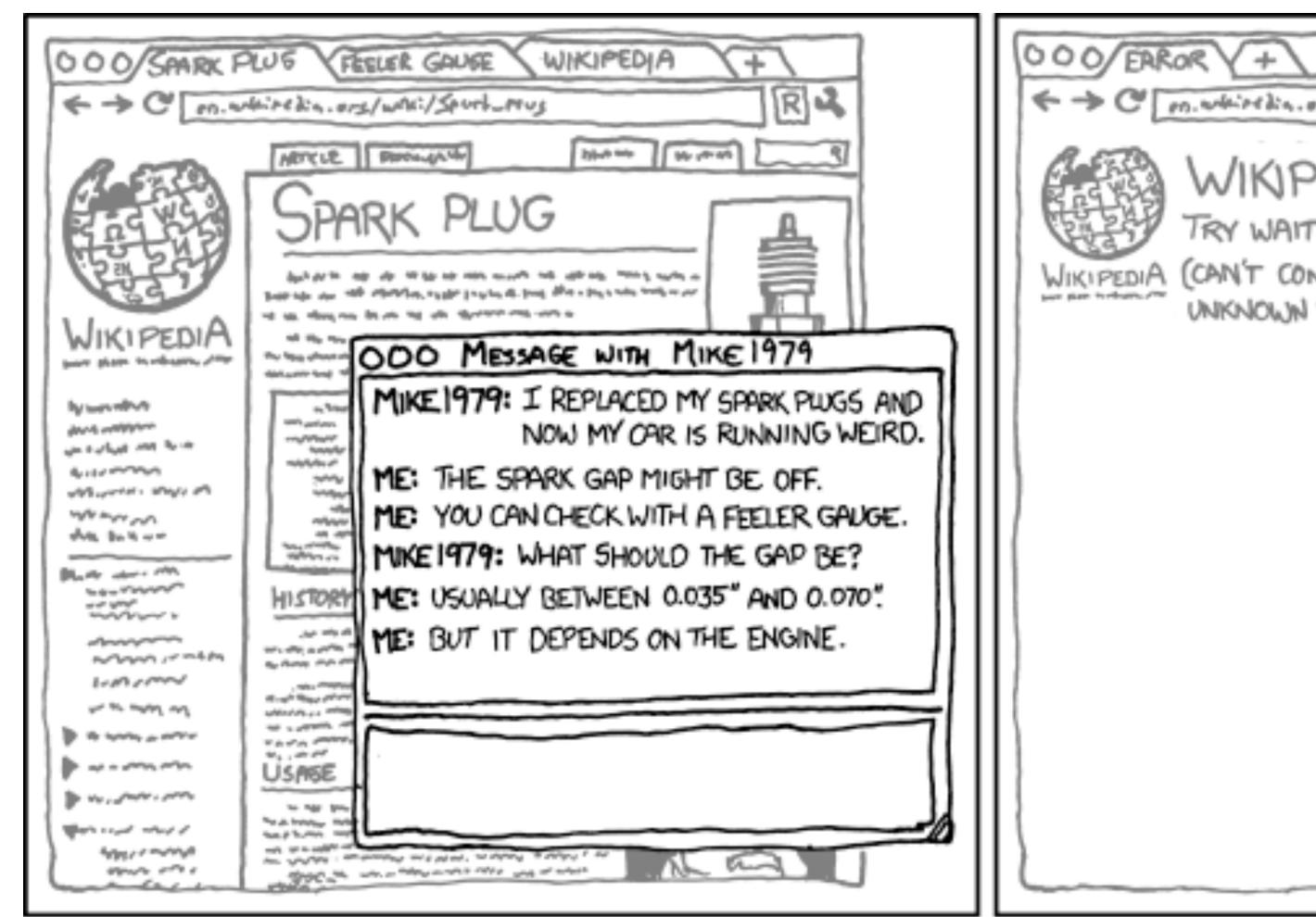
You create Google Maps!

You need to implement four different (but related) types of searches:

- Breadth First Search (today)
- Dijkstra (Friday)
- A* (Friday)
- Alternate (you must determine algorithm)



Wikipedia



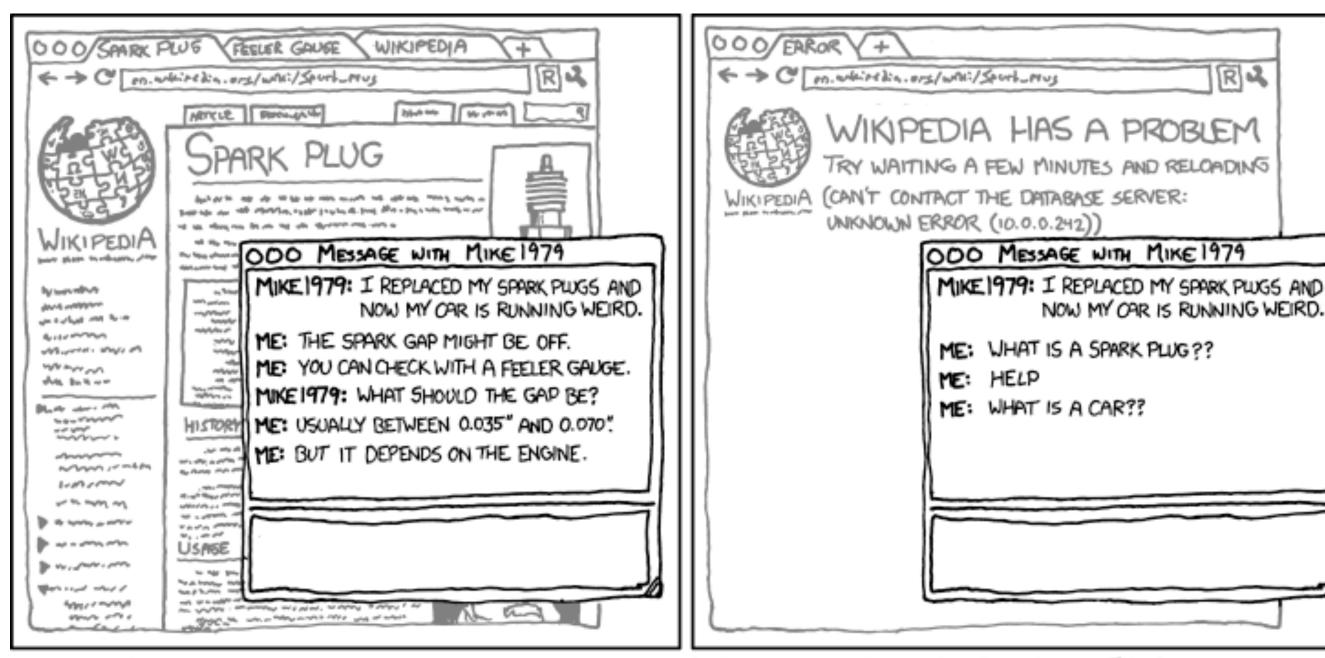


WHEN WIKIPEDIA HAS A SERVER OUTAGE, MY APPARENT IQ DROPS BY ABOUT 30 POINTS.

XKCD 903, Extended Mind, http://xkcd.com/903/



Wikipedia



WHEN WIKIPEDIA HAS A SERVER OUTAGE, MY APPARENT IQ DROPS BY ABOUT 30 POINTS.

When you hover over an XKCD comic, you get an extra joke:

Wikipedia trivia: if you take any article, click on the first link in the article text not in parentheses or italics, and then repeat, you will eventually end up at "Philosophy".



Wikipedia

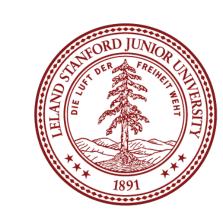
Wikipedia trivia: if you take any article, click on the first link in the article text not in parentheses or italics, and then repeat, you will eventually end up at "Philosophy".

Is this true??

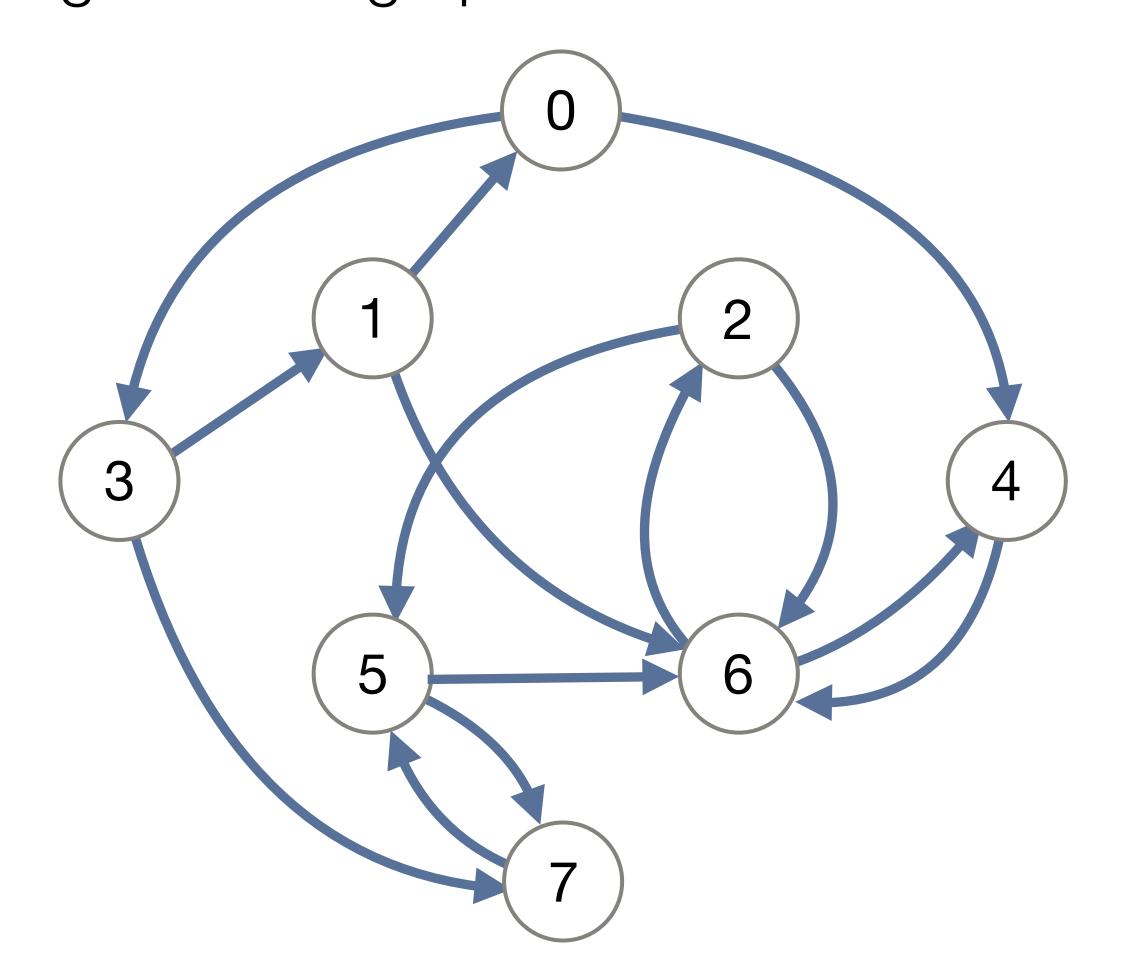
According to the Wikipedia article "Wikipedia:Getting to Philosophy" (so meta), (https://en.wikipedia.org/wiki/Wikipedia:Getting_to_Philosophy):

As of February 2016, 97% of all articles in Wikipedia eventually lead to the article Philosophy.

How can we find out? We shall see!



Recall that a *graph* is the "wild west of trees" — graphs relate *vertices* (nodes) to each other by way of *edges*, and they can be directed or undirected. Take the following directed graph:

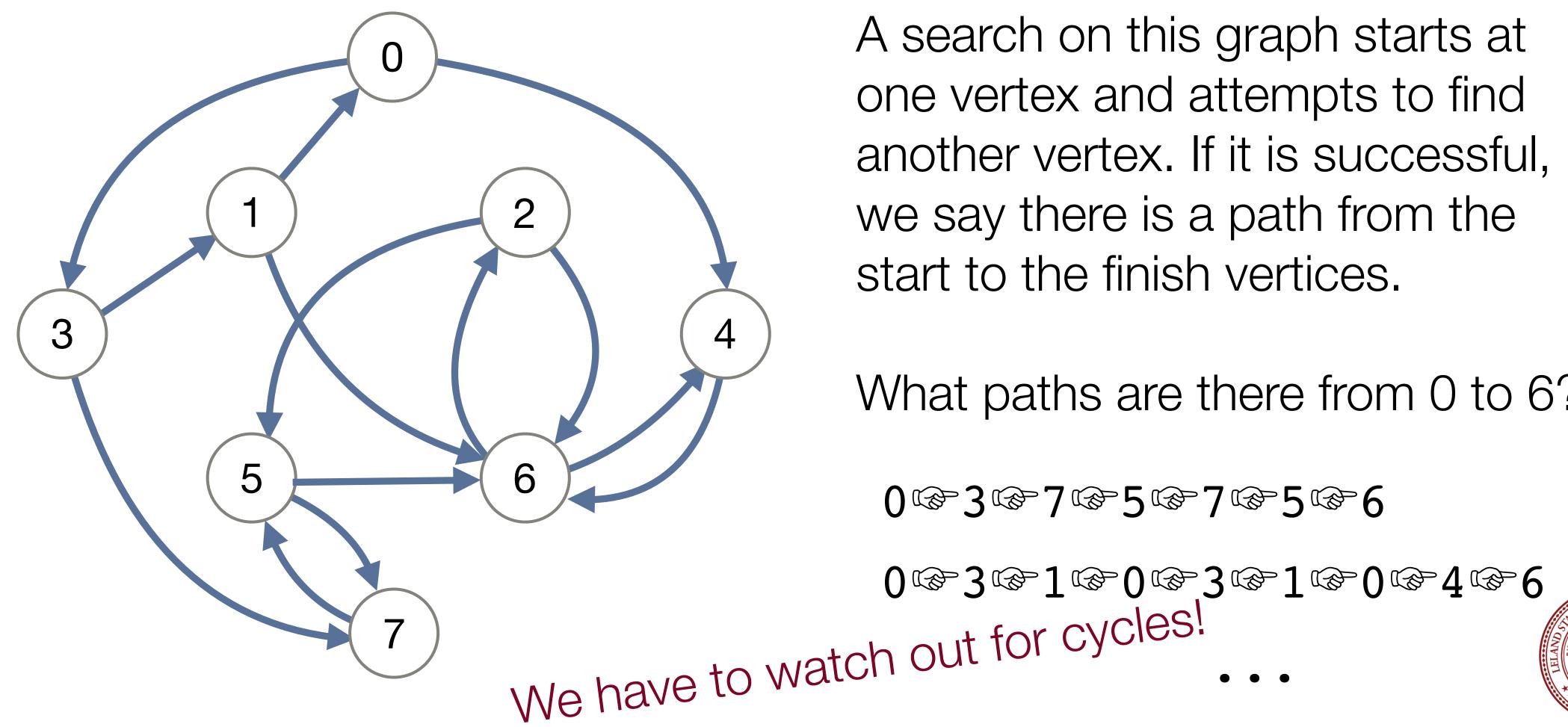


A search on this graph starts at one vertex and attempts to find another vertex. If it is successful, we say there is a path from the start to the finish vertices.

What paths are there from 0 to 6?



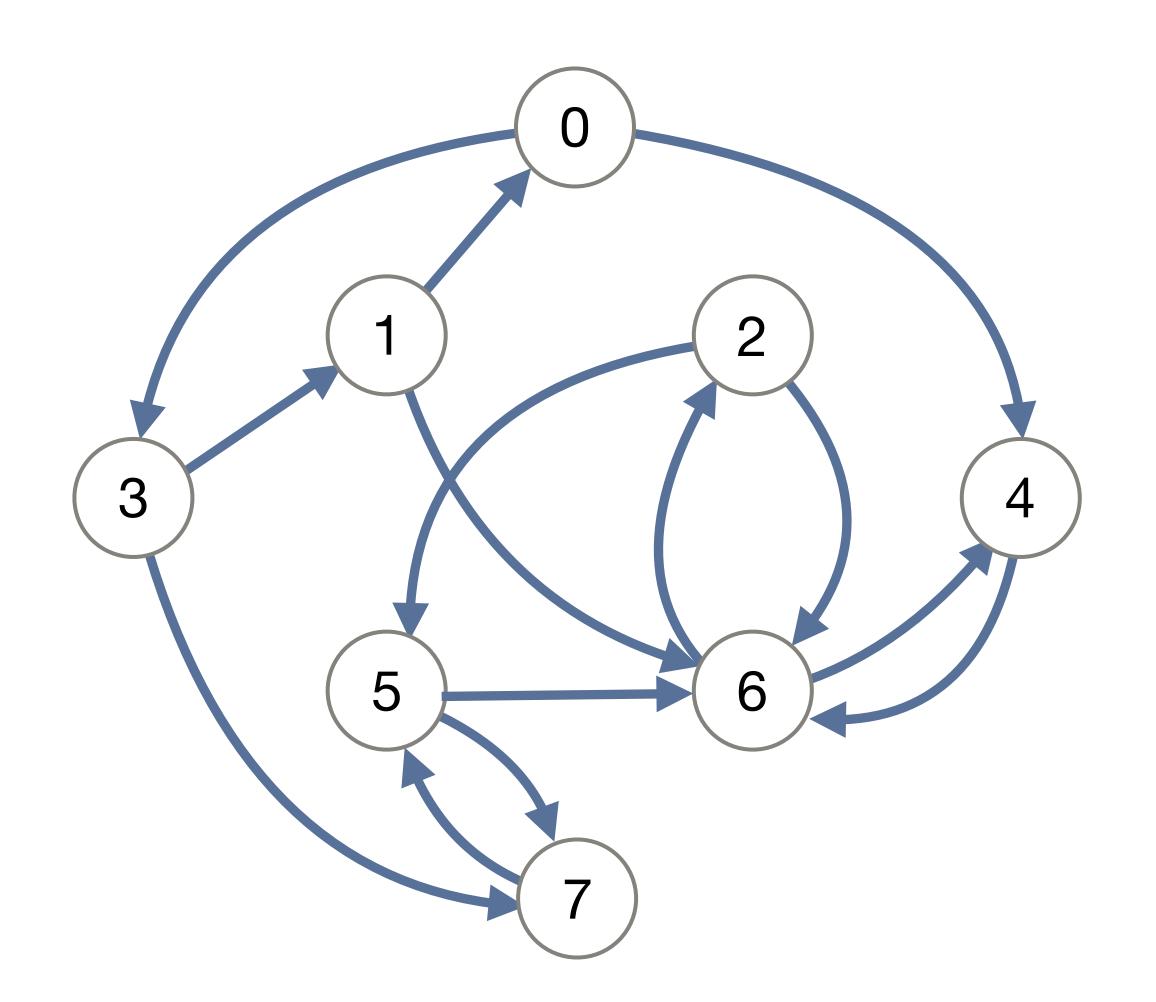
Recall that a *graph* is the "wild west of trees" — graphs relate vertices (nodes) to each other by way of edges, and they can be directed or undirected. Take the following directed graph:



A search on this graph starts at one vertex and attempts to find another vertex. If it is successful, we say there is a path from the start to the finish vertices.

What paths are there from 0 to 6?

What paths are there from 3 to 2?





What paths are there from 4 to 1?

```
None! :(
```

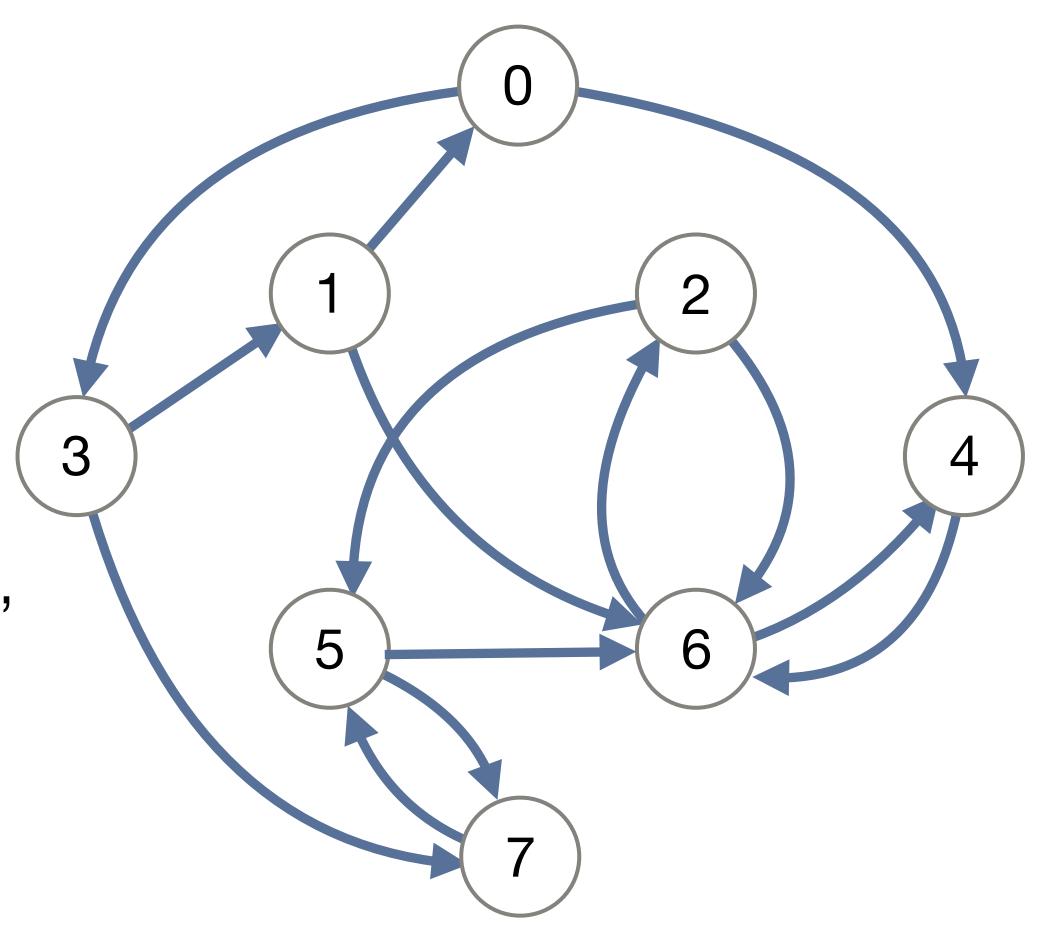


We have different ways to search graphs:

 Depth First Search: From the start vertex, explore as far as possible along each branch before backtracking.

 Breadth First Search: From the start vertex, explore the neighbor nodes first, before moving to the next level neighbors.

Both methods have pros and cons — let's explore the algorithms.





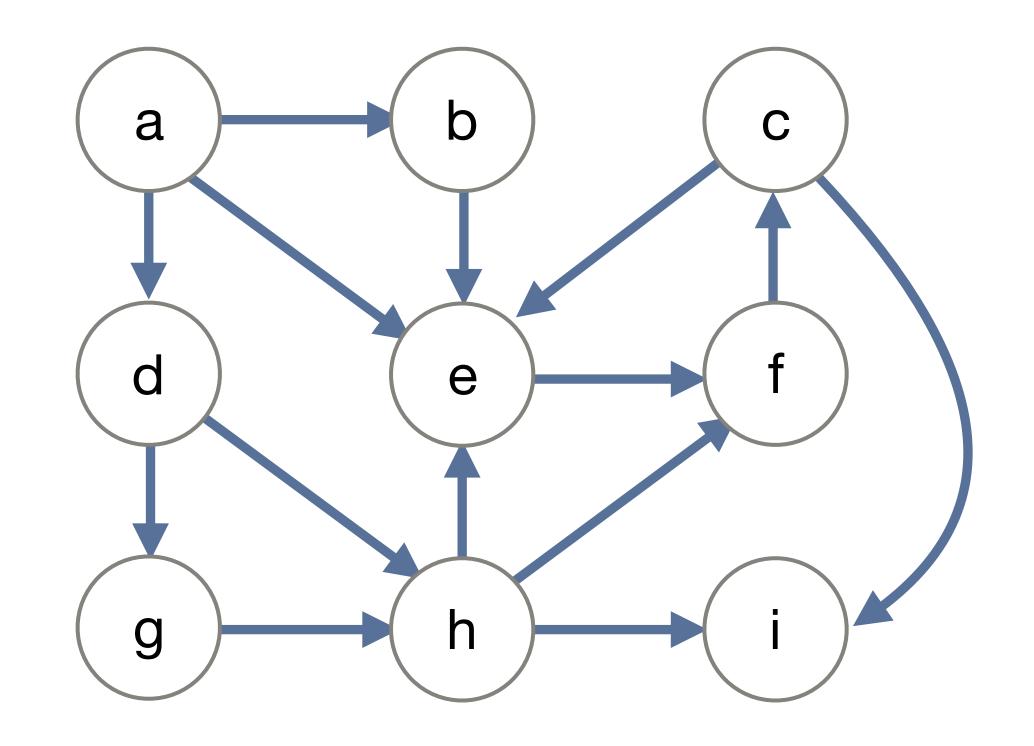
Depth First Search (DFS)

From the start vertex, explore as far as possible along each branch before backtracking.

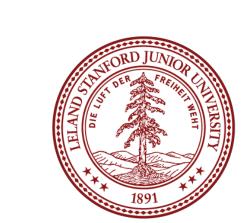
This is often implemented recursively. For a graph, you *must mark visited vertices*, or you might traverse forever (e.g., cefefece...)

DFS from a to h (assuming a-z order) visits:

```
a®
b®
e®
f®
d®
i (dead end — back to c,f,e,b,a)
N
g®
h path found: a®d®g®h ✓
```



Notice: not the shortest!



dfs from v_1 to v_2 :

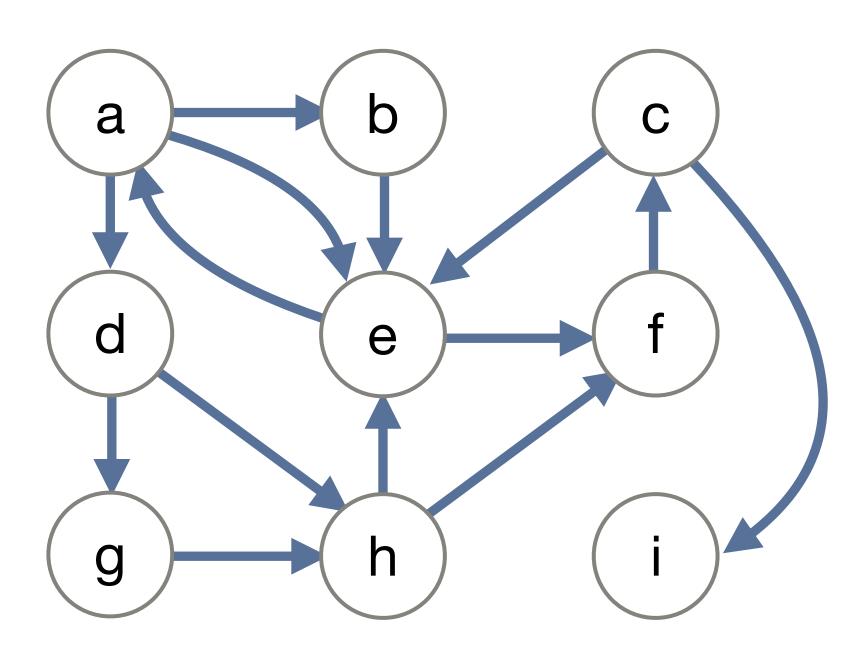
base case: if at v₂, found!

mark v₁ as visited.

for all edges from v₁ to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v₂).







dfs from v_1 to v_2 :

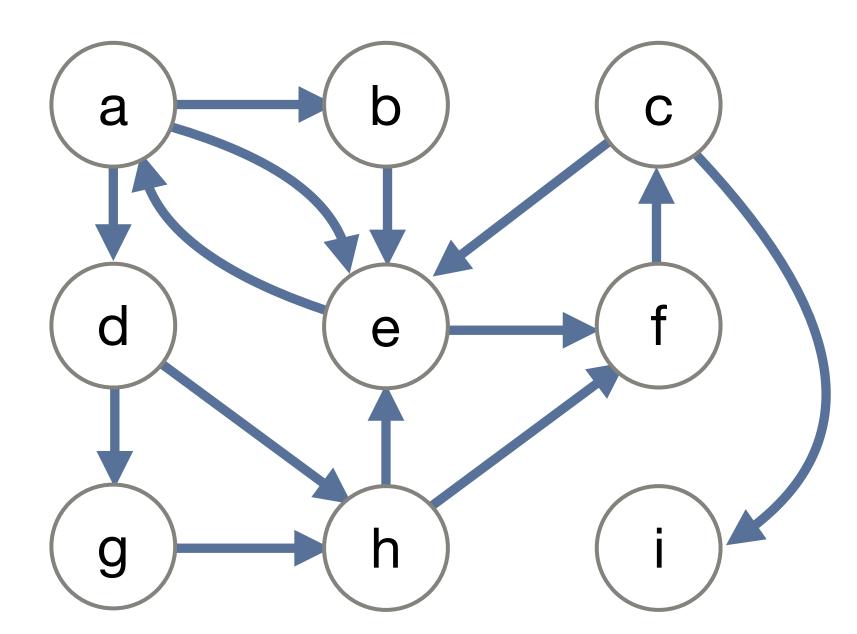
mark v₁ as visited.

for all edges from v₁ to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v₂).

Let's look at **dfs** from h to c:

Vertex	Visited?
a	false
b	false
C	false
d	false
е	false
f	false
g	false
h	false
İ	false





dfs from v_1 to v_2 :

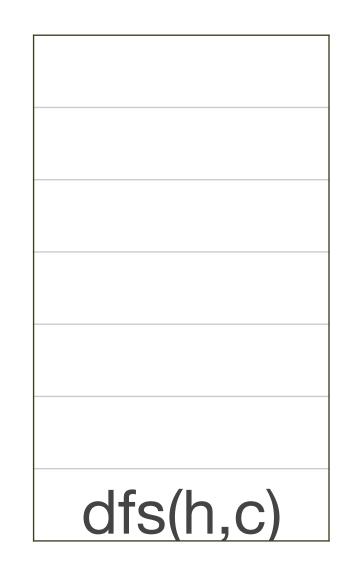
mark v₁ as visited.

for all edges from v₁ to its neighbors:

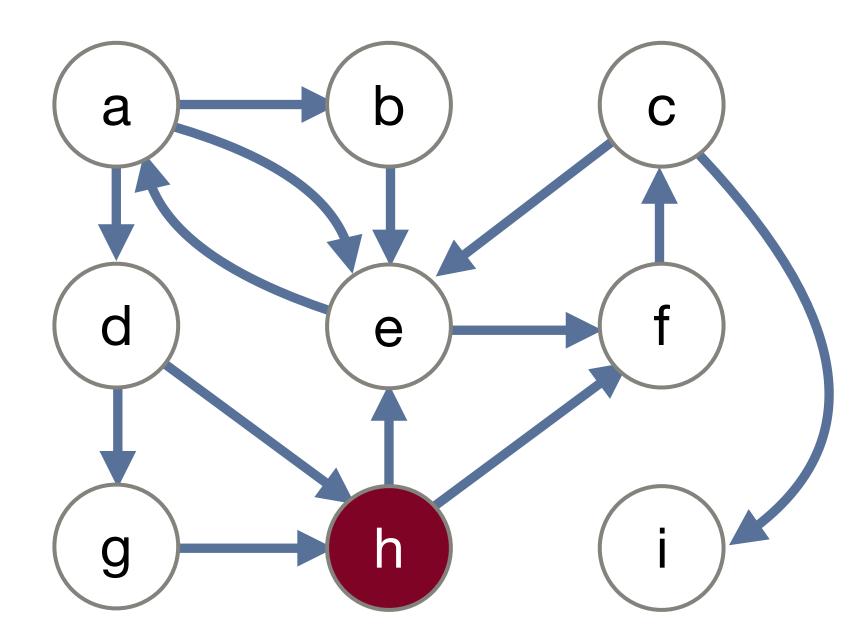
if neighbor n is unvisited, recursively call **dfs**(n, v₂).

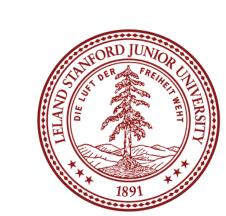
Let's look at dfs from h to c:

Vertex Map



Vertex	Visited?
a	false
b	false
C	false
d	false
е	false
f	false
g	false
h	true
i	false





dfs from v_1 to v_2 :

mark v₁ as visited.

for all edges from v₁ to its neighbors:

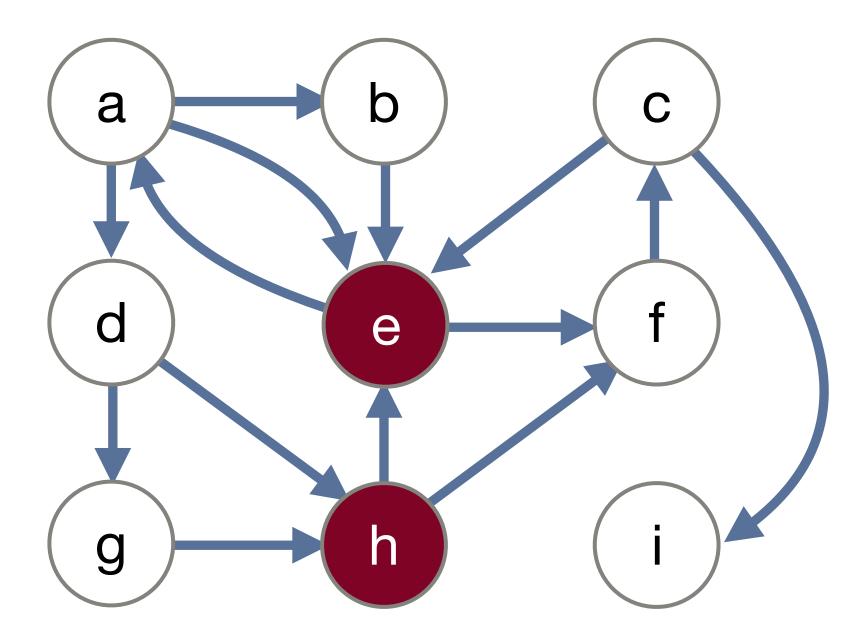
if neighbor n is unvisited, recursively call **dfs**(n, v₂).

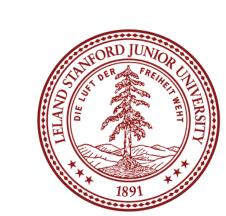
Let's look at **dfs** from h to c:

Vertex Map

olfo(o o)	
dfs(e,c)	
dfs(h,c)	

Vertex	Visited?
a	false
b	false
C	false
d	false
е	true
f	false
g	false
h	true
	false





dfs from v_1 to v_2 :

mark v₁ as visited.

for all edges from v₁ to its neighbors:

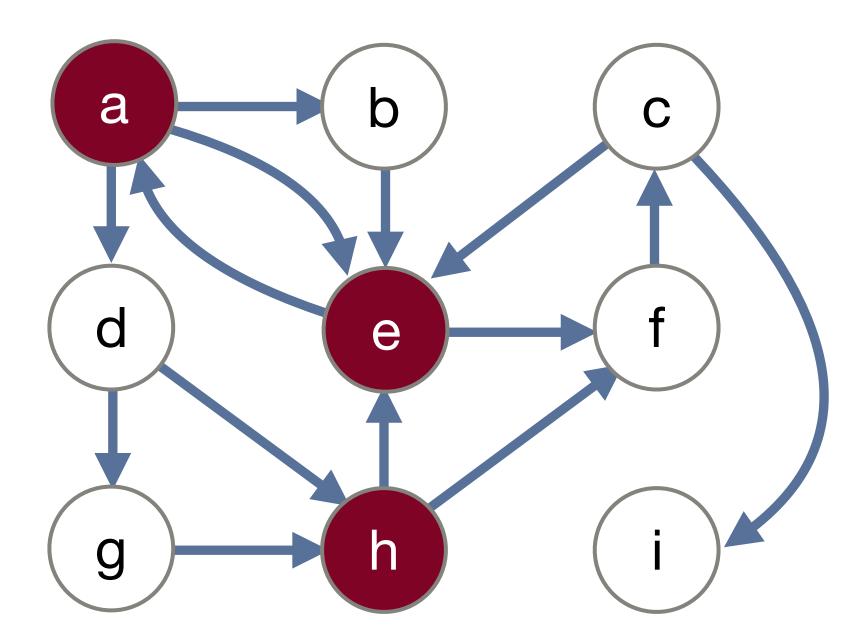
if neighbor n is unvisited, recursively call **dfs**(n, v₂).

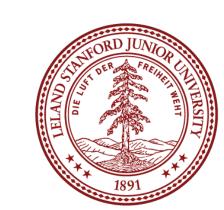
Let's look at dfs from h to c:

Vertex Map

dfs(a,c)
dfs(e,c)
dfs(h,c)

Vertex	Visited?
a	true
b	false
С	false
d	false
е	true
f	false
g	false
h	true
	false





dfs from v_1 to v_2 :

mark v₁ as visited.

for all edges from v₁ to its neighbors:

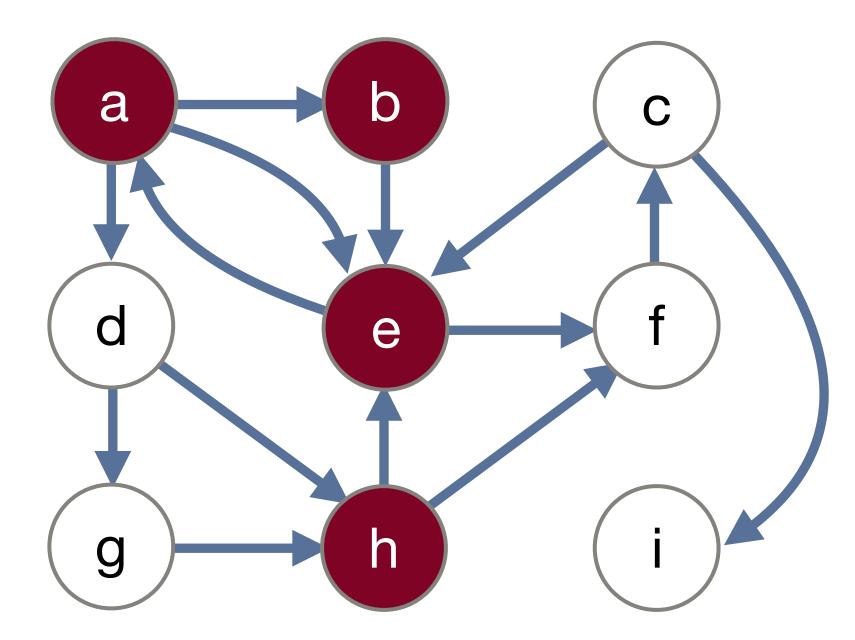
if neighbor n is unvisited, recursively call **dfs**(n, v₂).

Let's look at **dfs** from h to c:

Vertex Map

dfs(b,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)

Vertex	Visited?
a	true
b	true
C	false
d	false
е	true
f	false
g	false
h	true
i	false





dfs from v_1 to v_2 :

mark v₁ as visited.

for all edges from v₁ to its neighbors:

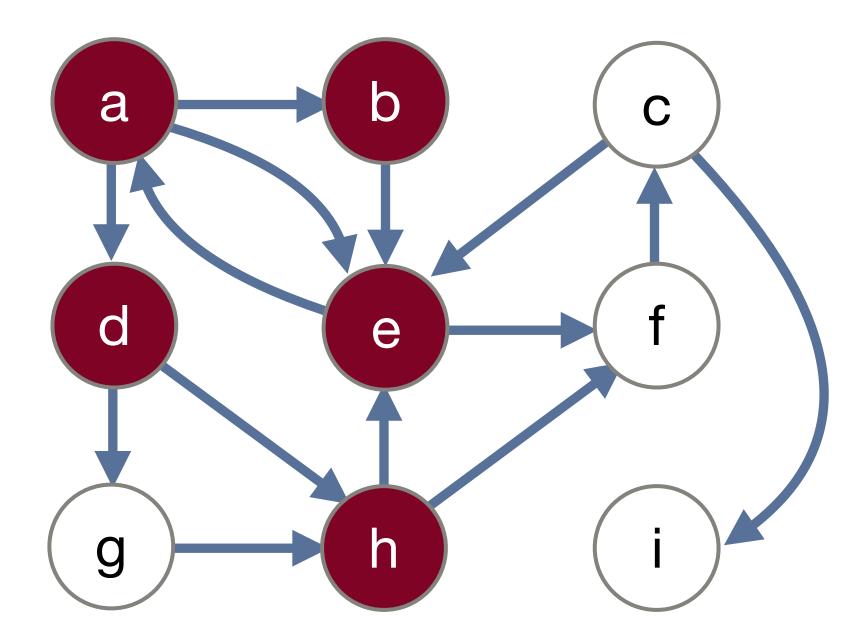
if neighbor n is unvisited, recursively call **dfs**(n, v₂).

Let's look at **dfs** from h to c:

Vertex Map

dfs(d,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)

Vertex	Visited?
a	true
b	true
C	false
d	true
е	true
f	false
g	false
h	true
i	false





dfs from v_1 to v_2 :

mark v₁ as visited.

for all edges from v₁ to its neighbors:

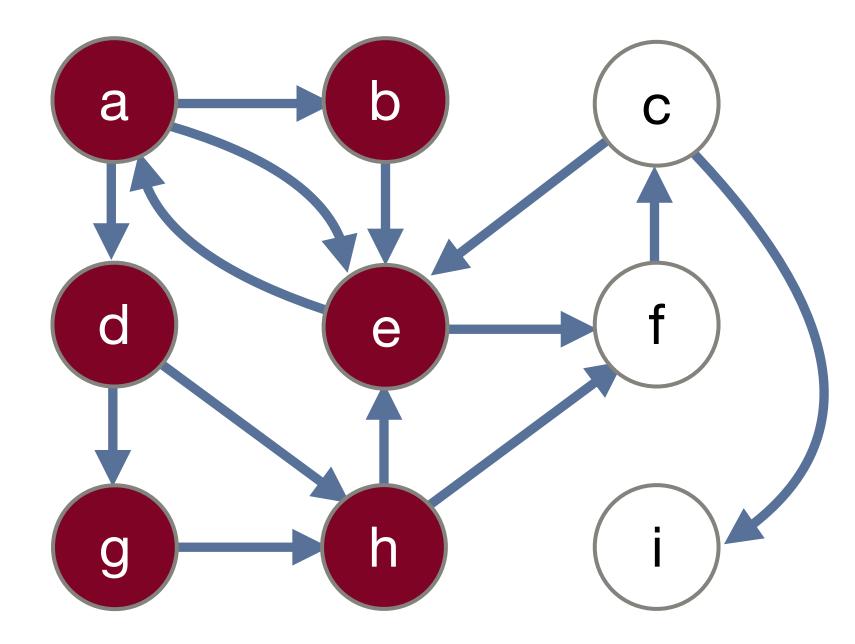
if neighbor n is unvisited, recursively call **dfs**(n, v₂).

Let's look at **dfs** from h to c:

Vertex Map

dfs(g,c)
dfs(d,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)

Vertex	Visited?
a	true
b	true
C	false
d	true
е	true
f	false
g	true
h	true
j	false





dfs from v_1 to v_2 :

mark v₁ as visited.

for all edges from v₁ to its neighbors:

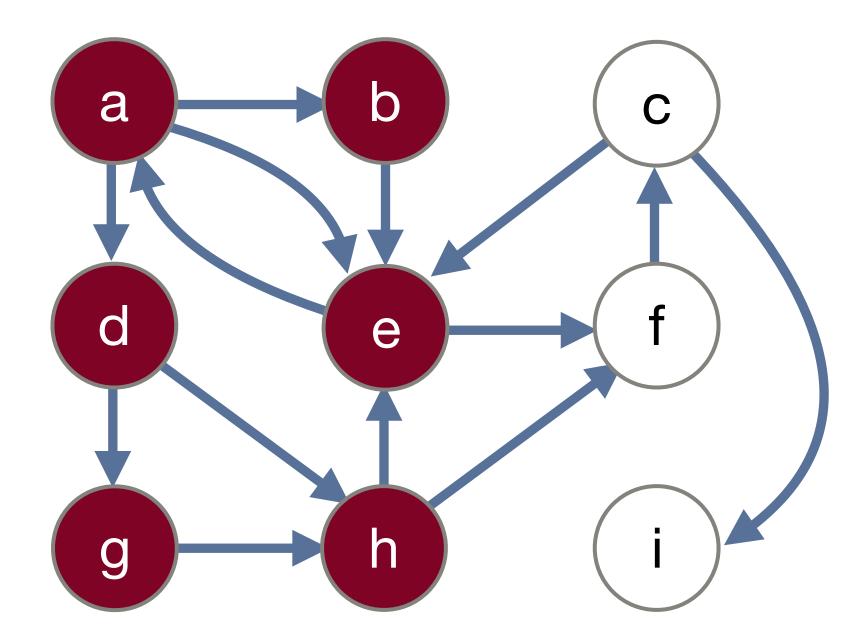
if neighbor n is unvisited, recursively call **dfs**(n, v₂).

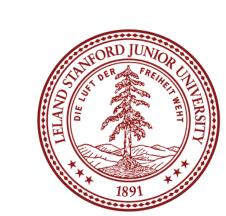
Let's look at **dfs** from h to c:

Vertex Map

dfs(g,c)
dfs(d,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)

Vertex	Visited?
a	true
b	true
C	false
d	true
е	true
f	true
g	true
h	true
i	false





dfs from v_1 to v_2 :

mark v₁ as visited.

for all edges from v₁ to its neighbors:

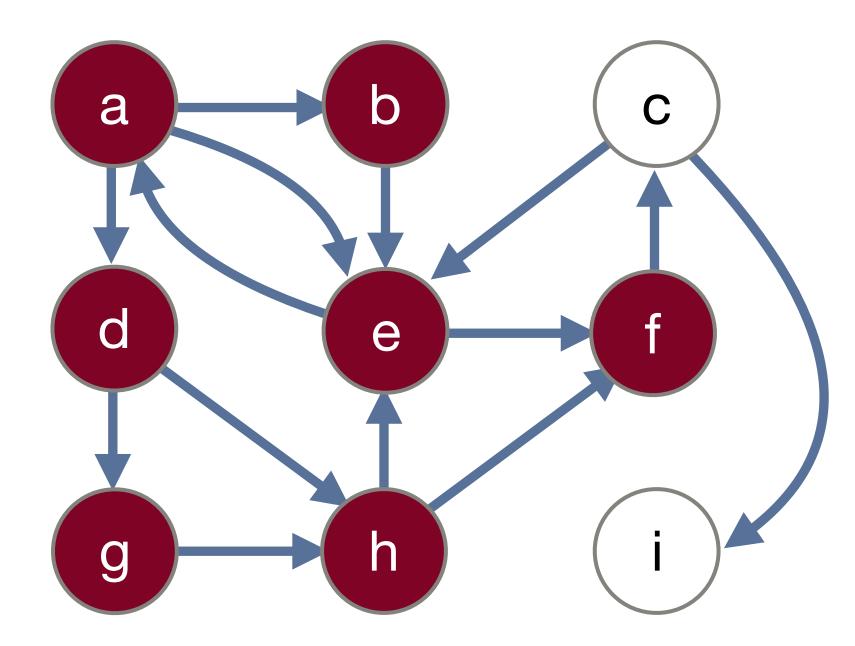
if neighbor n is unvisited, recursively call **dfs**(n, v₂).

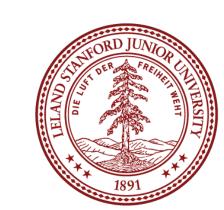
Let's look at dfs from h to c:

Vertex Map

dfs(f,c)
dfs(e,c)
dfs(h,c)

Vertex	Visited?
a	true
b	true
C	false
d	true
е	true
f	true
g	true
h	true
i	false





dfs from v_1 to v_2 :

mark v₁ as visited.

for all edges from v₁ to its neighbors:

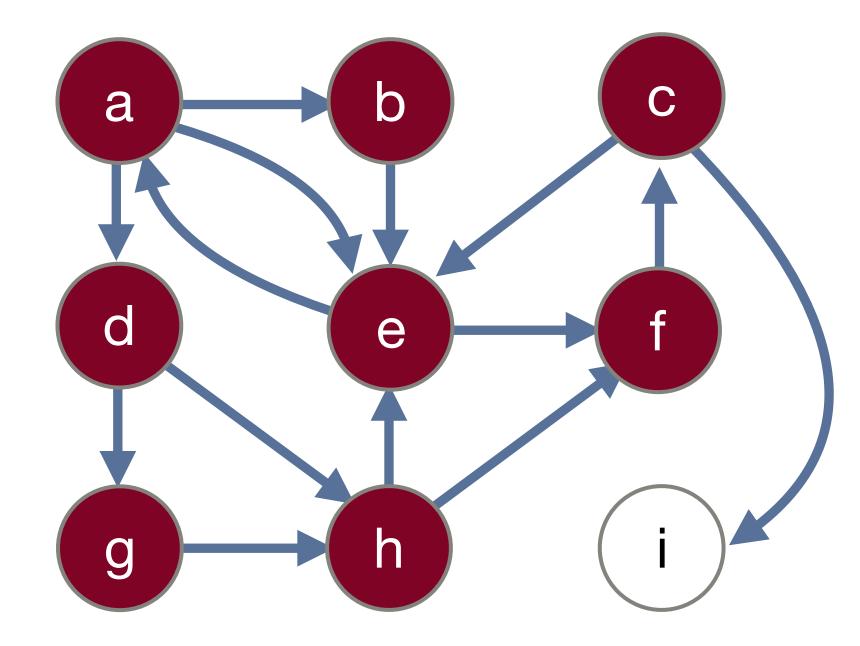
if neighbor n is unvisited, recursively call **dfs**(n, v₂).

Let's look at **dfs** from h to c:

Vertex Map

	dfs(c,c)
	dfs(f,c)
tonug;	dfs(e,c)
	dfs(h,c)

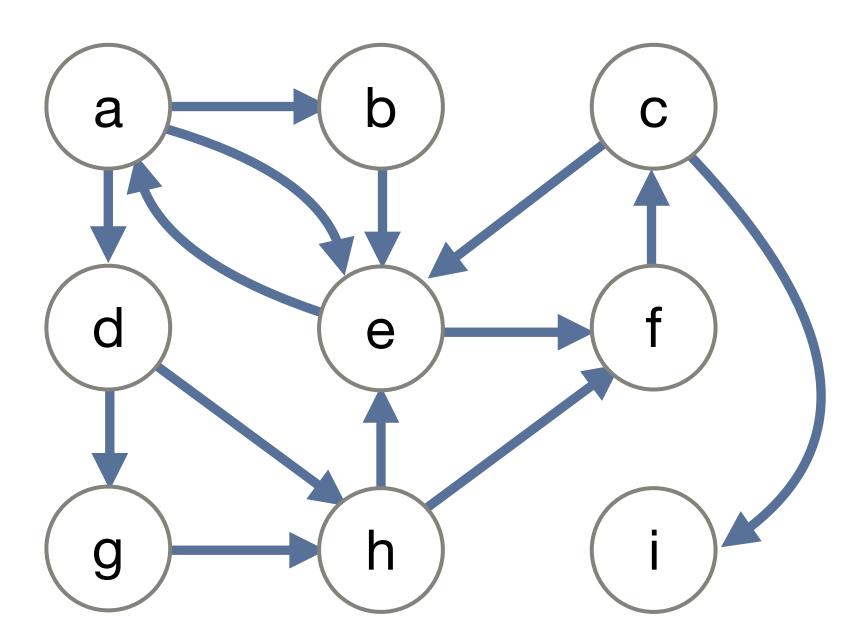
Vertex	Visited?
a	true
b	true
C	true
d	true
е	true
f	true
g	true
h	true
i	false





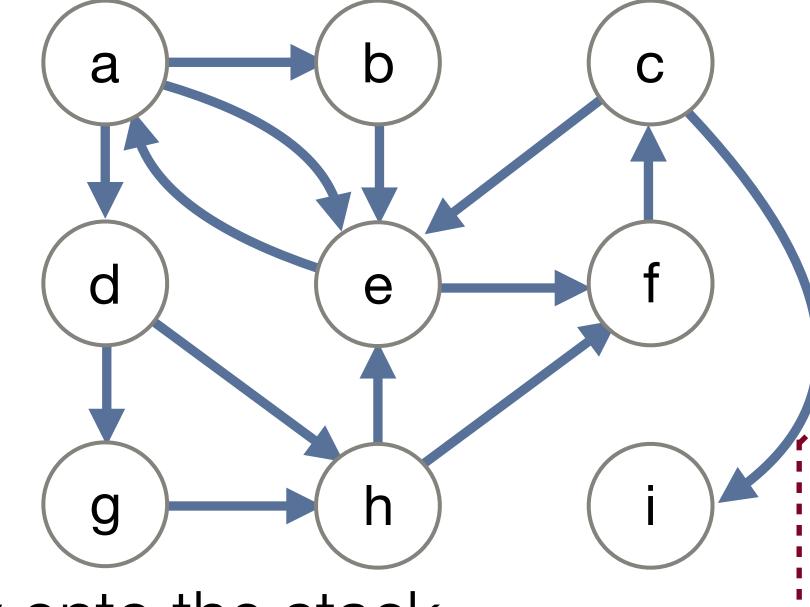
```
dfs from v<sub>1</sub> to v<sub>2</sub>:
    create a stack, s
    s.push(v<sub>1</sub>)
    while s is not empty:
    v = s.pop()
    if v has not been visited:
        mark v as visited
        push all neighbors of v onto the stack
```



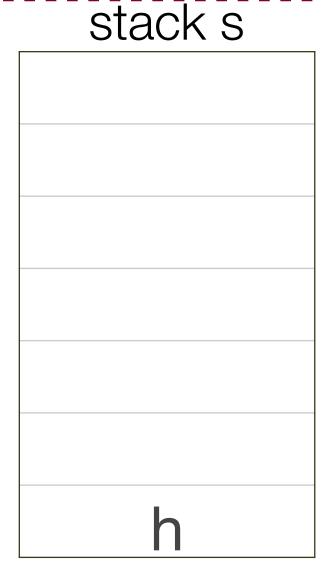




dfs from v_1 to v_2 : create a stack, s $s.push(v_1)$ while s is not empty: V = s.pop()if v has not been visited: mark v as visited push all neighbors of v onto the stack



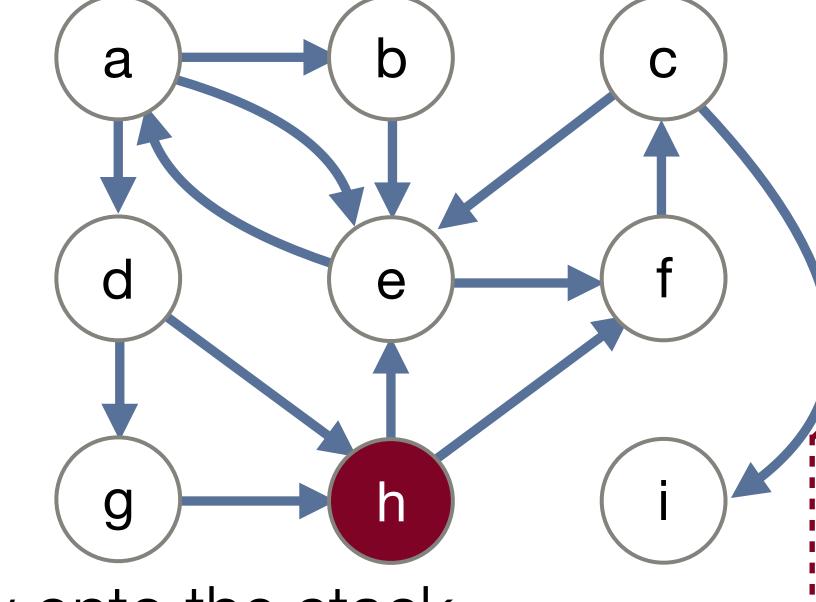
Let's look at **dfs** from h to c: push h



	Verte	ex Map	
Ve	rtex	Visited	d?
	a	false)
	b	false)
	С	false)
	d	false)
	е	false)
	f	false)
	g	false)
	h	false)
	i	false)



```
dfs from v_1 to v_2:
  create a stack, s
  s.push(v_1)
  while s is not empty:
     V = s.pop()
     if v has not been visited:
         mark v as visited
         push all neighbors of v onto the stack
```



Vertex Map

Vertex	Visited?
a	false
b	false
C	false
d	false
е	false
f	false
g	false
h	true

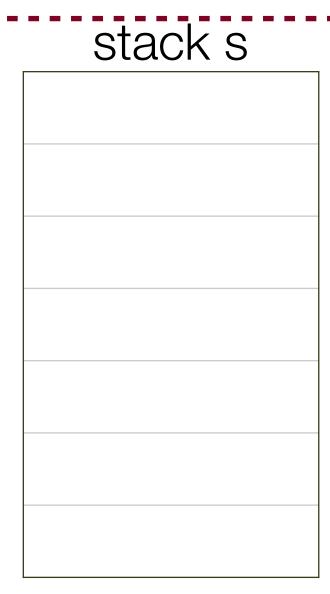
false

Let's look at **dfs** from h to c:

in while loop:

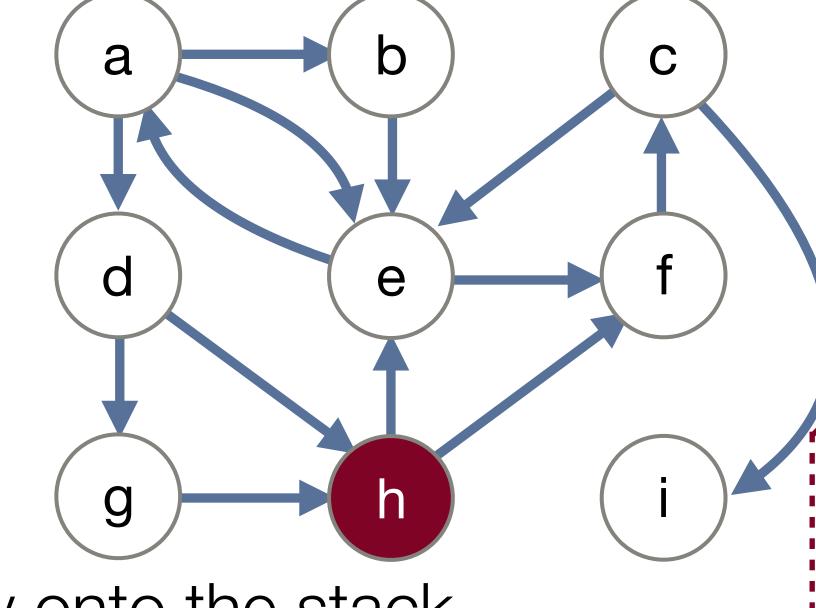
V = s.pop()

v: h



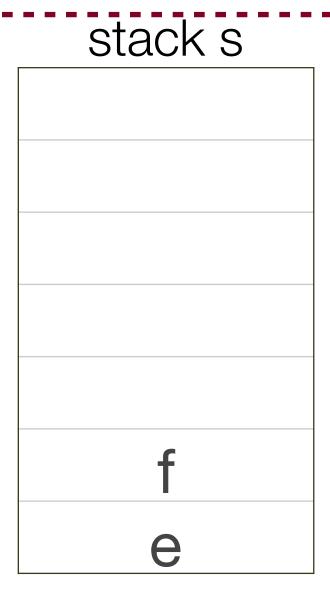


```
dfs from v_1 to v_2:
  create a stack, s
  s.push(v_1)
  while s is not empty:
     V = s.pop()
     if v has not been visited:
         mark v as visited
         push all neighbors of v onto the stack
```



Let's look at **dfs** from h to c:

in while loop: push all neighbors of h



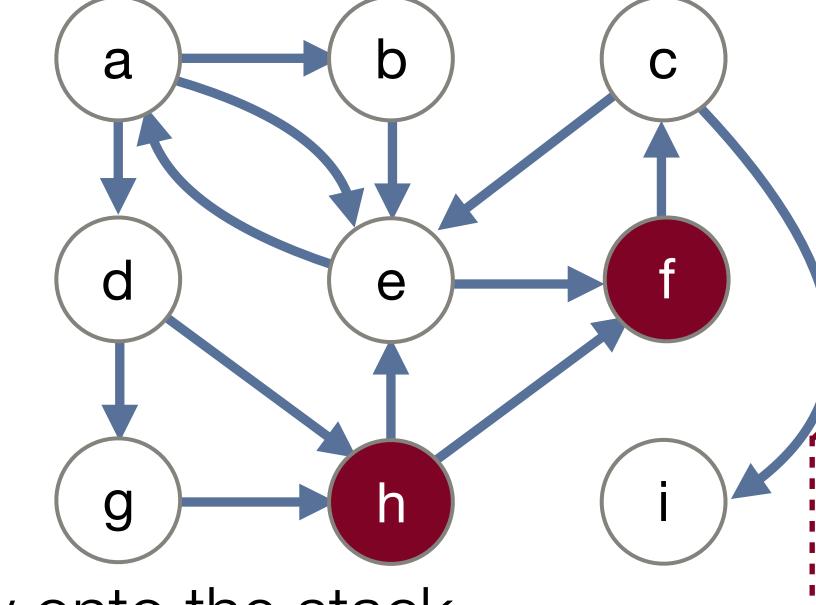
	verte	ex iviap	
Vert	ex	Visited	?
a		false)
b		false)
С		false)
d		false)
е		false)
f		false)
g		false)
h		true	

false

Vartay Man



```
dfs from v_1 to v_2:
  create a stack, s
  s.push(v_1)
  while s is not empty:
     V = s.pop()
     if v has not been visited:
         mark v as visited
         push all neighbors of v onto the stack
```



Vertex Map

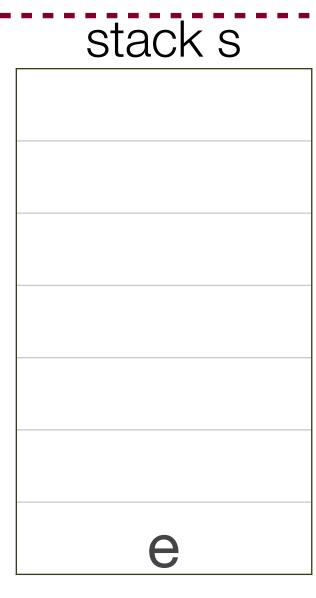
Vertex	Visited?
a	false
b	false
С	false
d	false
е	false
f	true
g	false
h	true
i	false

Let's look at **dfs** from h to c:

in while loop:

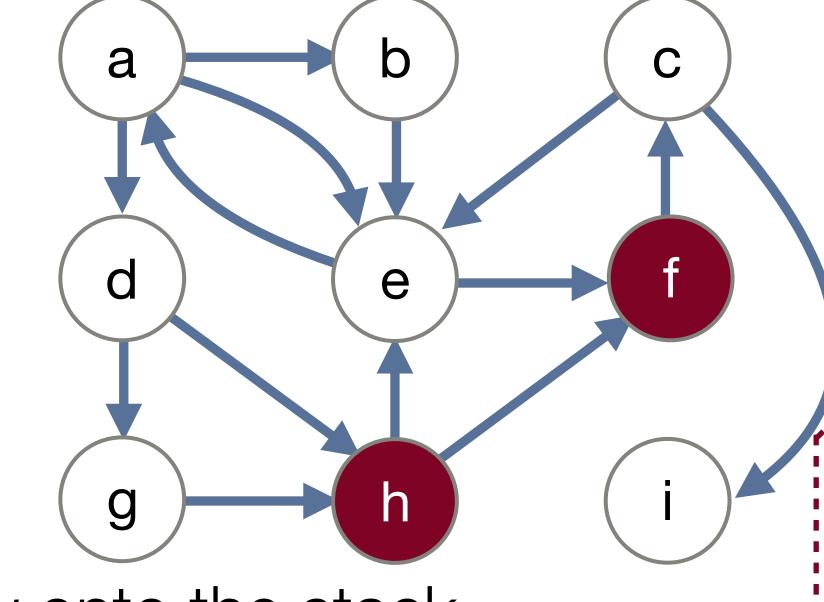
V = s.pop()

v: f





dfs from v_1 to v_2 : create a stack, s $s.push(v_1)$ while s is not empty: V = s.pop()if v has not been visited: mark v as visited push all neighbors of v onto the stack

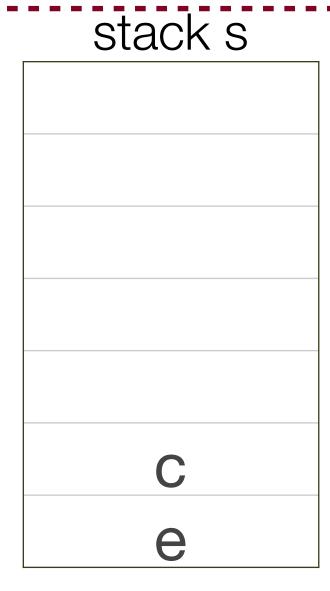


Vertex Map

	•
Vertex	Visited?
a	false
b	false
C	false
d	false
е	false
f	true
g	false
h	true
i	false

Let's look at **dfs** from h to c:

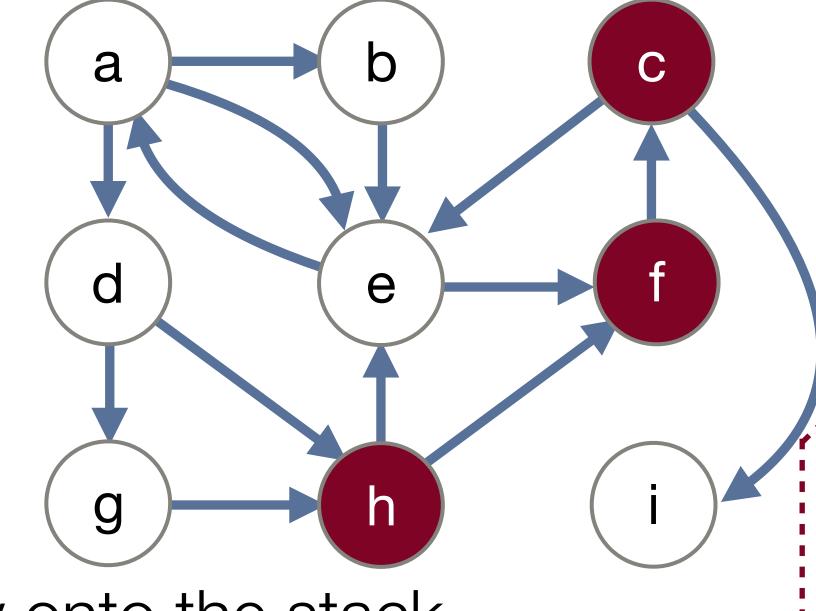
in while loop: push all neighbors of f





dfs from v_1 to v_2 : create a stack, s s.push(v_1) while s is not empty: v = s.pop()if v has not been visited:

v = s.pop()
if v has not been visited:
 mark v as visited
 push all neighbors of v onto the stack



Let's look at **dfs** from h to c:

in while loop: v = s.pop()

V: C found — stop!

stack s
С
е

	Voitox Map		
Vertex		Visited?	
a		false	
b		false	
C		false	
d		false	
е		false	
f		true	
g		false	
h		true	
i		false	

Vertex Map

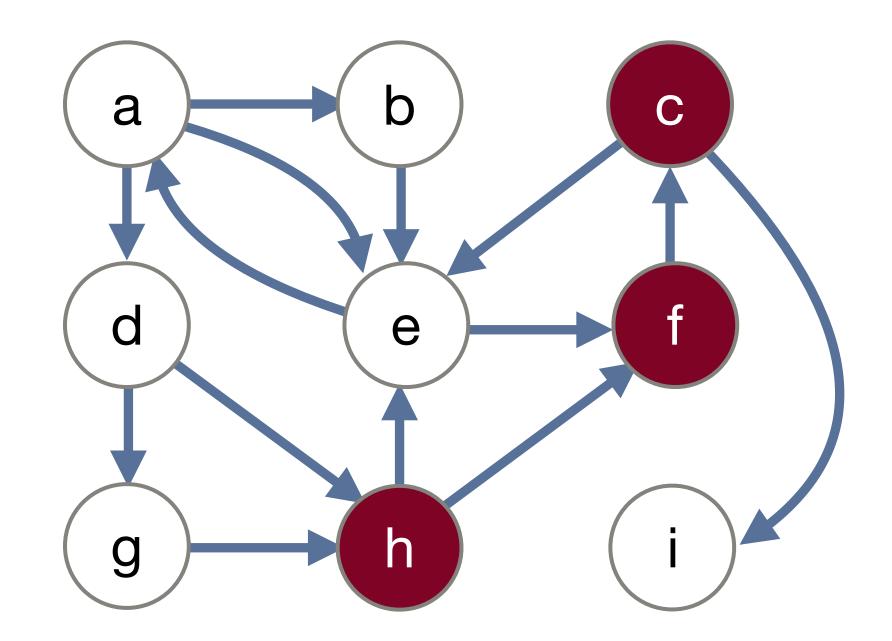


Depth First Search (DFS)

Both the recursive and iterative solutions to DFS were correct, but because of the subtle differences in recursion versus using a stack, they traverse the nodes in a different order.

For the h to c example, the iterative solution happened to be faster, but for different graphs the recursive solution may have been faster.

To retrieve the DFS path found, pass a collection parameter to each cell (if recursive) and chooseexplore-unchoose (our old friend, recursive backtracking!)

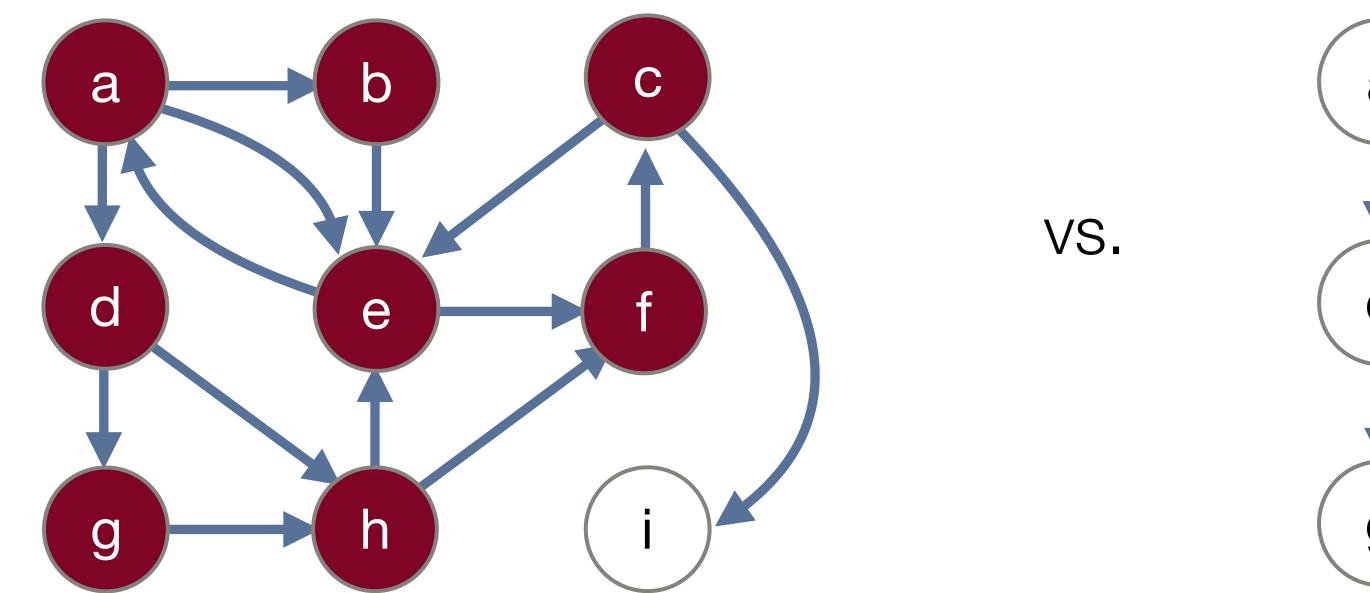


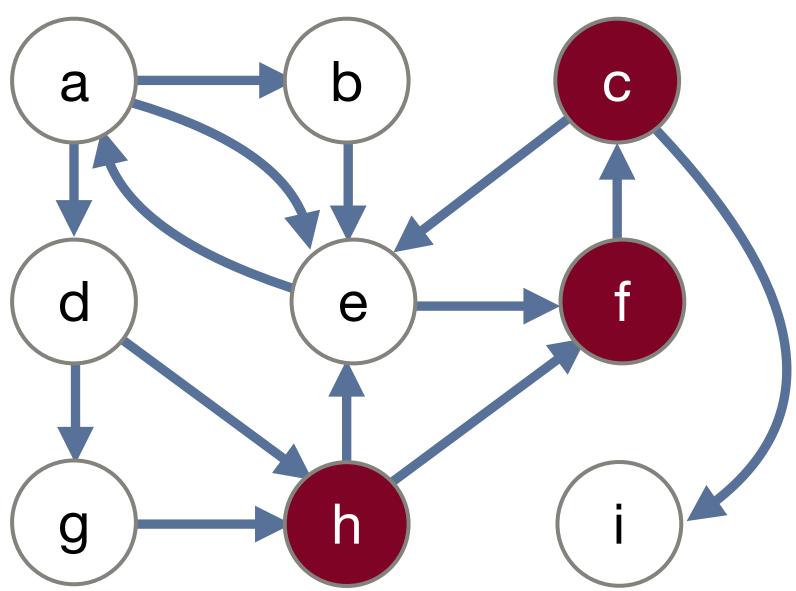


Depth First Search (DFS)

DFS is guaranteed to find a path if one exists.

It is *not* guaranteed to find the best or shortest path! (i.e., it is not optimal)







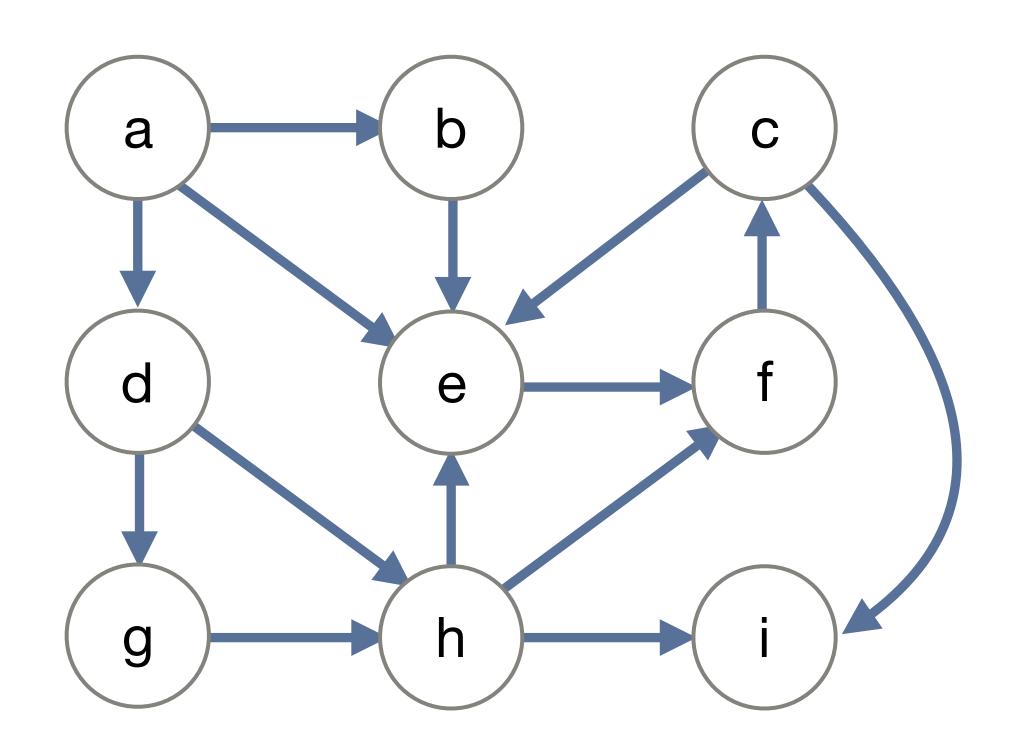
Breadth First Search (BFS)

 From the start vertex, explore the neighbor nodes first, before moving to the next level neighbors.

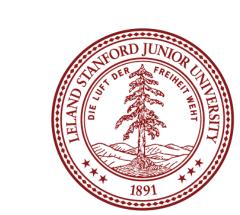
This isn't easy to implement recursively. The iterative algorithm is very similar to the DFS iterative, except that we use a queue.

BFS from a to i (assuming a-z order) visits:

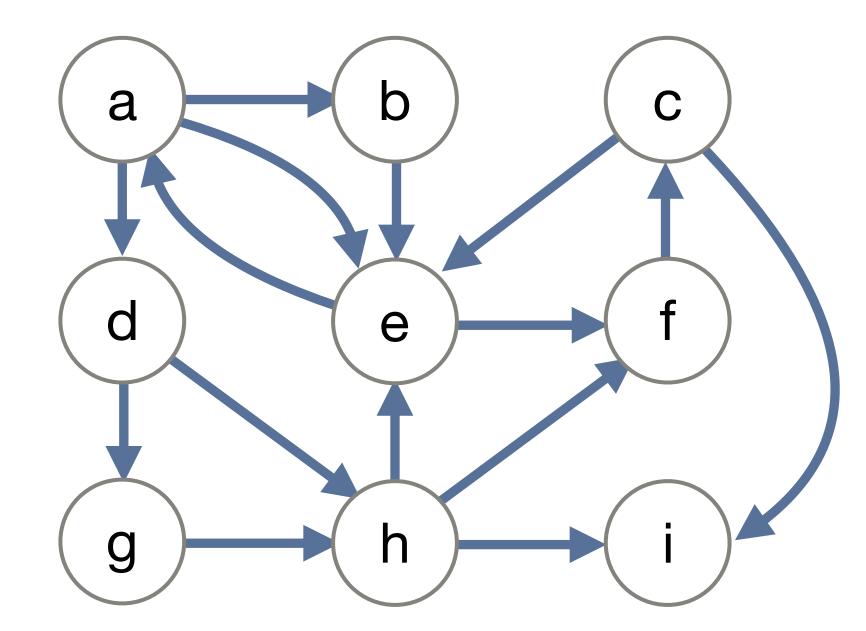




Notice: the shortest!



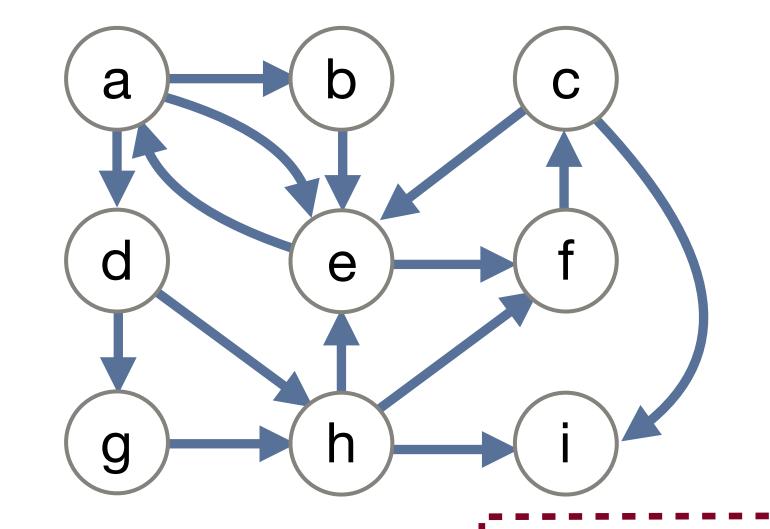
```
bfs from v<sub>1</sub> to v<sub>2</sub>:
   create a queue of paths (a vector), q
  q.enqueue(v<sub>1</sub> path)
   while q is not empty and v<sub>2</sub> is not yet visited:
      path = q.dequeue()
     v = last element in path
     if v is not visited:
       mark v as visited
       if v is the end vertex, we can stop.
       for each unvisited neighbor of v:
           make new path with v's neighbor as last element
           enqueue new path onto q
```







```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
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            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element enqueue new path onto q
```



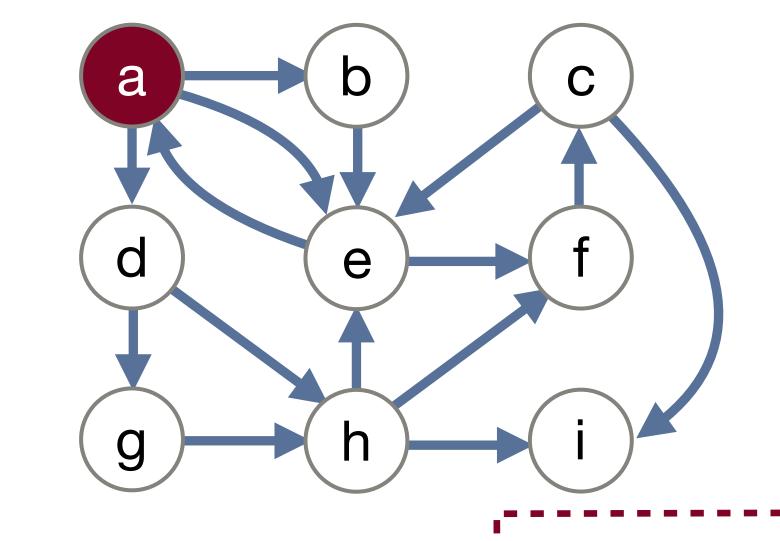
Let's look at **bfs** from a to i:

queue: front a

Vector<Vertex *> startPath startPath.add(a) q.enqueue(startPath) Visited Set: (empty)



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
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        v = last element in path
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            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element enqueue new path onto q
```



Let's look at **bfs** from a to i:

queue: front ae ad ab

in while loop:

curPath = q.dequeue() (path is a)

v = last element in curPath (v is a)

mark v as visited

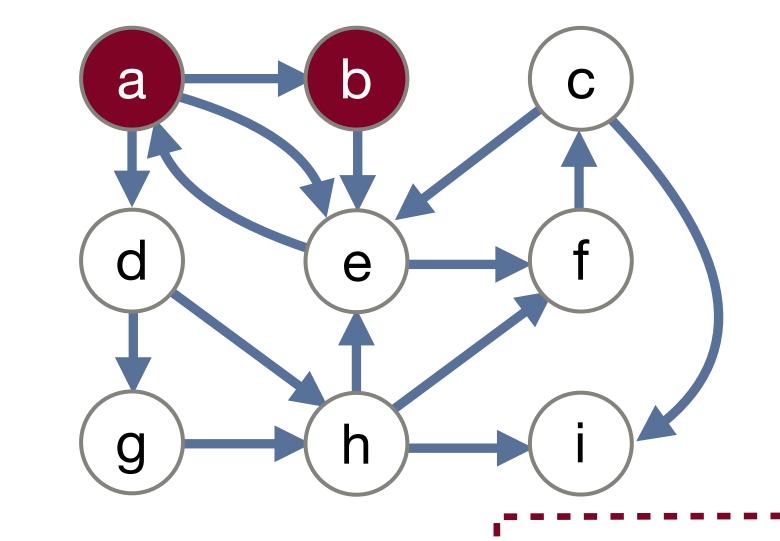
enqueue all unvisited neighbor paths onto q

Visited Set:

a



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
        path = q.dequeue()
        v = last element in path
        if v is not visited:
            mark v as visited
            if v is the end vertex, we can stop.
            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element enqueue new path onto q
```



Let's look at **bfs** from a to i:

queue: front abe ae ad

in while loop:

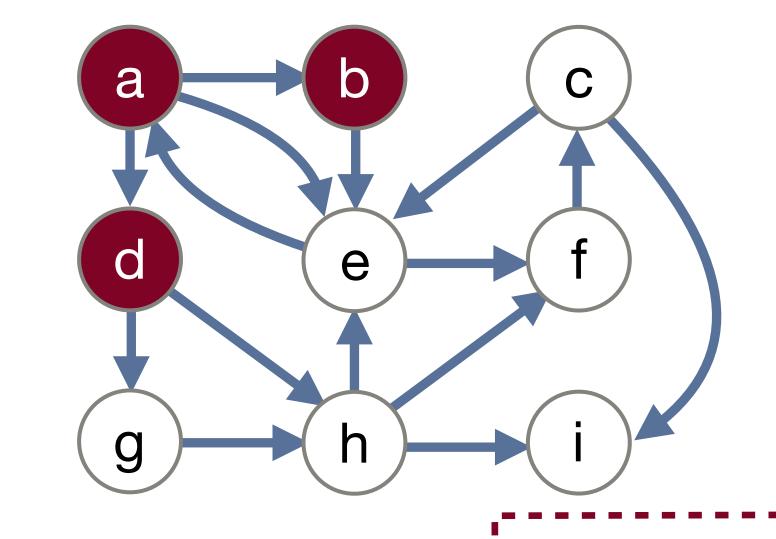
curPath = q.dequeue() (path is ab)
v = last element in curPath (v is b)
mark v as visited
enqueue all unvisited neighbor paths onto q

Visited Set:

a



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
        path = q.dequeue()
        v = last element in path
        if v is not visited:
            mark v as visited
            if v is the end vertex, we can stop.
            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element
                 enqueue new path onto q
```



Let's look at **bfs** from a to i:

queue:

						front
			adh	adg	abe	ae

in while loop:

curPath = q.dequeue() (path is ad)
v = last element in curPath (v is d)
mark v as visited
enqueue all unvisited neighbor paths onto q

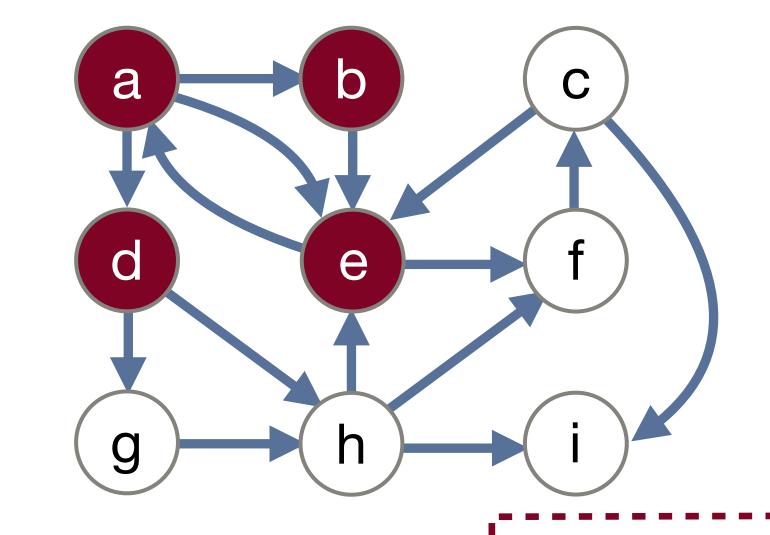
Visited Set:

a

d



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
        path = q.dequeue()
        v = last element in path
        if v is not visited:
            mark v as visited
            if v is the end vertex, we can stop.
            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element
                 enqueue new path onto q
```



Let's look at **bfs** from a to i:

in while loop:

curPath = q.dequeue() (path is ae)
v = last element in curPath (v is e)
mark v as visited
enqueue all unvisited neighbor paths onto q

Visited Set:

a

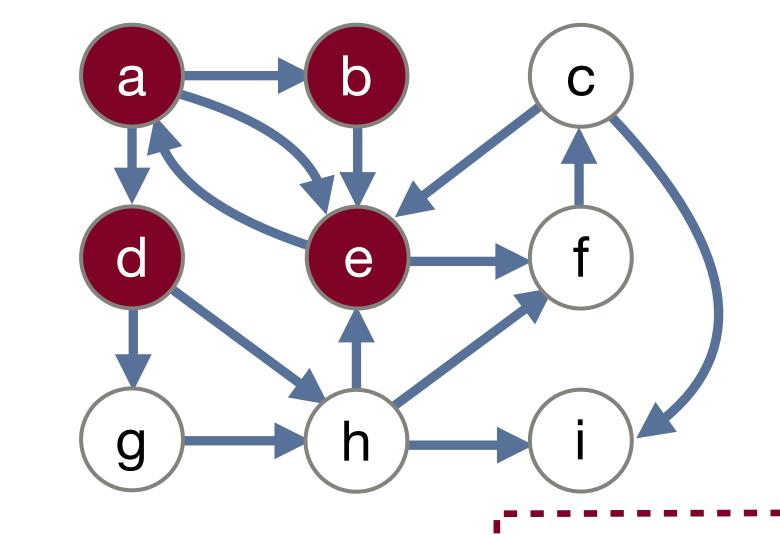
b

O

e



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
        path = q.dequeue()
        v = last element in path
        if v is not visited:
            mark v as visited
            if v is the end vertex, we can stop.
            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element
                 enqueue new path onto q
```



Let's look at **bfs** from a to i:

queue: front aef adh adg

in while loop:

curPath = q.dequeue() (path is abe)

v = last element in curPath (v is e)

mark v as visited (already been marked, no need to enqueue neighbors)

Visited Set:

a

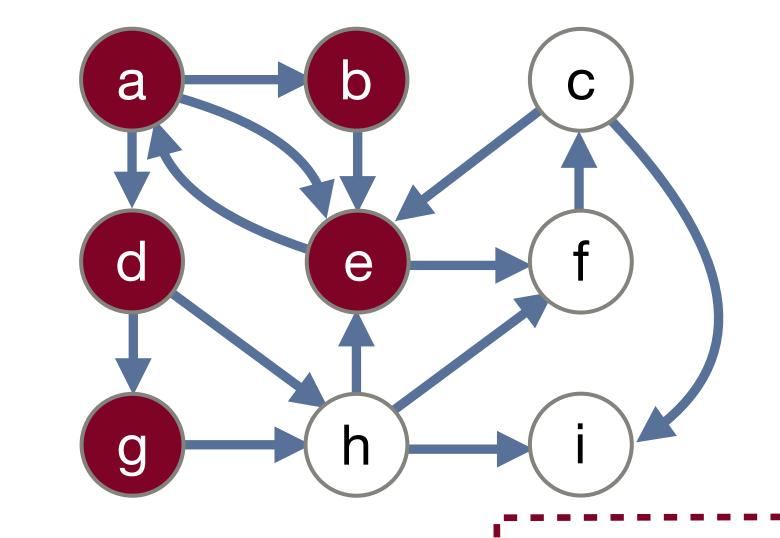
b

O

e



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
        path = q.dequeue()
        v = last element in path
        if v is not visited:
            mark v as visited
            if v is the end vertex, we can stop.
            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element
                 enqueue new path onto q
```



Let's look at **bfs** from a to i:

queue: front adgh aef adh

in while loop:

curPath = q.dequeue() (path is adg) v = last element in curPath (v is g) mark v as visited enqueue all unvisited neighbor paths onto q

Visited Set:

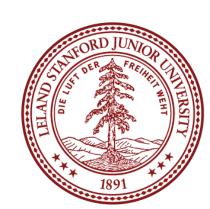
a

b

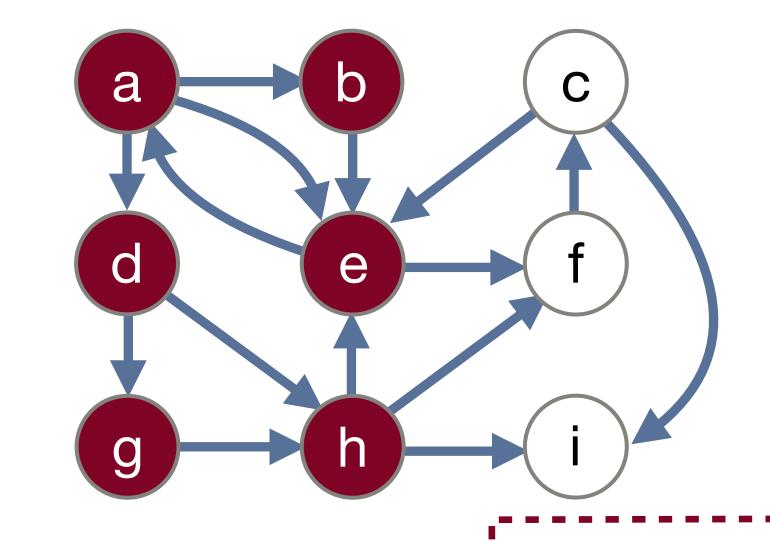
O

e

g



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
        path = q.dequeue()
        v = last element in path
        if v is not visited:
            mark v as visited
            if v is the end vertex, we can stop.
            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element enqueue new path onto q
```



Let's look at **bfs** from a to i:

in while loop:

curPath = q.dequeue() (path is adh)

v = last element in curPath (v is h)

mark v as visited

enqueue all unvisited neighbor paths onto q

Visited Set:

a

b

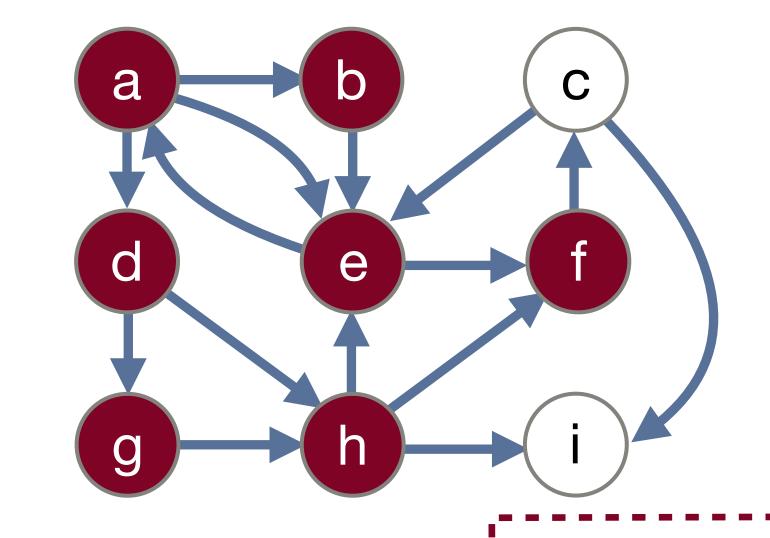
d

e

h



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
        path = q.dequeue()
        v = last element in path
        if v is not visited:
            mark v as visited
            if v is the end vertex, we can stop.
            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element enqueue new path onto q
```



Let's look at **bfs** from a to i:

queue: front aefc adhi adhf adgh

in while loop:

curPath = q.dequeue() (path is aef)

v = last element in curPath (v is f)

mark v as visited

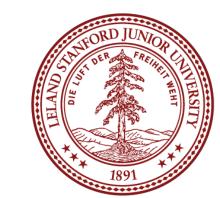
enqueue all unvisited neighbor paths onto q

Visited Set:

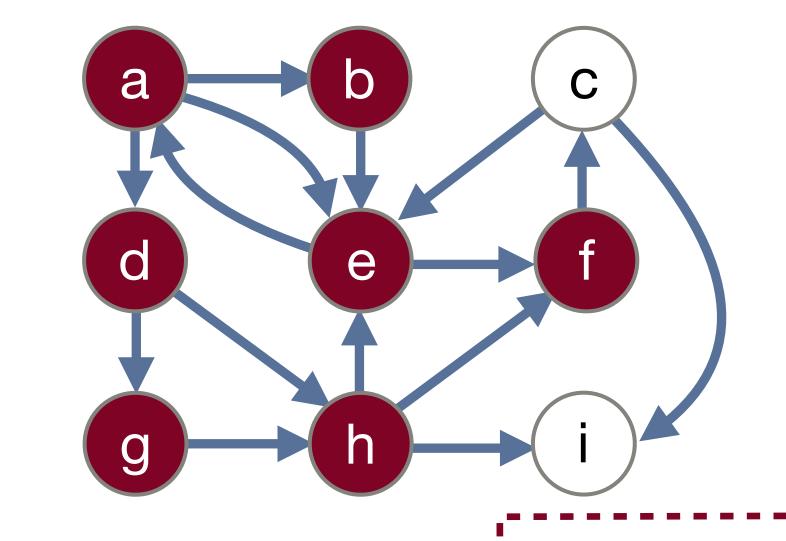
a b

d

t E



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
        path = q.dequeue()
        v = last element in path
        if v is not visited:
            mark v as visited
            if v is the end vertex, we can stop.
            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element
                 enqueue new path onto q
```



Let's look at **bfs** from a to i:

queue: front aefc adhi adhf

in while loop:

curPath = q.dequeue() (path is adgh)

v = last element in curPath (v is h)

mark v as visited (already been marked, no need to enqueue neighbors)

Visited Set:

a

b

C

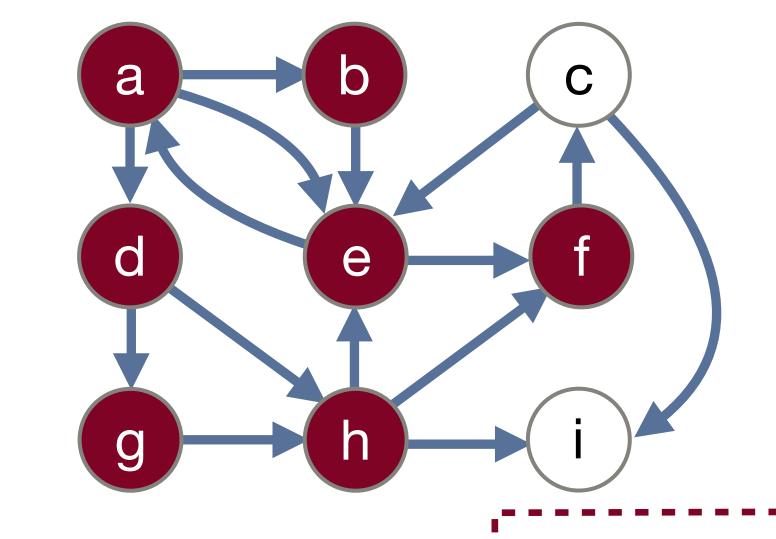
 ϵ

f

h



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
        path = q.dequeue()
        v = last element in path
        if v is not visited:
            mark v as visited
            if v is the end vertex, we can stop.
            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element
                  enqueue new path onto q
```



Let's look at **bfs** from a to i:

queue: front aefc adhi

in while loop:

curPath = q.dequeue() (path is adhf)

v = last element in curPath (v is f)

mark v as visited (already been marked, no need to enqueue neighbors)

Visited Set:

a

b

C

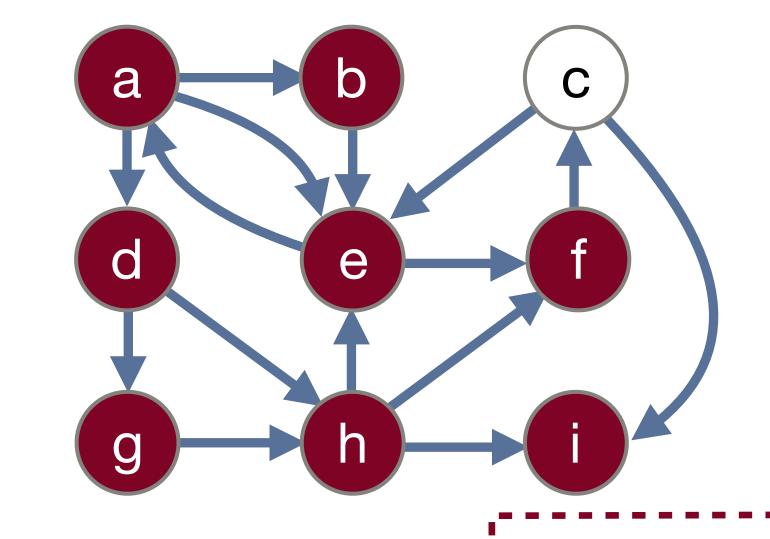
 ϵ

f

h



```
bfs from v<sub>1</sub> to v<sub>2</sub>:
    create a queue of paths (a vector), q
    q.enqueue(v<sub>1</sub> path)
    while q is not empty and v<sub>2</sub> is not yet visited:
        path = q.dequeue()
        v = last element in path
        if v is not visited:
            mark v as visited
            if v is the end vertex, we can stop.
            for each unvisited neighbor of v:
                 make new path with v's neighbor as last element
                  enqueue new path onto q
```



Let's look at **bfs** from a to i:

queue: front aefc adhi

in while loop:

curPath = q.dequeue() (path is adhi) v = last element in curPath (v is i)

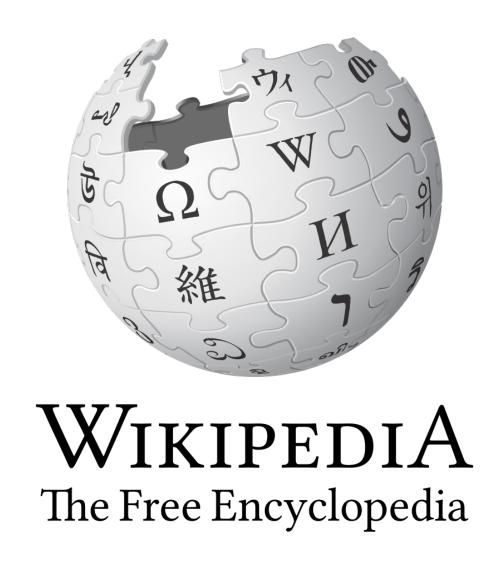
found!

Visited Set:

a b d e f h i



Wikipedia: Getting to Philosophy



So I downloaded Wikipedia...

It turns out that you *can* download Wikipedia, but it is > 10 Terabytes (!) uncompressed. The reason Wikipedia asks you for money every so often is because they have lots of fast computers with lots of memory, and this is expensive (so donate!)

But, the Internet is just a graph...so, Wikipedia pages are just a graph...let's just do the searching by taking advantage of this: download pages as we need them

Wikipedia: Getting to Philosophy



What kind of search is the "getting to philosophy" algorithm?

"Clicking on the first lowercase link in the main text of a Wikipedia article, and then repeating the process for subsequent articles, usually eventually gets one to the Philosophy article."

This is a depth-first search! To determine if a Wikipedia article will get to Philosophy, we just select the first link each time. If we ever have to select a second link (or if a first-link refers to a visited vertex), then that article doesn't get to Philosophy.



Wikipedia: Getting to Philosophy



We can also perform a Breadth First Search, as well. How would this change our search?

A BFS would look at all links on a page, then all links for each link on the page, etc. This has the potential of taking a long time, but it will find a shortest path.





References and Advanced Reading

References:

- Depth First Search, Wikipedia: https://en.wikipedia.org/wiki/Depth-first_search
- •Breadth First Search, Wikipedia: https://en.wikipedia.org/wiki/Breadth-first_search

Advanced Reading:

- •Visualizations:
- https://www.cs.usfca.edu/~galles/visualization/DFS.html
- https://www.cs.usfca.edu/~galles/visualization/BFS.html

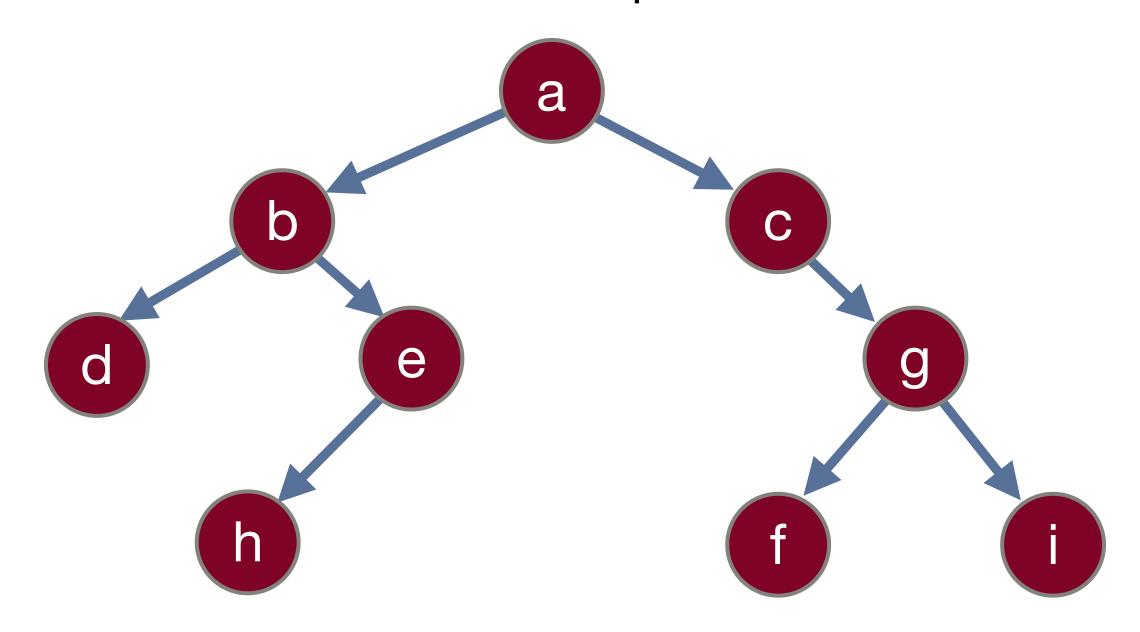


Extra Slides



Breadth First Search (BFS): Tree searching

A Breadth First Search on a tree will produce a "level order traversal":



Breadth First Search: a b c c d e e g h e f e i

This is necessary if we want to print the tree to the screen in a pretty way, such that it retains its tree-like structure.

