Catalan Numbers

1 Analytical Formula

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

2 Recursive Formula

2.1 Definition

$$C_0 = C_1 = 1$$
$$C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}, \quad n \geq 2$$

2.2 From the analytical formula

$$C_0 = C_1 = 1$$
$$C_n = \frac{4n-2}{n+1} C_{n-1}, \quad n \geq 2$$

3 Applications

- Number of correct bracket sequence consisting of n opening and n closing brackets.
- The number of triangulations of a convex polygon with $n+2$ sides
- The number of paths in a nxn grid that goes from the upper left corner to the lower right corner (only moving right or down) and that don't pass the main diagonal.
- The number of ways to connect the $2n$ points on a circle to form n disjoint chords

4 Reference

https://cp-algorithms.com/combinatorics/catalan-numbers.html