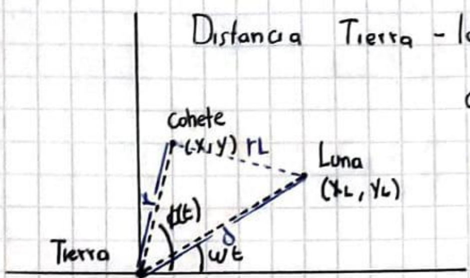
ASIGNATURA: MÉTODOS COMPUTACIONALES 2
ESTUDIANTE: DIEGO FERNANDO HERNANDEZ PORRAS 202014422
PROGRAMA DE ESTUDIOS: FÍSICA

NAVE-LUNA

0.1

Distancia Nave-Luna = r_L

Distancia Tierra-luna = d



Cohete-Luna

$$r_L = \sqrt{(x_L - x)^2 + (y_L - y)^2}$$

$$r_L = \sqrt{(d\cos(\omega t) - r(t)\cos(\phi(t)))^2 + (d\sin(\omega t) - r(t)\sin(\phi(t)))^2}$$

$x = r(t) \cdot \cos(\phi(t))$

$y = r(t) \cdot \sin(\phi(t))$

$x_L = d\cos(\omega t)$

$y_L = d\sin(\omega t)$

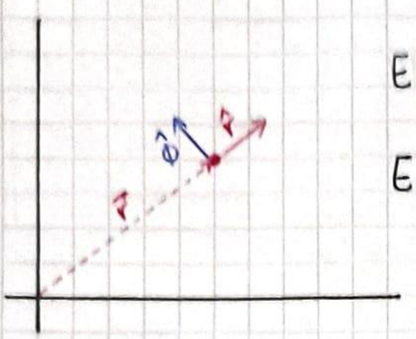
$$r_L^2 = d^2\cos^2(\omega t) - 2d\, r_{(t)}\cos(\omega t)\cos(\phi_{(t)}) + r_{(t)}^2\cos^2(\phi_{(t)}) + \cdots$$
$$\cdots + d^2\sin^2(\omega t) - 2d\, r_{(t)}\sin(\omega t)\sin(\phi(t)) + r_{(t)}^2\sin^2(\phi(t))$$

$$r_L^2 = d^2 + r_{(t)}^2 - 2d\, r_{(t)}\left[\cos(\omega t)\cos(\phi_{(t)}) + \sin(\omega t)\sin(\phi_{(t)})\right]$$

$$r_L^2 = d^2 + r_{(t)}^2 - 2d\, r_{(t)}\cos(\omega t - \phi_{(t)})$$

$$r_L = \sqrt{d^2 + r_{(t)}^2 - 2d\, r_{(t)}\cos(\phi - \omega t)}$$

d) $H = E_K + E_P \quad ; \quad E_P = -\dfrac{G m_T m}{r} - \dfrac{G m_L m}{r_L}$



$$E_K = \frac{1}{2} m (\dot{r})^2 + \frac{1}{2} I \omega^2$$

$$E_K = \left(\frac{1}{2}\right) m (\dot{r})^2 + \frac{1}{2} (m r^2) (\dot{\phi})^2$$

$$H = \frac{1}{2} m (\dot{r})^2 + \frac{1}{2} m r^2 (\dot{\phi})^2 - \frac{G m_T m}{r} - \frac{G m_L m}{r_L}$$

$P_r = m \dot{r} \qquad P_\phi = I\omega = m r^2 \dot{\phi}$

$$H = \frac{1}{2} P_r \dot{r} + \frac{1}{2} P_\phi \dot{\phi} - \frac{G m_T m}{r} - \frac{G m_L m}{r_L}$$

$$\frac{1}{2} P_r \dot{r} = P_r \dot{r} - \frac{1}{2} P_r \dot{r} = P_r \dot{r} - \frac{1}{2} m (\dot{r})^2$$

$$\frac{1}{2} P_\phi \dot{\phi} = P_\phi \dot{\phi} - \frac{1}{2} P_\phi \dot{\phi} = P_\phi \phi - \frac{1}{2} m r^2 (\dot{\phi})^2$$

$$H = P_r \dot{r} + P_\phi \dot{\phi} - \frac{1}{2} m (r)^2 - \frac{1}{2} m r^2 \dot{\phi}^2 - \frac{G m_T m}{r} - \frac{G m_L m}{r_L}$$

$$H = P_r \dot{r} + P_\phi \phi - \left[E_K - E_P\right] = P_r \dot{r} + P_\phi \dot{\phi} - L$$

$P_r = m \dot{r}$

$\dot{r} = \dfrac{P_r}{m}$

$P_\phi = m r^2 \dot{\phi}$

$\dot{\phi} = \dfrac{P_\phi}{m r^2}$

$$H = P_r \left(\frac{P_r}{m}\right) + P_\phi \left(\frac{P_\phi}{m r^2}\right) - L$$

$$H = \frac{P_r^2}{m} + \frac{P_\phi^2}{m r^2} - \overbrace{\frac{1}{2} \frac{P_r^2}{m}}^{\frac{1}{2} m (\dot{r})^2} - \overbrace{\frac{1}{2} \frac{P_\phi^2}{m r^2}}^{\frac{1}{2} m r^2 \dot{\phi}^2} - \frac{G m_T m}{r} - \frac{G m_L m}{r_L}$$

$$H = \frac{1}{2} \frac{P_r^2}{m} + \frac{1}{2} \frac{P_\phi^2}{m r^2} - \frac{G m_T m}{r} - \frac{G m_L m}{r_L}$$

(e) $H = P_r \dot{r} + P_\phi \dot{\phi} - L = \dfrac{P_r^2}{2m} + \dfrac{P_\phi^2}{2mr^2} - \dfrac{G m_T m}{r} - \dfrac{G m_L m}{r_L}$ | $P_r = m\dot{r}$, $P_\phi = mr^2\dot{\phi}$

$$\frac{\partial H}{\partial P_r} = \frac{2 P_r}{2m} = \frac{P_r}{m} = \dot{r} \qquad \Big| \; r_L = \sqrt{d^2 + r^2 - 2rd\cos(\phi - \omega t)}$$

$$\frac{\partial H}{\partial P_\phi} = \frac{2 P_\phi}{2mr^2} = \frac{P_\phi}{mr^2} = \dot{\phi}$$

$$\frac{\partial H}{\partial r} = \frac{-\cancel{2} P_\phi^2}{\cancel{2} m r^3} + \frac{G m_T m}{r^2} - G m_L m \left(\left(-\frac{1}{2}\right) \cdot \left(d^2 + r^2 - 2rd\cos(\phi - \omega t)\right)^{-3/2} \cdot \left(2r - 2d\cos(\phi - \omega t)\right) \right)$$

$$\frac{\partial H}{\partial r} = -\frac{P_\phi^2}{mr^3} + \frac{G m_T m}{r^2} + \frac{G m_L m}{\cancel{2} r_L^3} \cdot \cancel{2}\left(r - d\cos(\phi - \omega t)\right)$$

$$\frac{\partial H}{\partial \phi} = + G m_L m \left(\left(+\frac{1}{2}\right) \cdot \left(d^2 + r^2 - 2rd\cos(\phi - \omega t)\right)^{-3/2} \cdot \left(\cancel{2} rd\,(+\sin(\phi - \omega t))\right) \right)$$

$$\frac{\partial H}{\partial \phi} = \frac{G m_L m}{r_L^3} \cdot rd\sin(\phi - \omega t)$$

$$H = E_K + U \qquad \text{Por def} \quad F = -\frac{\partial U}{\partial x} = \dot{P}_x$$

$$\frac{\partial H}{\partial r} = \underbrace{\frac{\partial}{\partial r}(E_K)}_{0} + \frac{\partial}{\partial r}(U) = -F = -\dot{P}_r \qquad \frac{\partial H}{\partial \phi} = \underbrace{\frac{\partial}{\partial \phi}(E_K)}_{0} + \frac{\partial}{\partial \phi}(U) = -\dot{P}_\phi$$

(F) $\tilde{r} = \dfrac{r}{d} \qquad \tilde{P}_r = \dfrac{P_r}{md} \qquad \tilde{P}_\phi = \dfrac{P_\phi}{md^2}$

$$P_r = m\dot{r} \rightarrow \dot{r} = \frac{P_r}{m} = \tilde{P}_r d \qquad \dot{\tilde{r}} = \frac{\dot{r}}{d} \Rightarrow \dot{\tilde{r}} = \frac{\tilde{P}_r \cancel{d}}{\cancel{d}}$$

$$\dot{\phi} = \frac{P_\phi}{mr^2} = \frac{md^2 \tilde{P}_\phi}{m d^2 (\tilde{r})^2} = \frac{\tilde{P}_\phi}{(\tilde{r})^2}$$

$$\dot{\tilde{P}}_r = \frac{\dot{P}_r}{md} = \frac{-\frac{\partial H}{\partial r}}{md} = \frac{1}{md}\left[\left(\frac{P_\phi^2}{mr^3}\right) - \frac{G m_T m}{r^2} - \frac{G m_L m}{r_L^3}\left(r - d\cos(\phi - \omega t)\right)\right]$$

$$\dot{\tilde{P}}_r = \frac{1}{md}\,\frac{(md^2 \tilde{P}_\phi)^2}{m d^3 \tilde{r}^3} - \frac{1}{md}\cdot\frac{G m_T m}{d^2 \tilde{r}^2} - \frac{1}{md}\cdot\frac{G m_L m}{r_L^3}\left(r - d\cos(\phi - \omega t)\right)$$

$$\dot{\tilde{P}}_r = \frac{\tilde{P}_\phi^2}{\tilde{r}^3} - \frac{G m_T}{d^3}\left[\frac{1}{\tilde{r}^2} + \frac{m_L d^3}{d\, m_T\, r_L^3}\left(r - d\cos(\phi-\omega t)\right)\right]$$

$$\dot{\tilde{P}}_r = \frac{\tilde{P}_\phi^2}{\tilde{r}^3} - \Delta\left[\frac{1}{\tilde{r}^2} + \frac{m_L}{m_T}\cdot\frac{\cancel{d^4}}{\cancel{d^4}\,\tilde{r}'^3}\left(\overset{\tilde{r}}{\frac{r}{d}} - \cos(\phi-\omega t)\right)\right]$$

$$\tilde{r}' = \sqrt{1+\tilde{r}^2 - 2\tilde{r}\cos(\phi-\omega t)} = \sqrt{\frac{d^2}{d^2}+\frac{r^2}{d^2} - \frac{2r}{d^2}\,d\cos(\phi-\omega t)} = \frac{r_L}{d}$$

$$\dot{\tilde{P}}_r = \frac{\tilde{P}_\phi^2}{\tilde{r}^3} - \Delta\left[\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}'^3}\left(\tilde{r} - \cos(\phi-\omega t)\right)\right]$$

$$\dot{\tilde{P}}_\phi = \frac{\dot{P}_\phi}{m d^2} = -\frac{\frac{\partial H}{\partial \phi}}{m d^2} = -\frac{1}{\cancel{m} d^2}\left[\frac{G \cancel{m}\, m_L}{r_L^3}\, r d \sin(\phi-\omega t)\right]$$

$$\dot{\tilde{P}}_\phi = -\frac{1}{\cancel{d^2}}\,\frac{G m_L}{(\tilde{r}')^3\, d^3}\,\tilde{r}\,\cancel{d^2}\sin(\phi-\omega t)$$

$$\dot{\tilde{P}}_\phi = \frac{G m_L}{d^3}\,\frac{\tilde{r}}{(\tilde{r}')^3}\sin(\phi-\omega t) = -\frac{\Delta\,\mu\,\tilde{r}}{(\tilde{r}')^3}\sin(\phi-\omega t)$$

g)

$$\tilde{p}_r^{\,0} = \frac{p_r}{md} = \frac{\cancel{m}\,dr}{\cancel{m}\,d\,dt} = \frac{1}{d}\left(\frac{dr}{dt}\right) = \frac{1}{d}\left(\frac{d\sqrt{x^2+y^2}}{dt}\right)$$

$$\frac{d\sqrt{x^2+y^2}}{dt} = \frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{x^2+y^2}}\,\cancel{2}x\dot{x} + \cancel{2}y\dot{y} = \frac{x\dot{x}+y\dot{y}}{r}$$

$$\tilde{p}_r^{\,0} = \frac{x\dot{x}+y\dot{y}}{rd}$$ donde como $V = (\dot{x},\dot{y}) = (v\cos\theta, v\,\text{sen}\,\theta)$

$$= \frac{x\,v\cos\theta + y\,v\,\text{sen}\,\theta}{rd}$$ igualmente $x = r\cos\phi$, $y = r\,\text{sen}\,\phi$

$$= \frac{rv\cos\theta\cos\phi + rv\,\text{sen}\,\theta\,\text{sen}\,\phi}{rd}$$ con identidades trigono. $\Rightarrow$

$$= \frac{v}{d}\cdot\cos(\theta-\phi) = \tilde{v}_0\cos(\theta-\phi)$$

$$\tilde{p}_\phi = \frac{p_\phi}{md^2} = \frac{\cancel{m}r^2}{\cancel{m}d^2}\cdot\frac{d\phi}{dt} = \tilde{r}^2\frac{d\phi}{dt} \quad \Big| \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \tilde{r}^2\frac{d}{dt}\tan^{-1}(y/x) = \frac{\tilde{r}^2}{1+y^2/x^2}\frac{d}{dt}\left(\frac{y}{x}\right)$$

$$= \frac{\tilde{r}^2}{1+y^2/\cancel{x^2}}\cdot\frac{\dot{y}x-\dot{x}y}{x^2} = \frac{\tilde{r}^2}{r^2}(\dot{y}x-\dot{x}y)$$

$$= \frac{\tilde{r}^2}{r^2}(x v\,\text{sen}\,\theta - y v\cos\theta) = \frac{\tilde{r}^2}{r^2}\big(rv(\text{sen}\,\theta\cos\phi - \cos\theta\,\text{sen}\,\phi)\big)$$

$$= \frac{\tilde{r}^{\cancel{2}}}{r}v\sin(\theta-\phi) = \frac{r^{\cancel{2}}}{d^2}\cdot\frac{1}{\cancel{r}}\cdot v\sin(\theta-\phi)$$

$$= \frac{r}{d}\tilde{v}\,\text{sen}(\theta-\phi) = \tilde{r}_0\tilde{v}_0\,\text{sen}(\theta-\phi)$$