A Positive Theory of Dynamic Development Policies

Diego Huerta

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Abstract

How do different development policies come to exist across countries and over time? To address this question, this article develops a politico-economy theory of dynamic Ramsey policies. I adapt the workhorse macro model with heterogeneous agents to include: i) repeated elections, ii) endogenous credit constraints, and iii) occupational choice. Policies and inequality are jointly determined over time as a result of the political process. I characterize the transition dynamics of the equilibrium policy as a function of countries' initial wealth distribution. The different policy dynamics provide a political rationale for the differences in development policies pursued by developing and developed countries.

1 Introduction

There are large differences in development policies across countries and over time. In spite of this, the theoretical understanding of the politico-economy determinants of the dynamics of development policies is still limited. In simple terms, these policies shift resources from one market to another, creating a distributional conflict between different economic groups. The equilibrium policy results from the political process that aggregates these interests, which arise from economic inequality. Current policies in turn determine future inequality. In this sense, development policies and inequality are jointly determined over time, politics being the link between them. In this article, I apply this principle to develop a political theory of dynamic Ramsey policies that explains the observed international differences in development policies.

This paper is motivated by historical accounts showing that the direction of development policies largely varies across countries and over time. For instance, several East Asian countries that experienced episodes of rapid growth between the 60's and 80's implemented *pro-business* development policies, such as wage suppression or credit subsidies.[1] Around the 90's some of these countries changed the direction of these policies towards a more *pro-worker* stance. The East Asian case contrasts with the European experience. In fact, between the 60's and 80's, many countries from the European Union increasingly regulated labor markets by giving strong rights to workers. In the 2000's, many of these countries started a deregulation process and have been moving towards more pro-business policies.[2] How do different development policies come to exist across countries and over time?

To address this question, I adapt the workhorse macro model with heterogeneous agents and financial frictions to study dynamic Ramsey policies from a political economy perspective.[3] The new features of my model are i) repeated elections à la Persson and Tabellini (2000), ii) endoge-

[1] Examples of such countries include China, Japan, South Korea, Malaysia, Taiwan, Thailand and Singapore.

[2] For instance, Greece, Italy, France, Netherlands, Portugal, Romania, Spain , United Kingdom, among others.

[3] The workhorse macro model with heterogeneous agents and financial frictions is as in Buera and Shin (2013), Moll (2014), Buera and Moll (2015), or Itskhoki and Moll (2019). The common feature of these models is that heterogenous entrepreneurs face exogenous financial constraints (linear in wealth) that limit investment. In the tractable versions of these models the masses of workers and entrepreneurs are exogneous. Thus there is no occupational choice.

nous credit constraints, and iii) occupational choice. The economy is populated by a continuum of individuals that differ in their wealth. Policies and inequality affect credit constraints, which determine whether agents can become entrepreneurs or if they remain workers. Citizens vote anticipating the effects of a given policy on credit conditions, and thus, on occupational choice. The current distribution and the resulting equilibrium policy impact saving decisions, which in turn affect future inequality and policies. I study the transition dynamics of development policies as a function of countries' initial wealth distribution. The main policy instrument is a capital tax. Although the results apply to different types of policies such as a income tax and credit subsidies.

In order to obtain theoretical results, I make three simplifying assumptions. First, I use a constant risk aversion utility function. Secondly, firms are homogeneous. Specifically, there are separate production technologies for output and capital. Agents can invest a fixed amount in a capital-producing firm and become entrepreneurs. The output-good is produced by a representative firm that uses capital and labor. All agents work for the representative firm that pays a competitive wage. Those that become entrepreneurs sell their capital to the representative firm. Finally, there are no stochastic shocks in the economy.

In my model, there is a competitive banking system that provides credit to entrepreneurs. Due to credit market imperfections, banks limit access to credit by requiring a minimum collateral to obtain a loan. When the tax rate increases, the minimum collateral to obtain a loan increases, reducing the mass of agents that become entrepreneurs. Reduced competition in the capital market leads to a higher price of capital, but to a lower wage due to diminished productivity of labor.

The main results of the paper are as follows. First, there is an endogenous conflict for redistribution. Agents can be grouped into three categories according to their preferences for tax policies. The first includes poor agents that remain workers regardless of the tax rate. Thus, their preferred tax rate trades-off transfers against wages and is decreasing in wealth. The second group is composed by intermediate wealth agents that prefer a tax rate that allows them to start a firm but that excludes poorer agents from the credit market, leading to a high price at which they can sell their capital. The preferred tax rate function is increasing in wealth for this group.

A third group of wealthy agents is mainly concerned about capital income, and thus, supports a negative tax rate (subsidy).

Secondly, the transition dynamics of the policy platform that arise from aggregating these interests groups over time depend on the first and second moments of the initial wealth distribution. The first moment relates to capital constraints, while the second moment relates to inequality. Countries that start with binding credit constraints (*capital constrained*) and more unequal relative to the steady-state, exhibit a policy platform that moves towards a more *pro-worker* stance. On the other hand, in countries that start *capital unconstrained* and unequal, the policy platform becomes more pro-business over time. The results apply in the opposite direction when countries start more equal relative to the stationary wealth distribution.

Simplifying the complex interaction between the evolution of the wealth distribution, credit constraints and policies, the intuition for the transition dynamics of the equilibrium platform is as follows. When a country starts capital constrained and more unequal relative to the stationary distribution, the initial mass of agents that can become entrepreneurs for a given tax policy is large. Hence, initially, there is great political pressure to adopt a pro-business policy. I show that the initial tax rate is relatively high, thus the effective rate at which agents can save is low. In consequence, agents dissave over time shifting the wealth distribution to the left. As a result, for a given tax rate, the mass of agents that can start a firm decreases over time. Thus, the equilibrium policy becomes more pro-worker over time. These effects work on the opposite direction in an economy that starts unconstrained and unequal.

As far as I know, this paper is the first to introduce a political process in the workhorse model with heterogeneous agents and financial frictions to characterize policy dynamics. The transition dynamics of the equilibrium policy provide a political economy rationale for the different development policies pursued by developing and developed countries.

The paper is organized as follows. Section 2 presents the baseline model. Section 3 describes the competitive equilibrium when the policy platform is exogenous. Section 4 studies individual political preferences. Section 5 describes the political mechanism that determines the equilibrium policy. Section 6 characterizes the political equilibrium and transition dynamics. Section 7 concludes.

1.1 Literature review

This article relates to recent papers studying the Ramsey problem as in the traditional public finance literature (Barro, 1979; Lucas and Stokey, 1983), but in an environment that features a distributional conflict between capitalists and workers (Itskhoki and Moll, 2019; Straub and Werning, 2020). From a political economy perspective, there is a strand of the literature that studies Ramsey taxation under self-interested politicians (Acemoglu et al., 2008, 2010, 2011; Yared, 2010)

The baseline model is similar to the workhorse macro model with heterogeneous agents and collateral constraints as in Buera and Shin (2013); Moll (2014); Buera and Moll (2015), among others. While in my model I simplify the firm side by requiring that entrepreneurs invest a fixed amount of capital, credit conditions arise endogenously as a function of the equilibrium platform and wealth distribution. Collateral constraints in turn induce agents' occupational choice (as in Banerjee and Newman, 1993). This is a new feature that is not present in the aforementioned models and that is key to determine the evolution of individual political preferences.

The political mechanism relates to several papers that introduce probabilistic voting in dynamic macro models to study the political support for different types of policies. Song et al. (2012) analyze fiscal policy, while Farhi et al. (2012) examine taxation. Gonzalez-Eiras and Niepelt (2008) and Sleet and Yeltekin (2008) focus on social security, while Hassler et al. (2005) study income redistribution. More recently, Fischer and Huerta (2021) embed a static heterogeneous agent model with endogenous credit constraints in a probabilistic voting setup to study financial and labor regulations. Huerta (2022) works with a similar model to rationalize size-contingent labor policies.

2 Model

2.1 Preferences

Time is continuous. The economy is populated by a continuum of individuals that are heterogeneous in wealth a_t. At each point in time, the state of the economy is given by the wealth distribution $g_t(a)$. The cumulative wealth distribution is denoted by $G_t(a)$. Agents have standard preferences over utility flows from consumption c_t with a discount rate $\rho \geq 0$:

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_t) dt, \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \gamma \geq 1. \tag{1}$$

2.2 Technology

As in Bernanke and Gertler (1989), there are different production technologies for output and capital. Agents can decide to invest in a capital-producing firm (a 'project') and become entrepreneurs. The capital production technology requires $I > 0$ units of output to produce $R > 0$ units of capital.[4]

There is a representative output-producing firm that operates a standard homogeneous production function $F(K_t, L_t)$, where K_t and L_t are aggregate capital and labor at t, respectively. The production function is given by $F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$, with $\alpha \in (0, 1)$. Rewrite the production function in per capita terms, $f(k_t) = k_t^\alpha$, where k_t is capital per capita in period t.

All agents are endowed with $\bar{l}$ units of labor and work for the representative firm at a wage w_t. Thus aggregate labor is $L_t = \bar{l}$. Entrepreneurs sell their capital to the representative firm at a price p_t. Denote by η_t the mass of agents operating a firm at period t. Then output per capita is given by $\mathcal{Y}_t = f(k_t = R \cdot \eta_t)$. The factor markets are competitive. Thus, the price of capital is $p_t = \alpha k_t^{\alpha-1}$ and the wage rate is $w_t = (1-\alpha)k_t^\alpha$.

Alternatively, capital k_t can be interpreted as intermediate goods (e.g. machineries) that are used by the representative firm to produce the final good, $\mathcal{Y}_t$. Throughout the paper, I refer to p_t

[4] Matsuyama (2004) develops an overlapping generations model with a similar assumption regarding technologies.

as the price of the intermediate-output good or just as the 'price of output'.

2.3 Budgets

There is a time-dependent capital tax $\tau_t \in [\underline{\tau}, 1)$ which is the main policy intervention instrument in the economy. Thus post-tax assets are given by: $(1-\tau_t)a$. Each period, τ_t is determined through competitive elections Section 5 describes the political mechanism.

Individual wealth, a_t evolves according to

$$\dot{a}_t = \pi_t(a) + r(1-\tau_t)a + w_t\bar{l} + T_t - c_t. \tag{2}$$

where r is a fixed international interest rate, T_t are transfers from the government and $\pi_t(a_t)$ are firm profits of an entrepreneur with assets a_t. Thus savings, $\dot{a}_t$ equals firm profits plus net interest and labor income minus consumption.

Individuals also face a borrowing limit

$$a_t \geq \underline{a}, \tag{3}$$

where $-\infty < \underline{a} < 0$.

The government's budget constraint is balanced period by period:[5]

$$T_t = r \cdot \int \tau_t a dG_t(a). \tag{4}$$

In order to finance a firm, entrepreneurs may apply for a loan, $I-(1-\tau)a_t$. There is competitive banking system which provides loans and has unlimited access to international funds at fixed interest rate r. Agents asking for a loan must deposit their endowments in the banking system at the beginning of each period. There is a moral hazard problem in the credit market: agents may abscond with the loan instead of investing in a firm. As a result, banks limit access to credit by

[5]It is assumed that the government deposits its tax income at the international interest rate r.

imposing a minimum collateral to obtain a loan, $\hat{a}_t$ (see section 2.4). Thus firm profits as function of wealth a_t are given by

$$\pi_t(a_t) \equiv (p_t R - rI) \cdot \mathbb{1}_{a_t \geq \hat{a}_t}. \tag{5}$$

When an agent defaults with the loan, he loses the wealth used as collateral at the beginning of the period. Thus, his disposable income after default is given by $I - (1-\tau)a_t + w_t + T_t$.[6]

2.4 Credit conditions

Any agent that receives a loan at period t must satisfy the following incentive compatibility condition:

$$p_t R - rI + r(1-\tau_t)a + w_t + T_t \geq (I - (1-\tau_t)a_t) + w_t + T_t. \tag{6}$$

That is, his disposable income from investing in a firm must be higher or equal than what he would obtain if he defaults with the loan. This condition defines a minimum wealth to obtain a loan $\hat{a}_t$, which is implicitly defined by

$$\hat{a}_t = \left(I - \frac{p_t R}{1+r} \right) \frac{1}{1-\tau_t}. \tag{7}$$

Given $\hat{a}_t$ and the wealth distribution $G_t(a)$, the fraction of agents that access the credit market ('credit penetration') is $\eta_t = 1 - G_t(\hat{a}_t)$. Therefore production is $\mathcal{Y}_t = R^{\alpha}(1-G_t(\hat{a}_t))^{\alpha}$ and the output price is $p_t = \alpha R^{\alpha-1}(1-G_t(\hat{a}_t))^{\alpha-1}$. Hence, condition (7) implicitly defines $\hat{a}_t$ as a function of τ_t and the cumulative wealth distribution G_t, i.e $\hat{a}_t \equiv \hat{a}(\tau_t, G_t)$. The wage rate is $w_t = (1-\alpha)R^{\alpha}(1-G_t(\hat{a}_t))^{\alpha}$.

[6]Fischer et al. (2019) develop an overlapping generations model with a similar structure for financial markets.

2.5 Occupational choice

Any agent that has access to credit ($a \geq \hat{a}_t$) decides to invest in firm and to become an entrepreneur if the following participation constraint is satisfied:

$$\Pi_t \equiv p_t R - rI \geq 0, \tag{8}$$

which does not depend on the agent's wealth.[7] If this condition does not hold, then agents prefer to remain as workers even when they could start a firm. This condition asks that the net present value of the project is positive. Although not essential, to simplify the exposition I make the following assumption:

Assumption 1 $p(R)R > rI \Leftrightarrow R > \left(\frac{rI}{\alpha}\right)^{1/\alpha}$.

Since the output price p_t is decreasing in k_t, this is a sufficient condition for (8) to hold. Intuitively, it asks that the return of the project is sufficiently high (even under the minimum price $p(R)$) such that agents with access to credit are willing to invest in a firm.

To understand agents decisions, note that the budget constraint of entrepreneurs can be written in terms of debt, $d(a_t) = I - (1-\tau_t)a_t$ as follows:

$$\dot{a}_t = p_t R - rd(a_t) + w_t \bar{l} + T_t - c_t.$$

Observe that the amount of assets required to self-finance a firm are $\frac{I}{1-\tau_t}$. At each period t, the model sort agents into three groups. First, agents with $a_t < \hat{a}_t$ cannot invest in firm and remain as workers, while those with $a_t \in \left[\hat{a}_t, \frac{I}{(1-\tau_t)}\right)$ ask for a loan $d_t(a_t) = I - (1-\tau_t)a_t$ and become entrepreneurs.[8] Richer agents with $a_t \geq \frac{I}{(1-\tau_t)}$ self-finance a firm and deposit the rest in the banking system. For these agents the loan becomes negative, which captures the fact that

[7] Note that Π_t are firm profits conditional that the individual can start a firm ($a \geq \hat{a}_t$). More generally, equation (5) defines firm profits as a function of an agent wealth (a): $\pi_t(a) = (p_t R - r)\mathbb{1}_{a \geq \hat{a}_t} = \Pi_t \cdot \mathbb{1}_{a \geq \hat{a}_t}$.

[8] Since banks impose the incentive compatibility constraint (6) when setting $\hat{a}_t$, in equilibrium agents that ask for a loan honor the credit contract and invest in a firm.

they deposit their wealth at the interest rate r after investing I in a firm. Figure 1 illustrates these features.

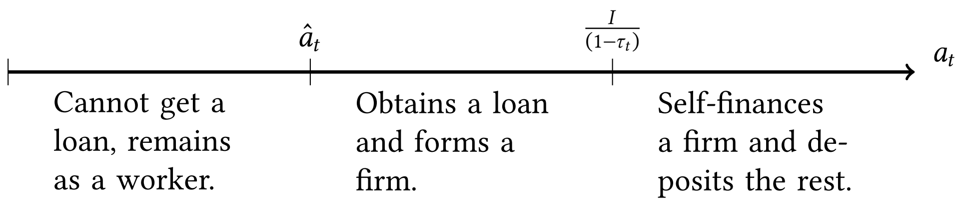


Figure 1: Occupational choice as a function of wealth.

The individual problem is written compactly as follows,

$$
\begin{aligned}
&\max_{\{c_t\}_{t=0}^{+\infty}} \left\{ \int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} \right\} \\
s.t \quad & \dot{a}_t = \pi_t(a_t) + r(1-\tau_t)a_t + w_t\bar{l} + T_t - c_t, \\
& \pi_t(a_t) = (p_t R - rI)\mathbb{1}_{a_t \geq \hat{a}_t}, \\
& a_t \geq \underline{a}.
\end{aligned}
$$

3 Competitive equilibrium

This section characterizes the equilibrium that arises by taking a fixed tax rate over time $\tau_t = \tau$. This is a useful starting point before analyzing the political equilibrium where $\{\tau_t\}_{t=0}^{\infty}$ is determined through repeated elections.

The timing of events in period t is as follows. 1) Agents observe a_t at the beginning of the period. 2) Banks observe the wealth distribution, G_t and define credit constraints, $\hat{a}_t$. 3) Given the minimum wealth required for a loan, agents make their occupational choice. 4) Workers and entrepreneurs make their saving consumption-decisions.[9]

[9]Since agents decide their occupational choice after banks have announced $\hat{a}(\tau, G_t)$, the only state variable in their utility maximization problem is their wealth a_t.

3.1 Saving and consumption decisions

The following lemma makes use of the CRRA utility function to derive closed form solutions for the saving policy, $s_t(a_t, \tau_t)$ and consumption policy function, $c_t(a_t, \tau_t)$.[10] In the rest of the paper, the partial derivative in terms of some variable x, $\frac{d}{dx}(\cdot)$ is denoted by $d_x(\cdot)$.

Lemma 1

1. *Individuals' consumption-saving decision arise from the Hamilton-Jacobi-Bellman (HJB) equation:*

$$\rho v(a_t) = \max_{c_t} \left\{ u(c_t) + d_a v(a_t)(y_t(a_t, \tau_t) - c_t) \right\}, \tag{9}$$

$$s.t \qquad \dot{a}_t = y_t(a_t, \tau_t) - c_t \tag{10}$$

$$a_t \geq \underline{a} \tag{11}$$

where $y_t(a_t, \tau_t) = \pi_t(a_t) + r(1 - \tau_t)a_t + w_t\bar{l} + T_t$.

2. *The consumption policy function is:*

$$c_t(a_t, \tau_t) = (1 - \theta_t) \cdot y_t(a_t, \tau_t). \tag{12}$$

3. *The saving policy function is*

$$s_t(a_t, \tau_t) = \theta_t \cdot y_t(a_t, \tau_t), \tag{13}$$

where $\theta_t \equiv \frac{1}{\gamma}\left(1 - \frac{\rho}{r(1-\tau_t)}\right)$.

Thus individuals consume and save a fraction of their disposable income $y_t(a_t, \tau_t)$. Note that I have defined θ_t as the saving rate out of disposable income at period t. Intuitively, when the 'effective rate' at which agents can save, $r(1 - \tau_t)$ is larger than the discount factor, ρ, they save a

[10] Recall that by now the tax rate is fixed $\tau_t = \tau$. However, in the rest of section 3, the results are written in terms of a path of tax rates $\{\tau_t\}_{t=0}^{+\infty}$. This will be useful in section 6 when the tax rate is endogenous and defined by competitive elections each period.

positive amount of their income. Otherwise, agents disaccumulate assets. Recall that firm profits of an entrepreneur with wealth a_t are given by $\pi_t(a_t) = (p_t R - rI)\mathbb{1}_{a_t \geq \hat{a}_t} \geq 0$. Hence, entrepreneurs ($a_t \geq \hat{a}_t$) consume and save/dissave more than those agents that remain as workers ($a_t < \hat{a}_t$).

3.2 Aggregation and equilibrium

I start by describing the evolution of aggregate assets $A_t \equiv \int a dG_t(a)$ given the cumulative distribution $G_t(a)$. Integrating the saving policy across all agents, aggregate wealth evolves according to:

$$\dot{A}_t = \theta_t[rA_t + w_t\bar{l} + (p_t R - rI)(1 - G_t(\hat{a}_t))], \tag{14}$$

where I have defined the saving rate out of disposable income: $\theta_t \equiv \frac{[r(1-\tau_t)-\rho]}{\gamma r(1-\tau_t)}$. Thus the change in aggregate wealth is just a fraction θ_t of aggregate income which corresponds to the sum of: total interest income rA_t, total labor income $w_t\bar{l}$, and total firm profits $(p_t R - rI)(1 - G_t(\hat{a}_t))$. The following lemma characterizes the evolution of the cumulative wealth distribution.

Lemma 2 *The evolution of the cumulative wealth distribution $G_t(a)$ is characterized by the Kolmogorov Forward (KF) equation:*

$$d_t G_t(a) = -G_t(\hat{a}_t) \cdot s_t^W(a, \tau_t) d_a G_t(a) - (1 - G_t(\hat{a}_t)) \cdot s_t^E(a, \tau_t) d_a G_t(a), \tag{15}$$

where $s_t^W(a, \tau_t) = \theta_t(a + w_t\bar{l} + T_t) \cdot \mathbb{1}_{a<\hat{a}_t}$, $s_t^E(a, \tau_t) = \theta_t(a + \Pi_t + w_t\bar{l} + T_t) \cdot \mathbb{1}_{a\geq\hat{a}_t}$ and $\hat{a}_t$ satisfies (7).

The first term of the KF equation captures the inflow and outflows due to continuous movements in the wealth of workers, the second term does the same for entrepreneurs.[11]

Given an initial assets distribution $G_0(a)$, a *competitive equilibrium* is such that: i) agents solve the HJB equation (9) by taking as given the price of output p_t and wages w_t, ii) prices are given by: $p_t = \alpha R^{\alpha-1}(1 - G_t(\hat{a}_t))^{\alpha-1}$ and $w_t = (1-\alpha)R^{\alpha}(1 - G_t(\hat{a}_t))^{\alpha}$, iii) the minimum collateral to obtain

[11] I express the KF equation in terms of the cumulative wealth distribution G_t, since the main endogenous variables of the model $(w_t, p_t, \hat{a}_t)$ are functions of credit penetration $1 - G_t(\hat{a}_t)$. Thus is easier to work with the CDF instead of using the density function, g_t.

credit $\hat{a}_t$ satisfies (7), and iv) the evolution of the cumulative wealth distribution is given by the KF equation (15).

3.3 Transition dynamics

In this section I characterize the transition dynamics of the economy given an initial wealth distribution $G_0(a)$. The following proposition describes the transition dynamics of the cumulative wealth distribution $G_t(a)$, the minimum collateral required to obtain a loan $\hat{a}_t$ and aggregate capital k_t.

Proposition 1 *Assume that the initial cumulative wealth distribution G_0 is such that initial credit penetration satisfies: $\eta_0 \in (0,1)$. Take $\tau < \frac{r-\rho}{r}$, then: i) $d_t G_t(a) \leq 0\ \forall a$, ii) $d_t \hat{a}_t \geq 0$, iii) $d_t k_t \geq 0$, and iv) $d_t A_t \geq 0$. Otherwise, if $\tau > \frac{r-\rho}{r}$ the signs of i) to iv) are reversed.*

In order to interpret proposition 1, consider momentarily a discrete time model where the length of a period is Δ. Suppose that $\tau < \frac{r-\rho}{r}$, then i) states that: $G_t(a) \geq G_{t+\Delta}(a), \forall a$. Thus $G_{t+\Delta}(a)$ first-order stochastic dominates (FOSD) $G_t(a)$. That is, the cumulative wealth distribution shifts to the right. Figures 6 and 7 in section 8.2 in the appendix illustrate this feature. The intuition is that when $\tau < \frac{r-\rho}{r}$, then the saving rate out of disposable income is positive, $\theta_t \geq 0$. Thus, $s_t(a,\tau) \geq 0, \forall a, \forall t$. Since agents are saving a positive fraction of assets each period, the wealth of each agent is weakly increasing over time and the cumulative wealth distribution shifts right. Moreover, aggregate wealth is increasing over time as stated by item iv).

Secondly, to understand ii) rewrite condition (7) as follows:

$$\hat{a}_t = \left(I - \frac{\alpha R^{\alpha}(1 - G_t(\hat{a}_t))^{\alpha-1}}{1+r} \right) \frac{1}{1-\tau_t}. \tag{16}$$

Note that the right-hand side of (16) increases if $G_t(a)$ shifts right. Since $G_{t+\Delta}$ FOSD G_t, $\hat{a}_t$ has to increase for (16) to hold, that is it must be that $\hat{a}_{t+\Delta} > \hat{a}_t$. The intuition is that when the wealth distribution shift right then the price of output at $t+\Delta$ is lower for any given $\hat{a}_t$. As a result, banks tighten credit conditions, i.e increase $\hat{a}$, to enforce the incentive compatibility constraint.

Finally, the evolution of capital k_t over time depends on the dynamics of credit penetration $\eta_t = 1 - G_t(\hat{a}_t)$. From previous discussion, there are two opposite effects over time: i) $G_t(a)$ shifts right and thus η_t increases for a given $\hat{a}_t$, and ii) $\hat{a}_t$ increases which reduces η_t for a given distribution $G_t(a)$. Proposition 1 shows that the distribution effect dominates and thus, $\eta_{t+\Delta} \geq \eta_t$. As a result, the economy accumulates wealth over time $k_{t+\Delta} \geq k_t$.

Proposition 1 yields the following corollary.

Corollary 1 *Assume that the initial cumulative wealth distribution G_0 is such that initial credit penetration satisfies: $\eta_0 \in (0, 1)$. Take $\tau < \frac{r-\rho}{r}$, then $k_\infty = R$. Otherwise, if $\tau > \frac{r-\rho}{r}$, then $k_\infty = 0$.*

Corollary 1 characterizes the steady-state level of capital. When $\tau < \frac{r-\rho}{r}$, credit penetration increases over time. At some point in time, $\eta = 1$ and thus the economy reaches it steady-state level of capital, $k_\infty = R$. In contrast, if $\tau > \frac{r-\rho}{r}$ capital decreases over time and $k_\infty = 0$.

4 Political Preferences

This section describes the political preferences for tax policies. At the beginning of each period t, agents observe their wealth endowments a_t and the economy's wealth distribution G_t. After observing (a_t, G_t) and before making consumption-saving decisions, agents define their preferred tax rate at period t, denoted by $\tau^*(a_t, G_t)$. When choosing $\tau^*(a_t, G_t)$, agents take into into account the impact that a given tax rate has on the minimum wealth required for a loan $\hat{a}_t$ and thus, on occupational choice at period t. Individual political preferences are defined on the basis of the impact of the tax rate on disposable income $y(a_t, \tau, G_t)$, while taking transfers T_t as given. Thus, given (a_t, G_t) and T_t, the preferred tax rate is chosen by maximizing the disposable income in period t.

4.1 Statics

Before describing individual political preferences, the following lemma presents some useful statics.

Lemma 3 *Given some wealth distribution G_t, an increase in the tax rate τ_t leads to:*

1. *A lower equilibrium wage w_t.*

2. *A higher price of output p_t.*

3. *A higher minimum wealth to obtain a loan $\hat{a}_t$.*

The lemma describes the effect of increasing τ_t on period's t factor prices and minimum wealth. The intuition for the results are as follows. There are two opposing effects. First, there is a direct effect of an increased tax rate on $\hat{a}_t$. Since a higher tax rate reduces the assets that can be used as a collateral (*collateral effect*), banks increase $\hat{a}_t$ to satisfy the incentive compatibility constraint. Secondly, tightened credit constraints means that less less capital is produced at t. Lower capital implies a higher marginal return of capital which increases p_t (*price effect*), pushing down $\hat{a}_t$. The lemma shows that this last effect cannot counteract the collateral effect. Thus, the minimum wealth to obtain a loan increases when the tax rate increases. As a result, less capital is produced, which increases p_t, but reduces w_t.

4.2 Individual political preferences

The preferred tax rate $\tau^*(a_t, G_t)$ of an agent with wealth a_t and given a cumulative wealth distribution G_t solves the following problem:

$$\tau^*(a_t, G_t) = \underset{\tau \in [\underline{\tau}, 1)}{\arg\max} \left\{ y(a_t, \tau, G_t) = y_t^W(a_t, \tau) \cdot \mathbb{1}_{a_t < \hat{a}(\tau, G_t)} + y_t^E(a_t, \tau) \cdot \mathbb{1}_{a_t \geq \hat{a}(\tau, G_t)} \right\}, \tag{17}$$

where $\hat{a}(\tau, G_t)$ solves equation (7), and $y_t^W(a_t, \tau) \equiv r(1-\tau)a_t + w_t + T_t$ and $y_t^E(a_t, \tau) \equiv \Pi_t + r(1-\tau)a_t + w_t + T_t$.

Proposition 2 *The preferred tax rate function $\tau^*(a_t, G_t)$ is as follows:*

$$\tau^*(a_t, G_t) = \begin{cases} \underline{\tau} & \text{if } a_t \leq \hat{a}(\underline{\tau}, G_t) \\ \Psi(a, G_t) & \text{if } a_t \in (\hat{a}(\underline{\tau}, G_t), \tilde{a}(G_t)) \\ \underline{\tau} & \text{if } a \geq \tilde{a}(G_t) \end{cases} \tag{18}$$

where $\Psi(a, G_t) \in [\underline{\tau}, 1)$ is a continuous and strictly increasing function in a implicitly defined by

$$a = \hat{a}(\Psi(a, G_t), G_t). \tag{19}$$

The threshold, $\tilde{a}(G_t) \in (\hat{a}(\underline{\tau}, G_t), +\infty)$ is given by

$$d_\tau(y_t^E(\tilde{a}(G_t), \tau)) = 0. \tag{20}$$

Proposition 2 characterizes the preferred tax rate function, $\tau^*(a_t, G_t)$ in terms of individual assets a_t. Figure 2 illustrates the proposition. In the figure, the minimum tax rate is $\underline{\tau} = -0.5$. In what follows I discuss the results of the proposition.

The proposition shows that there are three classes of agents whose political preferences depend on their wealth: i) *poor* agents with $a_t < \hat{a}_t(\underline{\tau}, G_t)$, ii) *intermediate-wealth* agents with assets $a_t \in (\hat{a}_t(\underline{\tau}, G_t), \tilde{a}(G_t))$, and iii) *wealthy* agents with $a_t > \tilde{a}(G_t)$.

Individual political preferences are as follows. First, *poor* agents prefer the minimum tax rate $\underline{\tau}$. Secondly, the proposition shows that the preferred tax rate is an increasing function in wealth for *intermediate-wealth* individuals, as shown in figure 2. Thirdly, *wealthy* agents are aligned with the *poor* class and prefer the minimum tax rate $\underline{\tau}$.

Figure 3 depicts the disposable income $y(a_t, \tau, G_t)$ as a function of τ and for four levels of wealth: low (a_L), lower-middle (a_{LM}), upper-middle (a_{UM}) and high (a_H). Agents with a_L belong to the *poor* class ($a_L < \hat{a}_t(\underline{\tau}, G_t)$), those with a_{LM} and a_{UM} are *intermediate-wealth* agents ($a_{LM}, a_{UM} \in (\hat{a}_t(\underline{\tau}, G_t), \tilde{a}(G_t))$) and individuals with a_H are from the *wealthy* class ($a_H \geq \tilde{a}(G_t)$).

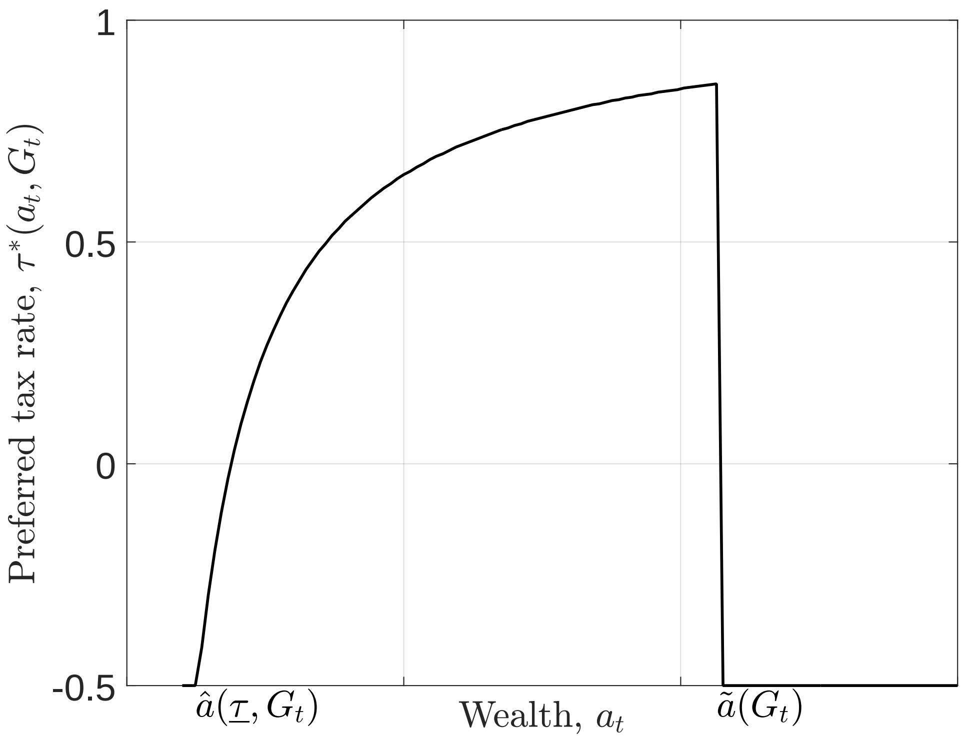


Figure 2: Preferred tax rate function, $\tau^*(a_t, G_t)$.

The black dots in figure 3 indicate the points at which the disposable income attains its maximum for the different wealth levels. Increasing the tax rate creates three different effects: i) it reduces capital income $r(1-\tau)a_t$ (*capital income effect*), ii) it decreases the equilibrium wage w_t (*wage effect*), and it increases the price at which firms can sell the intermediate output p_t (*price effect*). The shape of the disposable income curve depends on the magnitude of these three effects for the different wealth levels.

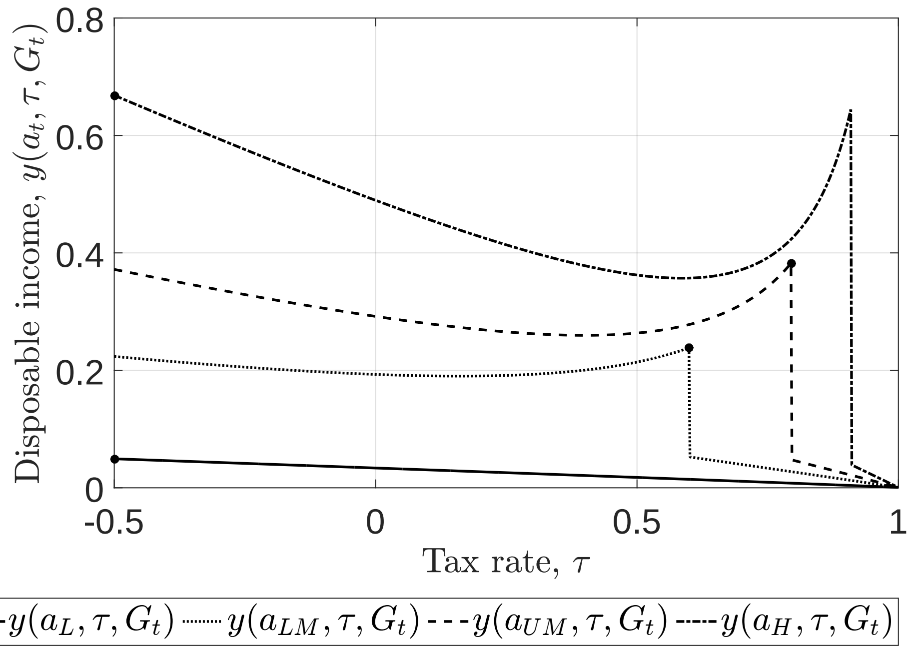


Figure 3: Disposable income, $y_t(a_t, \tau, G_t)$ as a function of τ for wealth levels: $a_L < a_{LM} < a_{UM} < a_H$.

First, the solid line represents an agent from *poor* class, $a_L < \hat{a}(\underline{\tau}, G)$. Even under the lowest possible tax rate he will not have sufficient collateral to obtain credit and form a firm. Inevitably, a *poor* agent remains as a worker, i.e. $y_t = y_t^W$. The *wage* and *capital income* effects mean that his disposable income is decreasing in τ, as shown in the figure. In consequence, his preferred tax rate is $\tau^*(a_L, G_t) = \underline{\tau}$.

Agents with $a_t > \hat{a}(\underline{\tau}, G_t)$ will have access to credit if the tax rate is not too high. By assumption 1, forming a firm is profitable. Thus agents with wealth $a_t > \hat{a}(\underline{\tau}, G_t)$ prefer a tax rate that allows them to become entrepreneurs, i.e. $\tau^*(a_t, G_t)$ belongs to the set of tax rates that satisfy: $a_t \geq \hat{a}(\tau, G_t)$. The question that remains is whether these individuals prefer a low or a high tax rate in this set.

Note that y_t is convex to the left of the interior peak. On the one hand, for high values of τ only few agents have access to credit and can start a firm ($\hat{a}_t$ is high), i.e. k_t is low. Recall that the price of the intermediate good is equal to the marginal productivity of capital, which is decreasing in k_t at a decreasing rate. Therefore, a marginal increase in the tax rate when τ is high means that more agents are excluded from the credit market (i.e. k_t becomes even lower), which leads to a substantial increase in p_t (*price effect*). This *price effect* is more pronounced for higher tax rates, as can be inferred by comparing the interior peaks of y_t for agents with assets a_{LM}, a_{UM} and a_H. On the other hand, for low values of τ many agents have access to credit (k_t is high). Thus an increase in the tax rate does not produce a significant increase in p_t. As a result, the *capital income effect* dominates and a lower τ means a higher y_t.

The dotted and dashed lines correspond to agents in the *intermediate-wealth* class, $a_{LM}, a_{UM} \in (\hat{a}(\underline{\tau}, G_t), \tilde{a}(G_t))$. As long as τ is not too high (to the left of the black dots), these agents have access to credit and become entrepreneurs, i.e. $y_t = y_t^E$. The interior peaks of the curves correspond to the points at which τ is such that $a_{LM} = \hat{a}(\tau, G_t)$ and $a_{UM} = \hat{a}(\tau, G_t)$. To the right of those rates these agents lose access to credit ($y_t = y_t^W$) and thus y_t becomes decreasing in τ, as shown in the figure.

For agents with *intermediate-wealth* levels the *price effect* dominates the *capital income* and *wage* effects. Thus these agents prefer the maximum tax rate that allows them to become entrepreneurs, i.e. $\tau^*(a, G_t)$ is such that $a_{LM} = \hat{a}(\tau^*, G_t)$ and $a_{UM} = \hat{a}(\tau^*, G_t)$. Out of agents with *intermediate-wealth* levels, wealthier ones can afford to pay a higher tax rate to benefit from a larger 'jump' in their incomes due to the *price effect.* That is, the interior peak of y_t for an agent with a_{UM} is higher than the one of agents with assets a_{LM}. Therefore, the preferred tax function $\tau^*(a, G_t)$ is increasing in wealth for *intermediate wealth* agents and is characterized by the function $\Psi(a, G_t)$.

Lastly, the dotted-dashed line represents a *wealthy* agent with $a_H > \tilde{a}(G_t)$. In this case, the *capital income effect* dominates the *price effect.* Therefore, his preferred tax rate is the minimum tax rate $\underline{\tau}$, as shown by the black dot in the figure. By continuity, it can be shown that there exists some wealth threshold $\tilde{a}(G_t) \in (\hat{a}(\underline{\tau}, G_t), +\infty)$ such that agents with $a_t \in (\hat{a}(\underline{\tau}, G_t), \tilde{a}(G_t))$ prefer a tax rate characterized by $\Psi(a_t, G_t)$ as defined by (19), while individuals with $a_t \geq \tilde{a}(G_t)$ prefer $\underline{\tau}$.

To conclude, note that the individual political preferences depicted in figure 3 do not satisfy the single-peaked property. In consequence, I cannot apply the median-voter theorem to find an equilibrium policy platform. Therefore, I opt for a different modelling strategy to aggregate individual political preferences, replacing the median-voter approach for a probabilistic voting model along the lines of Persson and Tabellini (2000).

In this setting, voters support a party not only for its proposed policy, but also for other characteristics (like ideology) that are orthogonal to tax policies. Parties announce their policy platforms under uncertainty about the preferences of each voter, that is there is uncertainty about the outcome of elections. Thus, probabilistic voting smooths the objective function of the parties by making the expected number of votes a smooth function of the tax policy. In section 5, I describe in detail the political mechanism that endogenizes the tax rate over time.

4.3 Intrepretation: pro-business versus pro-worker policies

This section discusses the orientation of the different tax policies. As shown in previous section, a high tax rate suppresses wages and increases the price of the intermediate output good, favoring businesses. Thus a high τ is interpreted as a *pro-business* regulatory framework.[12] In contrast, a low tax rate means high wages, but a low price at which businesses can sell their output. Hence, in this case the regulatory framework is said to be *pro-worker.*

This interpretation is consistent with the one commonly used in the macro literature. For

[12]Recall however that not all entrepreneurs prefer a high tax rate. The richest ones support a low tax rate to increase their capital income. From the point of view of firms, a higher tax rate increases profits Π_t as long as firm's assets are such that $a_t \geq \hat{a}(\tau, G_t)$.

instance, Itskhoki and Moll (2019) study optimal Ramsey interventions in an environment with financial frictions that features a distributional conflict between workers and capitalists. The optimal policy is stage-contingent, pro-worker at the early stages of development, but pro-business close to the steady state. The political orientation of the policy intervention is also defined in terms of whether it restricts wages, pro-business, or pro-worker if it boosts wages.

In my model, there is an additional endogenous measure that defines the nature of the policy intervention: the price of intermediate output good p_t. A higher p_t benefits only businesses, while workers are made worse off due to a lower wage.

5 Political Mechanism

In this section I present a politico-economy mechanism to aggregate the conflicting political interests described in section 4. I embed the basic environment presented in section 2 into a political economy framework with probabilistic voting along the lines of Persson and Tabellini (2000).

Several papers implement probabilistic voting in dynamic models to study the political support for different types of policies. Hassler et al. (2005) study income redistribution, while Gonzalez-Eiras and Niepelt (2008) and Sleet and Yeltekin (2008) focus on social security. Farhi et al. (2012) analyze taxation and Song et al. (2012) examine fiscal policy. In what follows I detail how the equilibrium tax rate τ_t is determined each period t through probabilistic voting.

5.1 Politicians

Each period t, the electoral competition takes place between two office-seeking parties, A and B. Both parties simultaneously and noncooperatively announce their electoral platforms, $\tau_t^A \in [\underline{\tau}, 1)$ and $\tau_t^B \in [\underline{\tau}, 1)$. At the beginning of each period, parties observe the wealth distribution G_t and make their announcements to maximize their probability of wining the election. Parties' only relevant state variable is the current wealth distribution, G_t. As in Song et al. (2012), there are new elections every period, thus parties cannot make credible commitments over future tax rates. Probabilistic voting smooths the political objective function by introducing uncertainty from the parties point of view (Lindbeck and Weibull, 1987). The specific sources of uncertainty are described in next section.

5.2 Voters

At the beginning of each period t, agents observe their assets a_t and the economy's wealth distribution, G_t. Given the state (a_t, G_t), agents can anticipate the effects that a given tax policy τ has on occupational choice and thus on their disposable income $y_t(a_t, \tau, G_t)$. They vote before making consumption-saving decisions and to maximize $y_t(a_t, \tau, G_t)$. To simplify the exposition, in the rest of this section I omit the dependence of y_t on G_t.

Voters have heterogeneous political preferences not only over tax policies (as shown in section 4.2), but also over other policy dimensions that are orthogonal to tax policies. Specifically, each voter with assets a_t has ideological preferences denoted by v_t. Thus, each period there is a continuum of agents indexed by (a_t, v_t).

Voter (a_t, v_t) votes for party A if:

$$y_t(a_t, \tau_t^A) > y_t(a_t, \tau_t^B) + \delta_t + \sigma_t(a_t, v_t), \tag{21}$$

where δ_t reflects the general popularity of party B, which I assume to be uniformly distributed on $[-1/(2\psi), 1/(2\psi)]$. The value of δ_t is revealed after the parties announce their platforms. Therefore, parties announce their policies under uncertainty about the results of the election.

The variable $\sigma_t(a_t, v_t)$ corresponds to the ideological preference of voter (a_t, v_t) for party B. The distribution of $\sigma_t(a_t, v_t)$ is assumed to be uniform on $[-1/(2\phi), 1/(2\phi)]$. Parties know the ideological distributions before announcing their policy platforms. The term $\delta_t + \sigma_t(a_t, v_t)$ captures the relative 'appeal' of party B in period t. That is, the inherent bias of voter v_t with wealth a_t in period t for party B, irrespective of the proposed policy platforms.

Probabilistic voting has been used in similar static models with heterogeneous agents, endogenous credit constraints and occupational choice, where individuals vote based on an *ex-ante* position in society (*ex-ante* occupation) (see Fischer and Huerta, 2021; Huerta, 2022). This paper incorporates two additional challenges. First, agents vote before knowing their occupations, which means that the wealth distribution G_t is a state variable.[13] Thus when voting they anticipate which position in society they will occupy given some τ_t. Secondly, even when new elections take place each period t, the equilibrium platform depends on the endogenous wealth distribution G_t, which is a function of the entire history of tax policies.

5.3 Timing

The timing of events at period t is as follows: 1) Party A and B simultaneously and noncooperatively announce their electoral platforms, τ_t^A and τ_t^B after observing G_t. 2) Elections are held, in which voters choose between the two parties after observing their wealth a_t and G_t. 3) The elected party implements his announced tax rate. 4) Given the winning platform τ_t and wealth distribution G_t, banks define the minimum wealth required for a loan, $\hat{a}_t$. 5) After observing τ_t and $\hat{a}_t$, agents make their occupational choice and their consumption-saving decisions.

5.4 The political objective function

I study the policy outcome under a proportional electoral system. Thus a party needs more than 50% of total votes to win the election. In what follows I describe the problem of politicians.

It is useful to start by identifying the 'swing voter' ($v_t = V_t$) for each level of wealth a_t. That is, the voter with wealth a_t who is indifferent between the two parties:

$$\sigma_t(a_t, v_t = V_t) = y_t(a_t, \tau_t^A) - y_t(a_t, \tau_t^B) - \delta_t.$$

[13] This is a distinctive feature of my model. Typically, the papers that use probabilistic voting models take the different groups of agents (e.g. old/young, worker/entrepreneur) as given before elections take place. Usually the different groups differ in the parameter that regulates the dispersion of the ideological preferences (ϕ). In contrast, in my setting these groups are defined endogenously after the equilibrium platform has been set. Interestingly, when voting agents anticipate the implications of a given policy platform for occupational choice and choose their preferred party accordingly.

All agents with wealth a_t whose ideology is such that $\sigma_t(a_t, v_t) < \sigma_t(a_t, V_t)$ vote for party A, while the rest vote for B. Thus conditional on the popularity of party B, δ_t, the fraction of agents with wealth a_t that vote for party A in period t is given by:

$$\pi_t^A(a_t, \tau_t^A|\delta_t) = Prob[\sigma_t(a_t, v_t) < \sigma_t(a_t, V_t)] = \phi \cdot [y_t(a_t, \tau_t^A) - y_t(a_t, \tau_t^B) - \delta_t] + \frac{1}{2}.$$

The probability that party A wins the election when it announces τ_t^A in period t, $p^A(\tau_t^A)$ is obtained by integrating in a_t and δ_t:

$$p^A(\tau_t^A) = Prob\left[\int \pi_t^A(a, \tau_t^A|\delta_t)dG_t(a) \geq \frac{1}{2}\right],$$

where the probability is taken with respect to δ_t. Rearranging terms:

$$\begin{aligned} p^A(\tau_t^A) &= Prob\left[\delta_t \leq \int_{a_t<\hat{a}(\tau_t^A,G_t)} (y_t^W(a, \tau_t^A) - y_t^W(a, \tau_t^B))dG_t(a) + \int_{a_t\geq\hat{a}(\tau_t^A,G_t)} (y_t^E(a, \tau_t^A) - y_t^E(a, \tau_t^B))dG_t(a)\right], \\ &= \psi\left[\int_{a_t<\hat{a}(\tau_t^A,G_t)} (y_t^W(a, \tau_t^A) - y_t^W(a, \tau_t^B))dG_t(a) + \int_{a_t\geq\hat{a}(\tau_t^A,G_t)} (y_t^E(a, \tau_t^A) - y_t^E(a, \tau_t^B))dG_t(a)\right] + \frac{1}{2}. \end{aligned}$$

Since both parties maximize the probability of wining the election in period t, the Nash equilibrium is characterized by:

$$\begin{aligned} \tau_t^A &= \arg\max_{\tau^A\in[\underline{\tau},1)} \left\{\int_{a<\hat{a}(\tau^A,G_t)} (y_t^W(a, \tau^A) - y_t^W(a, \tau^B))dG_t(a) + \int_{a\geq\hat{a}(\tau^A,G_t)} (y_t^E(a, \tau^A) - y_t^E(a, \tau^B))dG_t(a)\right\}, \\ \tau_t^B &= \arg\max_{\tau^B\in[\underline{\tau},1)} \left\{\int_{a<\hat{a}(\tau^B,G_t)} (y_t^W(a, \tau^B) - y_t^W(a, \tau^A))dG_t(a) + \int_{a\geq\hat{a}(\tau^B,G_t)} (y_t^E(a, \tau^B) - y_t^E(a, \tau^A))dG_t(a)\right\}. \end{aligned}$$

The parties' problems are symmetric. Thus the policy platforms converge in equilibrium to the same tax rate, τ_t that maximizes the weighted income–the *political objective function* ($W(\tau, G_t)$)– given the cumulative wealth distribution, G_t:

$$\tau_t = \arg\max_{\tau\in[\underline{\tau},1)} \left\{W(\tau, G_t) \equiv \int_{a_t<\hat{a}(\tau,G_t)} y_t^W(a, \tau)dG_t(a) + \int_{a_t\geq\hat{a}(\tau,G_t)} y_t^E(a, \tau)dG_t(a)\right\}. \quad (22)$$

Note that when choosing τ_t parties take into account the effects on the minimum collateral to obtain a loan, $\hat{a}(\tau_t, G_t)$ which in turn, determines occupational choice and the demographic weights of workers and entrepreneurs.

The political objective function is written as follows:

$$
\begin{aligned}
W(\tau, G_t) &= \int_{a<\hat{a}(\tau,G_t)} (r(1-\tau)a + w_t\bar{l} + T_t)dG_t(a) + \int_{a\geq\hat{a}(\tau,G_t)} (p_tR - rI + r(1-\tau)a + w_t\bar{l} + T_t)dG_t(a), \\
&= w_t\bar{l} + rA_t + (p_tR - rI)[1 - G_t(\hat{a}(\tau, G_t))]. \qquad (23)
\end{aligned}
$$

Thus $W(\tau, G_t)$ is just aggregate labor income $w_t\bar{l}$, plus capital income rA_t and total firm profits $\Pi_t[1-G_t(\hat{a}_t)]$. Recall that the wealth distribution is taken as given in period t. Thus the aggregate wealth A_t in (23) works as a constant in the parties' maximization problem. Thus when choosing the equilibrium tax rate, parties trade-off the effects on labor income and firm profits.

To illustrate the forces at play, suppose that a party decreases τ_t. First, the equilibrium wage rises, thus labor income increases. Secondly, banks relax credit constraints by reducing $\hat{a}_t$, which increases credit penetration ($\eta_t = 1 - G_t(\hat{a}_t)$) and production. Finally, because more agents start a firm the price of output p_t goes down, which reduces firm profits ($\Pi_t = p_tR - rI$).

Note that the effect of τ on total firm profits is ambiguous. The parties may be tempted to increase Π_t by setting a more pro-business policy, i.e. a higher tax rate, but at the cost of fewer agents producing capital (lower η_t). In section 6.1, I study the solution to problem (22).

6 Political Equilibrium

6.1 Equilibrium tax at t

I start by characterizing the equilibrium tax policy τ_t that solves (22) given a wealth distribution G_t. In order for the problem to be well defined, I require the following assumption:

Assumption 2 $R \leq \left(\frac{rI}{(1-\alpha)\bar{l}+\alpha}\right)^{1/\alpha}$.

Combining assumptions 1 and 2, I require the return of the project R to satisfy:

$$R \in \left[\left(\frac{rI}{\alpha} \right)^{1/\alpha}, \left(\frac{rI}{(1-\alpha)\bar{l} + \alpha} \right)^{1/\alpha} \right].$$

This condition leads to the following lemma.

Lemma 4 *Given a cumulative wealth distribution G_t, the equilibrium tax rate τ_t that solves* (22) *satisfies:*

$$1 - G_t(\hat{a}(\tau_t, G_t)) = \Theta, \tag{24}$$

where $\Theta \equiv \left(\frac{\alpha R^{\alpha}[(1-\alpha)\bar{l}+\alpha]}{rI} \right)^{\frac{1}{1-\alpha}}$.

Condition (24) states that the equilibrium tax rate is such that the fraction of agents that become entrepreneurs given G_t is fixed over time and equal to $\Theta \in [0, 1]$.

Using condition (24) in the KF equation under a competitive equilibrium (15), leads to the KF equation under the political equilibrium:

$$d_t G_t(a) = (\Theta - 1) s_t^W(a) d_a G_t(a) - \Theta s_t^E(a) d_a G_t(a). \tag{25}$$

Since politicians cannot commit to a tax rate beyond a given period t, voters do not anticipate the evolution of τ_t over time. That is, when making their consumption-saving decisions they do not take into account the changes in future taxes. Thus the HJB equation under electoral competition is unchanged and given by (9).

6.2 Stationary political equilibrium

In the stationary-equilibrium the KF equation takes the form:

$$(\Theta - 1) s^W(a) d_a G(a) = \Theta s^E(a) d_a G(a). \tag{26}$$

where $G(a)$ is the stationary wealth distribution. Since $\Theta \in [0,1]$ and $d_a G_t(a) \geq 0$, this condition holds only if $s_t^W(a) = s_t^E(a)$. Recall that the saving policy function can be written compactly as: $s_t(a) = \theta_t \cdot y_t(a)$. Thus it must be that the saving rate out of disposable income in steady state (θ_∞) satisfies: $\theta_\infty = 0$. This gives that the stationary tax rate must be $\tau_\infty = \frac{r-\rho}{r}$.

A *stationary political equilibrium* is such that: i) the tax rate is $\tau_\infty = \frac{r-\rho}{r}$, ii) agents solve the HJB equation (9) given τ_∞ and factor prices, iii) prices are given by: $p_\infty = \alpha R^{\alpha-1}\Theta^{\alpha-1}$ and $w_\infty = (1-\alpha)R^\alpha\Theta^\alpha$, iv) the minimum collateral to obtain a loan $\hat{a}_\infty$ satisfies (7), and v) given an initial wealth distribution $G_0(a)$, the stationary wealth distribution $G(a)$ satisfies the KF equation (25).[14]

6.3 Political transition dynamics

The main interest of this paper is to study the transition dynamics of the policy platform when the initial wealth distribution does not satisfy (25). That is, when the initial equilibrium tax rate, τ_0 that solves (24) given $G_0(a)$ is different from the stationary tax rate $\tau_\infty = \frac{r-\rho}{r}$.

Given an initial wealth distribution $G_0(a)$, a *political equilibrium* is such that: i) the time path of tax rates $\{\tau_t\}_{t=0}^\infty$ solves (24) each period t given $G_t(a)$, ii) agents solve the HJB equation (9) given the tax rate and factor prices, iii) prices are given by: $p_t = \alpha R^{\alpha-1}\Theta^{\alpha-1}$ and $w_t = (1-\alpha)R^\alpha\Theta^\alpha$, iv) the minimum collateral to obtain credit $\hat{a}_t$ satisfies (7) given τ_t and $G_t(a)$, and v) the evolution of the cumulative wealth distribution is given by the KF equation (25).

Next proposition characterizes the transition dynamics of the tax rate given an initial wealth distribution G_0.

Proposition 3 *Assume that the initial wealth distribution G_0 is such that the tax rate that solves* (24) *satisfies: $\tau_0 \neq \frac{r-\rho}{r}$. If $\tau_0 < \frac{r-\rho}{r}$, then i) $d_t G_t(a) \leq 0, \forall a$, ii) $d_t \tau_t \geq 0$, iii) $d_t \hat{a}_t \geq 0$, and iv) $d_t A_t \geq 0$. Otherwise, if $\tau_0 > \frac{r-\rho}{r}$ the signs of i) to iv) are reversed.*

[14]Note that the stationary wealth distribution depends on the initial distribution of wealth, G_0. Given G_0, the cumulative wealth distribution evolves according to the KF equation (15) until the economy achieves the stationary equilibrium, $d_t G_t(a) = 0$.

Proposition 3 shows that the transition dynamics of the equilibrium tax rate depends on initial conditions. When the initial wealth distribution G_0 is such that τ_0 is below the stationary tax rate, the equilibrium tax rate increases over time until it reaches τ_∞. That is, the tax policy becomes more pro-business over time. In contrast, if $\tau_0 > \tau_\infty$, then the equilibrium tax rate decreases over time, i.e. the tax policy becomes more pro-worker over time.

The goal now is to characterize the initial wealth distributions that lead to the different political transition dynamics. The relevant question is: what initial conditions create the different tax policy patterns over time?

For that end, define by $\mathcal{G}_0$ the set of initial wealth distributions that are continuous and with support in $[a_{min}, a_{max}]$, where $a_{max} \in (a_{min}, +\infty)$ and $a_{min} \geq 0$. Take now some distribution $G_0 \in \mathcal{G}_0$ an iterate forward the KF equation (15) to obtain the stationary distribution G that satisfies (25). This procedure creates a unique mapping from an initial wealth distribution G_0 to its stationary distribution G. Denote by $\mathcal{G}$ the set of stationary wealth distributions that arise from applying this process to all distributions in $\mathcal{G}_0$.

In what follows I take distributions in $\mathcal{G}$ and apply different transformations to construct distributions in $\mathcal{G}_0$. Given the different initial distributions that arise from these transformations, I explore the transition dynamics of the equilibrium tax rate. This procedure can be interpreted as an MIT shock. The economy is initially in its steady state and there is an unanticipated shock that shifts the equilibrium distribution. The goal is to study the transition back to the steady state.

Section 6.3.1 studies the transition dynamics when the distribution shifts according to FOSD. Section 6.3.2 analyzes the dynamics of the tax policy after applying mean preserving spreads to the distributions in $\mathcal{G}$. This last approach admits an interpretation in terms of Lorentz curve dominance and thus, in terms of initial inequality. Interestingly, the evolution of the policy platform also depends on how binding are credit constraints under the initial distribution.

6.3.1 First order stochastic dominance

For the following lemma, define the set of initial distributions that FOSD the steady-state distribution $G \in \mathcal{G}$ as follows:

$$\mathcal{G}_{FOSD}(G) \equiv \{G_0 \,:\, G_0(a) \leq G(a), \forall a, G \in \mathcal{G}\}.$$

Similarly, define the set of initial distributions that reverse-first order stochastic dominate (reverse-FOSD) the steady-state distribution $G \in \mathcal{G}$ as:

$$\mathcal{G}_{FOSD_{rev}}(G) \equiv \{G_0 \,:\, G_0(a) \geq G(a), \forall a, G \in \mathcal{G}\}.$$

Lemma 5 *Consider some stationary distribution $G \in \mathcal{G}$.*

1. *Take $G_0 \in \mathcal{G}_{FOSD}(G)$ then: i) $d_t G(a) \geq 0, \forall a$, ii) $d_t(\tau_t) \leq 0$, iii) $d_t \hat{a}_t \leq 0$, and iv) $d_t A_t \leq 0$.*

2. *Take $G_0 \in \mathcal{G}_{FOSD_{rev}}(G)$ then the signs of i) to iv) are reversed.*

In order to understand lemma 5, suppose that the economy initially has some stationary wealth distribution $G \in \mathcal{G}$. At period $t = 0$, there is an unanticipated shock such that the new distribution FOSD G.

First, since the distribution shifts right, the price of output at $t = 0$ is lower for any given $\hat{a}$. As a result, banks tighten credit constraints to enforce the incentive compatibility constraint, i.e. it must be that $\hat{a}_0 > \hat{a}_\infty$. Secondly, the impact on credit penetration, $\eta = 1 - G(\hat{a})$, depends on two opposing forces: i) G shifts right, therefore η increases for a given $\hat{a}$, and ii) $\hat{a}$ increases which reduces η for a given distribution G. In proposition 1, I showed that the distribution effect dominates and thus, $\eta_0 \geq \eta_\infty$. Finally, the equilibrium tax rate at $t = 0$ must satisfy (24). Without taking into account the impact on τ, credit penetration increases. Hence, for (24) to hold, η has to go down. That is, $\hat{a}$ must increase even more, but through an increase in τ. In consequence, it must be that $\tau_0 > \tau_\infty$.

In conclusion, given any initial wealth distribution $G_0 \in \mathcal{G}_{FOSD}(G)$, the tax rate is such that $\tau_0 > \tau_\infty$. Therefore, from proposition 3, $d_t G_t(a) \geq 0$, $d_t(\tau_t) \leq 0$, $d_t(\hat{a}_t) \leq 0$. and $d_t A_t \leq 0$. That is, the tax policy becomes more pro-worker over time and credit conditions are relaxed. The transition dynamics of an economy with $G_0 \in \mathcal{G}_{FOSD_{rev}}(G)$ are just the opposite.

6.3.2 Mean preserving spread

In this section, I explore the political transition dynamics when applying a mean preserving spread (MPS) on a given stationary distribution $G \in \mathcal{G}$. The advantage of using MPSs is that I can isolate the pure impact of higher inequality on the evolution of the equilibrium policy platform. An MPS is equivalent to second order stochastic dominance while keeping the mean unchanged. Shorrocks (1983) shows that second order stochastic dominance is equivalent to an ordering according to the generalized Lorentz curve. In particular, an MPS is analogous to Lorentz dominance, which allows to interpret the results in terms of standard inequality measures.

I restrict attention to MPS distributions that intersect only once (Rothschild and Stiglitz, 1971) at the mean, A.

Definition 1 *Consider two distributions G_1 and G_2 with mean A and support in $[0, +\infty)$, G_2 is said to be an MPS of G_1, denoted as $G_2 >_{MPS} G_1$, if:*

1. *$G_2(a) > G_1(a)$ if $a < A$.*

2. *$G_2(a) \leq G_1(a)$ if $a > A$.*

In order to state next proposition, I define the set of initial distributions that are an MPS of the stationary distribution $G \in \mathcal{G}$ as:

$$\mathcal{G}_{MPS}(G) \equiv \{G_0 \ : \ G_0 >_{MPS} G, G \in \mathcal{G}\}.$$

Note that $G_0 \in \mathcal{G}_{MPS}(G)$ is equivalent to say that G Lorentz dominates G_0. That is, the Lorentz curve of G lies everywhere above that of G_0. Hence , G_0 is more unequal than G.

Analogously, I define the set of reverse-mean preserving spreads (reverse-MPS) of some steady-state distribution $G \in \mathcal{G}$ as follows:

$$\mathcal{G}_{MPS_{rev}}(G) \equiv \{G_0 \,:\, G_0 <_{MPS} G, G \in \mathcal{G}\}.$$

Figure 4 illustrates the properties of the MPSs curves in definition 1. Figure 4a depicts the cumulative distribution for $G \in \mathcal{G}$ and $G_0 \in \mathcal{G}_{MPS}(G)$, i.e. when G_0 (red solid-line) is an MPS of G (blue dashed-line). As shown in the figure, the cumulative distributions intersect in the interior only once at the mean A (single-crossing property) . Figure 4b shows the density functions which intersect in the interior only twice (double-crossing) at some wealth levels a_1 and a_2, with $a_1 < a_2$. As shown in figure 4c, the first cross a_1 corresponds to the wealth level at which $G_0 - G$ is maximized, while the second cross, a_2 minimizes this difference. Graphically, the MPS distribution G_0 shifts the frequencies of G from the middle towards the tails, while keeping the mean unchanged.

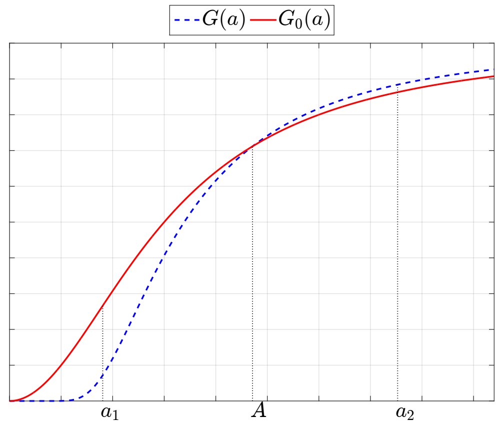
Proposition 4 *Consider some stationary distribution $G \in \mathcal{G}$ with mean A. Take $G_0 \in \mathcal{G}_{MPS}(G)$, then:*

1. *If $A < \hat{a}_\infty$, then: i) $d_t G(a) \geq 0$, ii) $d_t(\tau_t) \leq 0$, iii) $d_t \hat{a}_t \leq 0$, and iv) $d_t A_t \leq 0$.*
2. *If $A > \hat{a}_\infty$, then the signs of i) to iv) are reversed.*

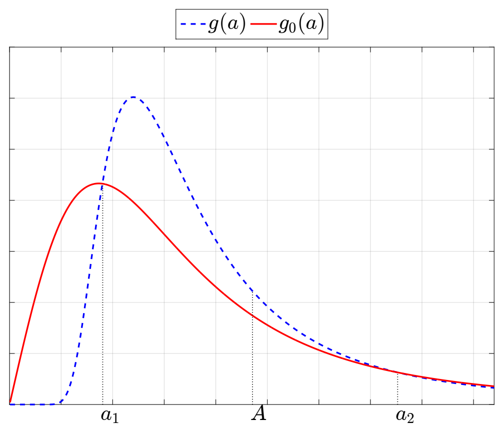
where $\hat{a}_\infty = \frac{rI}{\rho}\left(1 - \frac{r}{(1+r)[(1-\alpha)\bar{l}+\alpha]}\right)$. The transition dynamics are reversed if $G_0 \in \mathcal{G}_{MPS_{rev}}(G)$.

Proposition 4 characterizes the transition dynamics of the political equilibrium as a function of initial inequality and wealth. It sorts economies into terms of their initial wealth according to $\hat{a}_\infty$, which depends only on the primitives of the model. Countries that start with $A < \hat{a}_\infty$ can be interpreted as *poor* countries, while those with $A > \hat{a}_\infty$ are *rich* ones.

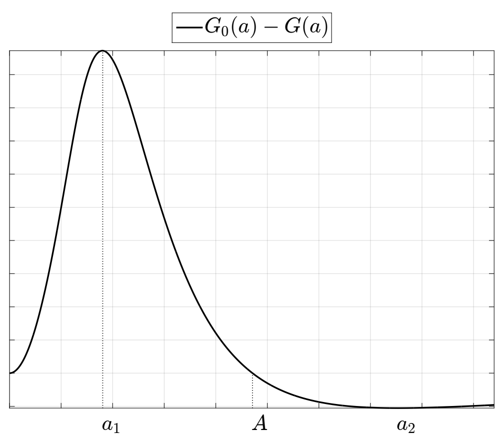
The proposition states the following. 1) Countries that start *poor* and with higher inequality relative to the steady-state, exhibit a policy transition that moves from a more *pro-business* policy platform towards a more *pro-worker* one. 2) Countries that are initially *rich* and more unequal, start with a more *pro-worker* tax rate and move towards a more *pro-business* platform over time.



(a) Cummulative distribution functions, G and G_0



(b) Densitiy functions, g and g_0.



(c) $G_0(a) - G(a)$.

Figure 4: MPS distributions, $G \in \mathcal{G}$ and $G_0 \in \mathcal{G}_{MPS}(G)$

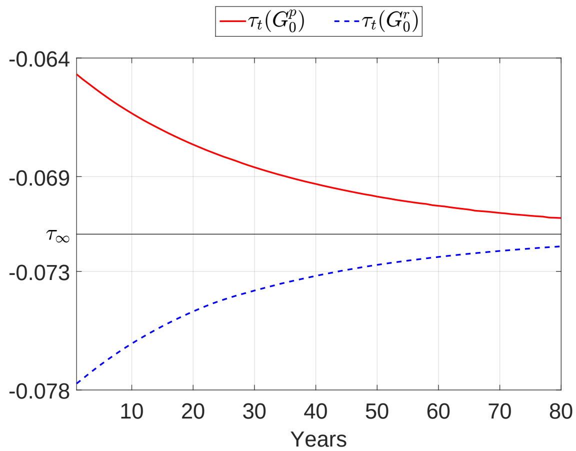
Items 1) and 2) apply in the opposite direction in case of starting with a more equal distribution, i.e. when $G_0 \in \mathcal{G}_{MPS_{rev}}(G)$.

Figure 5 depicts the transition dynamics of an initially *poor* (red solid-line) and initially *rich* (blue dashed-line) economies. I denote their initial cumulative distributions by G_0^p and G_0^r with means $A^p < \hat{a}_\infty$ and $A^r > \hat{a}_\infty$, respectively. Both distributions are an MPS of some stationary distributions $G^p \in \mathcal{G}$ and $G^r \in \mathcal{G}$, i.e. $G_0^p \in \mathcal{G}_{MPS}(G^p)$ and $G_0^r \in \mathcal{G}_{MPS}(G^r)$.

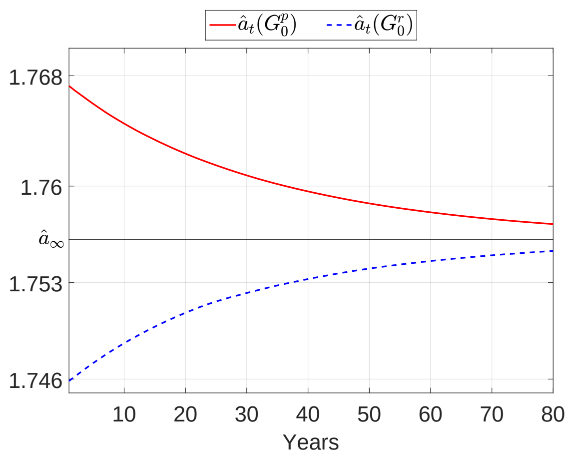
In order to construct the red solid-lines, I select a stationary distribution G^p with mean $A^p < \hat{a}_\infty$ and apply an MPS to build the initial distribution G_0^p. Given G_0^p, I use the equilibrium tax rate condition (24) and the saving policy (13) to iterate forward the KF equation (25) and obtain the transition dynamics. To obtain the blue dashed-lines, I apply the same procedure but starting from a different stationary distribution G^r with mean $A^r > \hat{a}_\infty$. Figure 5a presents the evolution of the tax rate as a function of the initial distribution, figure 5b presents the minimum wealth to obtain a loan and figure 5c the aggregate wealth.

In what follows I explain the intuition for item 1 in proposition 4. Assume that the economy is initially at some stationary wealth distribution $G^p \in \mathcal{G}$ with mean $A^p < \hat{a}_\infty$, i.e. a *poor* country. At period $t = 0$, there is an unanticipated shock that shifts the initial distribution towards an MPS distribution G_0^p. That is, at $t = 0$ there is a shock that increases wealth inequality by keeping the same mean.

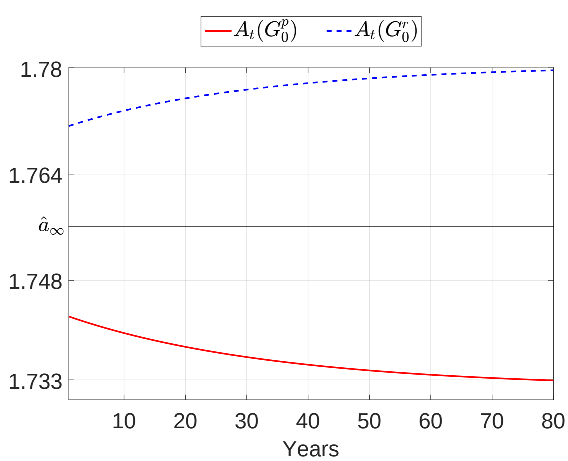
First, since $A^p < \hat{a}_\infty$, an MPS decreases the mass of agents below $\hat{a}$. Graphically, the red solid-line in figure 4a is below the blue dashed-line. Thus credit penetration $\eta = 1 - G_0^p(\hat{a}_\infty)$ increases. Secondly, the new equilibrium tax rate at $t = 0$ must satisfy (24). For this condition to hold, η has to decline, i.e. $\hat{a}$ must increase through an increase in the tax rate. Intuitively, since the mass of agents running a firm increases, they exert higher political pressure towards a more *pro-business* policy platform. Therefore, as shown by the red solid-line in figures 5a and 5b, it must be that $\tau_0^p > \tau_\infty$ and $\hat{a}_0^p > \hat{a}_\infty$. To sum up, an MPS in an economy that starts *poor* increases the equilibrium tax rate and the minimum wealth to obtain credit. From proposition 3, the economy exhibits a policy transition that moves towards a more *pro-worker* one over time. That is, as shown by the



(a) Tax rate, τ_t.



(b) Minimum wealth, $\hat{a}_t$



(c) Aggregate wealth, A_t.

Figure 5: Political transition dynamics.

red solid-line in figure 5a, the tax rate decreases over time. The intuition works in the opposite direction for an economy that is initially *rich*.

As depicted in figure 5b, the minimum wealth to obtain a loan under $G_0^p \in \mathcal{G}_{MPS}(G)$ is larger than under G^p when the economy starts *poor*, i.e. $\hat{a}_0^p > \hat{a}_\infty$. Thus the economy with G_0^p satisfies that $A^p < \hat{a}_0^p$. This condition implies that an agent with the average wealth A^p cannot access the credit market and start a firm. That is, credit constraints are binding in the economy with G_0^p; a *capital constrained* economy. On the other hand, it can be shown that an MPS in a *rich* economy decreases the minimum collateral ($\hat{a}_0^r < A^r$); a *capital unconstrained* country. As shown by the blue dashed-line in figure 4b.

Hence, countries can be sorted in terms of how binding are initial credit constraints.[15] Thus proposition 4 admits a different interpretation. 1) Countries that start more unequal relative to the steady state and *capital constrained* exhibit a policy platform that becomes more *pro-worker* over time. 2) Countries that start more unequal and *capital unconstrained* exhibit a tax rate policy that moves towards a more *pro-business* one over time. These results apply in the opposite direction if the initial distribution is more equal relative to the stationary distribution.

[15]Note that whether a country is capital constrained or not depends on the initial (at $t = 0$) minimum wealth to obtain a loan $\hat{a}_0$, which is endogenously defined as a function of G_0. Proposition 4 could be stated in terms of $\hat{a}_0$. However, since this is an endogenous measure, I prefer to state the proposition in terms of $\hat{a}_\infty$ which is a function of the exogenous parameters of the model.

7 Conclusions

There are historical accounts showing that the direction of development policies largely differs across countries and over time. This article develops a political theory of dynamic Ramsey policies that can rationalize the international differences in development policies.

The baseline model is an adaptation of the workhorse macro model with heterogeneous agents and financial frictions that includes: i) repeated elections, ii) endogenous credit constraints, and iii) occupational choice. Wealth heterogeneity means that agents are differently affected by tax policies. Poorer agents are excluded from the credit market and remain workers, while richer agents can obtain a loan to finance a firm and become entrepreneurs. Occupational choice creates a distributional conflict between the different classes of agents. Each period, the policy platform arises from a political process that aggregates these interests, which depend on current inequality. Current policies in turn affect savings decisions that determine future inequality and policies. Thus inequality and policies are jointly determined over time as a result of elections.

The tractability of my model allows for a sharp characterization of the evolution of the equilibrium policy in terms of countries' initial inequality and capital constraints. Countries that start capital constrained and more unequal relative to the stationary wealth distribution, exhibit a policy platform that moves towards a more *pro-worker* stance. On the other hand, in countries that start unconstrained and unequal, the equilibrium policy becomes more *pro-business* over time. These results apply in the opposite direction in countries that start more equal relative to the steady-state. These different equilibrium dynamics provide a rationale to the differences in development policies pursued by developed and developing countries.

References

Acemoglu, Daron, Michael Golosov, and Aleh Tsyvinski, "Political Economy of Mechanisms," *Econometrica*, 2008, *76* (3), 619–641.

— , **Mikhail Golosov, and Aleh Tsyvinski**, "Dynamic Mirrlees Taxation under Political Economy Constraints," *The Review of Economic Studies*, 2010, *77* (3), 841–881.

— , — , **and** — , "Political Economy of Ramsey Taxation," *Journal of Public Economics*, 2011, *95* (7-8), 467–475.

Banerjee, Abhijit and Andrew Newman, "Occupational Choice and the Process of Development," *Journal of Political Economy*, 1993, *101* (2), 274–298.

Barro, Robert J, "On the Determination of the Public Debt," *Journal of Political Economy*, 1979, *87* (5, Part 1), 940–971.

Bernanke, Ben and Mark Gertler, "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 1989, *79* (1), 14–31.

Buera, Francisco J and Benjamin Moll, "Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity," *American Economic Journal: Macroeconomics*, 2015, *7* (3), 1–42.

— **and Yongseok Shin**, "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 2013, *121* (2), 221–272.

Farhi, Emmanuel, Christopher Sleet, Ivan Werning, and Sevin Yeltekin, "Non-Linear Capital Taxation Without Commitment," *Review of Economic Studies*, 2012, *79* (4), 1469–1493.

Fischer, Ronald and Diego Huerta, "Wealth Inequality and the Political Economy of Financial and Labour Regulations," *Journal of Public Economics*, 2021, *204*, 104553.

— , — , **and Patricio Valenzuela**, "The Inequality-Credit Nexus," *Journal of International Money and Finance*, 2019, *91*, 105 – 125.

Gonzalez-Eiras, Martin and Dirk Niepelt, "The Future of Social Security," *Journal of Monetary Economics*, 2008, *55* (2), 197–218.

Hassler, John, Per Krusell, Kjetil Storesletten, and Fabrizio Zilibotti, "The Dynamics of Government," *Journal of Monetary Economics*, 2005, *52* (7), 1331–1358.

Huerta, Diego, "The Political Economy of Labor Policy," 2022.

Itskhoki, Oleg and Benjamin Moll, "Optimal Development Policies with Financial Frictions," *Econometrica*, 2019, *87* (1), 139–173.

Lindbeck, Assar and Jörgen W Weibull, "Balanced-Budget Redistribution as the Outcome of Political Competition," *Public Choice*, 1987, *52* (3), 273–297.

Lucas, Robert E and Nancy L Stokey, "Optimal Fiscal and Monetary Policy in an Economy Without Capital," *Journal of Monetary Economics*, 1983, *12* (1), 55–93.

Matsuyama, Kiminori, "Financial Market Globalization, Symmetry-Breaking, and Endogenous Inequality of Nations," *Econometrica*, 2004, *72* (3), 853–884.

Moll, Benjamin, "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?," *American Economic Review*, 2014, *104* (10), 3186–3221.

Persson, Torsten and Guido Tabellini, *Political Economics: Explaining Economic Policy*, The MIT Press, 2000.

Rothschild, Michael and Joseph E Stiglitz, "Increasing Risk II: Its Economic Consequences," *Journal of Economic Theory*, 1971, *3* (1), 66–84.

Shorrocks, Anthony F., "Ranking Income Distributions," *Economica*, 1983, *50* (197), 3–17.

Sleet, Christopher and Şevin Yeltekin, "Politically Credible Social Insurance," *Journal of Monetary Economics*, 2008, *55* (1), 129–151.

Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti, "Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt," *Econometrica*, 2012, *80* (6), 2785–2803.

Straub, Ludwig and Iván Werning, "Positive Long-Run Capital Taxation: Chamley-Judd Revisited," *American Economic Review*, 2020, *110* (1), 86–119.

Yared, Pierre, "Politicians, Taxes and Debt," *The Review of Economic Studies*, 2010, *77* (2), 806–840.

8 Appendix

8.1 Proofs

Lemma 1

1. *Individuals' consumption-saving decision arise from the Hamilton-Jacobi-Bellman (HJB) equation:*

$$\rho v(a_t) = \max_{c_t} \left\{ u(c_t) + d_a v(a_t)(y_t(a_t, \tau_t) - c_t) \right\},$$

$$\text{s.t} \qquad \dot{a}_t = y_t(a_t, \tau_t) - c_t$$

$$a_t \geq \underline{a}$$

where $y_t(a_t, \tau_t) = \pi_t(a_t) + r(1 - \tau_t)a_t + w_t\bar{l} + T_t$.

2. *The consumption policy function is:*

$$c_t(a_t, \tau_t) = (1 - \theta_t) \cdot y_t(a_t, \tau_t).$$

3. *The saving policy function is*

$$s_t(a_t, \tau_t) = \theta_t \cdot y_t(a_t, \tau_t),$$

where $\theta_t \equiv \frac{1}{\gamma}\left(1 - \frac{\rho}{r(1-\tau_t)}\right)$.

Proof: Consider a problem in discrete time where periods are of the length of Δ and with discount factor $e^{-\rho\Delta}$. The Bellman equation is:

$$v(a_t) = \max_{c}\{\Delta u(c) + e^{-\rho\Delta} v(a_{t+\Delta})\}$$

$$\text{s.t} \qquad a_{t+\Delta} = \Delta(y(a) - c) + a_t,$$

$$a_{t+\Delta} \geq \underline{a}.$$

Take $\Delta \to 0$, so $e^{-\rho\Delta} \to 1 - \rho\Delta$. Then the first equation reads as,

$$\begin{aligned} v(a_t) &= \max_c \{\Delta u(c) + (1 - \rho\Delta) v(a_{t+\Delta})\}, \\ \Leftrightarrow \rho\Delta v(a_t) &= \max_c \{\Delta u(c) + (1 - \rho\Delta) v(\Delta(y(a) - c) + a_t) - v(a_t)\}, \end{aligned} \tag{27}$$

where in the second line I subtracted $(1 - \rho\Delta)v(a_t)$ from both sides. Note that,

$$\lim_{\Delta \to 0} \frac{v(\Delta(y(a) - c) + a_t) - v(a_t)}{\Delta(y(a) - c)} \cdot (y(a) - c) = \frac{dv(a)}{da}(y(a) - c).$$

Divide (27) by Δ and take $\Delta \to 0$ to obtain (9).

In order to obtain $c(a)$ and $s(a)$, guess that the value function is $v(a) = B \cdot \frac{\left(a + \frac{\overline{w}}{r(1-\tau)}\right)^{1-\gamma}}{1-\gamma}$, where $\overline{w} = \pi(a) + w + T$. Thus, $v'(a) = B\left(a + \frac{\overline{w}}{r(1-\tau)}\right)^{\gamma}$.

The FOC of (9) is,

$$u'(c) = v'(a) \Rightarrow c(a) = B^{\frac{1}{\gamma}}\left(a + \frac{\overline{w}}{r(1-\tau)}\right).$$

Then the HJB equation reads as,

$$\begin{aligned} \rho B \frac{\left(a + \frac{\overline{w}}{r(1-\tau)}\right)^{1-\gamma}}{1-\gamma} &= B^{-\frac{1-\gamma}{\gamma}} \frac{\left(a + \frac{\overline{w}}{r(1-\tau)}\right)^{1-\gamma}}{1-\gamma} + B\left(a + \frac{\overline{w}}{r(1-\tau)}\right)^{-\gamma}\left(\overline{w} + r(1-\tau)a - B^{\frac{-1}{\gamma}}\left(a + \frac{\overline{w}}{r(1-\tau)}\right)\right), \\ &= B^{-\frac{1-\gamma}{\gamma}} \frac{\left(a + \frac{\overline{w}}{r(1-\tau)}\right)^{1-\gamma}}{1-\gamma} + B\left(r(1-\tau) - B^{\frac{-1}{\gamma}}\right)\left(a + \frac{\overline{w}}{r(1-\tau)}\right)^{1-\gamma}, \\ \Leftrightarrow \frac{\rho B}{1-\gamma} &= \frac{B^{\frac{-(1-\gamma)}{\gamma}}}{1-\gamma} + Br(1-\tau) - B^{\frac{-1}{\gamma}}. \end{aligned}$$

Therefore, $B^{\frac{-1}{\gamma}} = \frac{\rho-(1-\gamma)r(1-\tau)}{\gamma}$. Replacing in the FOC: $c(a) = \frac{\rho-(1-\gamma)r(1-\tau)}{\gamma}\left(a + \frac{\overline{w}}{r(1-\tau)}\right)$. Finally, the budget constraint (10) implies: $s(a) = \frac{r(1-\tau)-\rho}{\gamma}\left(a + \frac{\overline{w}}{r(1-\tau)}\right)$. Use that $y(a) = \bar{w} + r(1-\tau)a$ to write the policy functions as in items 2 and 3. ■

Lemma 2 *The evolution of the cumulative wealth distribution $G_t(a)$ is characterized by the Kol-*

mogorov Forward (KF) equation:

$$d_tG_t(a) = -G_t(\hat{a}_t)\cdot s_t^W(a,\tau_t)d_aG_t(a) - (1-G_t(\hat{a}_t))\cdot s_t^E(a,\tau_t)d_aG_t(a),$$

where $s_t^W(a,\tau_t) = \theta_t(a + w_t\bar{l} + T_t)\cdot \mathbb{1}_{a<\hat{a}_t}$, $s_t^E(a,\tau_t) = \theta_t(a + \Pi_t + w_t\bar{l} + T_t)\cdot \mathbb{1}_{a\geq\hat{a}_t}$ *and* $\hat{a}_t$ *satisfies* (7).

Proof: Assets evolve according to

$$da = s(a)dt, \tag{28}$$

where $s(a)$ is given by equation (13). Consider now a discrete time model where the length of a period is Δ. The analogue of (28) is:

$$a_{t+\Delta} - a_t = \int_t^{t+\Delta} s(a_h)dh.$$

For Δ small, $\int_t^{t+\Delta} s(a_h)dh \approx \Delta s(a_t)$ and also $\int_t^{t+\Delta} s(a_h)dh \approx \Delta s(a_{t+\Delta})$. Using the second approximation and rearranging terms,

$$a_{t+\Delta} = a_t + \Delta s(a_{t+\Delta}). \tag{29}$$

Define $s_t^W(a) = \theta_t(a + w_t + T_t)\cdot \mathbb{1}_{a<\hat{a}_t}$ and $s_t^E = \theta_t(a + \pi_t + w_t + T_t)\cdot \mathbb{1}_{a\geq\hat{a}_t}$. Assume that agents dissave, $s(a)\leq 0$ (the case with $s(a) > 0$ is symmetric). Then, from equation (29) the fraction of agents with wealth below a at period $t+\Delta$ is:

$$\begin{aligned} Pr(a_{t+\Delta}\leq a) &= Pr(a_t < \hat{a}_t)\cdot Pr(a_t \leq a - \Delta s_t^W(a)) + Pr(a_t \geq \hat{a}_t)\cdot Pr(a_t\leq a-\Delta s_t^E(a)),\\ G_{t+\Delta}(a) &= G_t(\hat{a}_t)\cdot G_t(a-\Delta s_t^W(a)) + (1-G_t(\hat{a}_t))\cdot G_t(a-\Delta s_t^E(a)). \end{aligned}$$

Subtracting $G_t(a)$ from both sides and dividing by Δ:

$$\frac{G_{t+\Delta}(a) - G_t(a)}{\Delta} = G_t(\hat{a}_t)\left(\frac{G_t(a-\Delta s_t^W(a)) - G_t(a)}{\Delta}\right) + (1-G_t(\hat{a}_t))\left(\frac{G_t(a-\Delta s_t^E(a)) - G_t(a)}{\Delta}\right). \tag{30}$$

Note that,

$$\lim_{\Delta\to 0} \frac{G_t(a - \Delta s_t^j(a)) - G_t(a)}{\Delta s_t^j(a)} \cdot s_t^j(a) = -d_a G_t(a) \cdot s_t^j(a). \tag{31}$$

Taking $\Delta \to$ in (30) and using (31) leads to (15). ■

Proposition 1 *Assume that the initial cumulative wealth distribution G_0 is such that initial credit penetration satisfies: $\eta_0 \in (0, 1)$. Take $\tau < \frac{r-\rho}{r}$, then: i) $d_t G_t(a) \leq 0$ $\forall a$, ii) $d_t \hat{a}_t \geq 0$, iii) $d_t k_t \geq 0$, and iv) $d_t A_t \geq 0$. Otherwise, if $\tau > \frac{r-\rho}{r}$ the signs of i) to iv) are reversed.*

Proof:

In what follows I assume that $\theta_t > 0$, that is $\tau_t < \frac{r-\rho}{r}$. From equation (14), this implies item iv) that: $d_t A_t \geq 0$. The proof proceeds analogously (but with opposite signs) when $\theta_t < 0$, i.e if $\tau_t > \frac{r-\rho}{r}$. Note that I allow the tax rate to depend on time. Thus the proof applies for a path of taxes $\{\tau_t\}_{t=0}^{\infty}$ satisfying $\tau_t < \frac{r-\rho}{r}, \forall t > 0$ or $\tau_t > \frac{r-\rho}{r}, \forall t > 0$. As will be clear later, this more general case is useful to study the transition dynamics when the tax rate is defined through a political process.

Proof of i)

The saving policy function (13) can be written as follows:

$$s_t(a) = \begin{cases} \theta_t a + \theta_t(w_t \bar{l} + T_t) & \text{if } a < \hat{a}_t, \\ \theta_t a_t + \theta_t(\pi_t + w_t \bar{l} + T_t) & \text{if } a \geq \hat{a}_t. \end{cases}$$

Define $\underline{\omega}_t \equiv \theta_t(w_t \bar{l} + T_t)$ and $\overline{\omega}_t \equiv \theta_t(\pi_t + w_t \bar{l} + T_t)$. Consider a discrete time model where the length of a period is Δ, then:

$$\begin{aligned} a_{t+\Delta} &= a_t + \Delta s_t(a_t), \\ &= a_t(1 + \Delta\theta) + \Delta \underline{\omega}_t \mathbb{1}_{a_t < \hat{a}_t} + \overline{\omega}_t \mathbb{1}_{a_t \geq \hat{a}_t}. \end{aligned} \tag{32}$$

Consider the density function at period t, $g_t(a)$ with support in $[\underline{a}_t, +\infty)$. Then the density function at period $t+\Delta$, $g_{t+\Delta}(a)$ is defined by:

$$g_{t+\Delta}(a) = \begin{cases} 0 & \text{if } a < \underline{a}_{t+\Delta}, \\ g_t\left(\frac{a-\Delta\underline{\omega}_t}{1+\Delta\theta_t}\right) & \text{if } a \in [\underline{a}_{t+\Delta}, a^1_{t+\Delta}), \\ 0 & \text{if } a \in [a^1_{t+\Delta}, a^2_{t+\Delta}), \\ g_t\left(\frac{a-\Delta\overline{\omega}_t}{1+\Delta\theta_t}\right) & \text{if } a \geq a^2_{t+\Delta}, \end{cases} \tag{33}$$

where I have defined:

$$\underline{a}_{t+\Delta} \equiv \underline{a}_t(1+\Delta\theta) + \Delta\underline{\omega}_t,$$
$$a^1_{t+\Delta} \equiv \hat{a}_t(1+\Delta\theta) + \Delta\underline{\omega}_t,$$
$$a^2_{t+\Delta} \equiv \hat{a}_t(1+\Delta\theta) + \Delta\overline{\omega}_t.$$

Figure 6 in section 8.2 illustrates the properties of g_t and $g_{t+\Delta}$.

From equation (33), the difference between the cumulative wealth distribution in period $t+\Delta$, $G_{t+\Delta}(a)$ and the current cumulative distribution, $G_t(a)$ is written as follows:

$$G_{t+\Delta}(a) - G_t(a) = \begin{cases} -G_t(a) & \text{if } a < \underline{a}_{t+\Delta}, \\ G_t\left(\frac{a-\Delta\underline{\omega}_t}{1+\Delta\theta_t}\right) - G_t(a) & \text{if } a \in [\underline{a}_{t+\Delta}, a^1_{t+\Delta}), \\ G_t(\hat{a}_t) - G_t(a) & \text{if } a \in [a^1_{t+\Delta}, a^2_{t+\Delta}), \\ G_t\left(\frac{a-\Delta\overline{\omega}_t}{1+\Delta\theta_t}\right) - G_t(a) & \text{if } a \geq a^2_{t+\Delta}. \end{cases} \tag{34}$$

Note that,

$$\lim_{\Delta\to 0} \frac{G_t\left(\frac{a-\Delta\underline{\omega}_t}{1+\Delta\theta_t}\right) - G_t(a)}{\Delta} = \lim_{\Delta\to 0} \frac{G_t\left(a-\Delta\underline{\omega}_t\right) - G_t(a)}{\Delta\underline{\omega}_t} \cdot \underline{\omega}_t = -d_a G_t(a) \cdot \underline{\omega}_t \tag{35}$$

Analogously,

$$\lim_{\Delta \to 0} \frac{G_t\left(\frac{a-\Delta\overline{\omega}_t}{1+\Delta\theta_t}\right) - G_t(a)}{\Delta} = -d_a G_t(a) \cdot \overline{\omega}_t \tag{36}$$

Divide (34) by Δ and take $\Delta \to 0$ to obtain:

$$d_t G_t(a) = \begin{cases} -\infty & \text{if } a < \underline{a}_{t+\Delta}, \\ -d_a G_t(a)\underline{\omega}_t & \text{if } a \in [\underline{a}_{t+\Delta}, a^1_{t+\Delta}), \\ -\infty & \text{if } a \in [a^1_{t+\Delta}, a^2_{t+\Delta}), \\ -d_a G_t(a)\overline{\omega}_t & \text{if } a \geq a^2_{t+\Delta}, \end{cases}$$

where I have used conditions (35) and (36), and that $d_a G_t(a) \geq 0$. I conclude that $d_t G_t(a) \leq 0$. Figure 7 in section 8.2 depicts G_t and $G_{t+\Delta}$.

Proof of ii)

Condition (7) can be rewritten as:

$$\hat{a}_t(1+r)(1-\tau_t) = I(1+r) - p_t R = I(1+r) - \alpha R^\alpha (1 - G_t(\hat{a}_t))^{\alpha-1}. \tag{37}$$

Differentiating in terms of t,

$$\begin{aligned} d_t(\hat{a}_t) \cdot (1+r)(1-\tau_t) &= -\alpha(1-\alpha)R^\alpha(1-G_t(\hat{a}_t))^{\alpha-2} \frac{d}{dt}\left(G_t(\hat{a}_t)\right) \\ &= -\alpha(1-\alpha)R^\alpha(1-G_t(\hat{a}_t))^{\alpha-2}\left(d_t G_t(\hat{a}_t) + d_a G_t(\hat{a}_t) \cdot d_t(\hat{a}_t)\right) \end{aligned}$$

Rearranging terms and solving for $d_t(\hat{a}_t)$,

$$d_t(\hat{a}_t) = \frac{-\alpha(1-\alpha)R^{-2}k_t^{\alpha-2} d_t G_t(\hat{a}_t)}{(1+r)(1-\tau_t) + \alpha(1-\alpha)R^{-2}k_t^{\alpha-2} d_a G_t(\hat{a}_t)}. \tag{38}$$

Note that the denominator of (38) is positive since $\tau_t < 1$ and $d_a G_t(a) \geq 0$. From property i), $d_t G_t(\hat{a}_t) \leq 0$ if $\tau_t < \frac{r-\rho}{r}$ and then $d_t(\hat{a}_t) \leq 0$.

Proof of iii)

Rewrite condition (37) as follows:

$$\hat{a}_t(1+r)(1-\tau_t) = I(1+r) - \alpha R^{\alpha}\eta_t^{\alpha-1}.$$

Differentiation in terms of t leads to:

$$d_t(\hat{a}_t)(1+r)(1-\tau_t) = \alpha(1-\alpha)R^{\alpha}(\eta_t)^{\alpha-2}d_t(\eta_t).$$

From property ii), $d_t(\hat{a}_t) \leq 0$ if $\tau_t < \frac{r-\rho}{r}$. Thus $d_t(\eta_t) \leq 0$ and $d_t k_t = R \cdot d_t(\eta_t) \leq 0$, which concludes the proof. ■

Lemma 3 *Given some wealth distribution G_t, an increase in the tax rate τ_t leads to:*

1. *A lower equilibrium wage w_t.*
2. *A higher price of output p_t.*
3. *A higher minimum wealth to obtain a loan $\hat{a}_t$.*

Proof: Differentiate (37) with respect to τ_t to obtain:

$$d_{\tau_t}(\hat{a}_t)(1+r)(1-\tau_t) - \hat{a}_t(1+r) = -\alpha(1-\alpha)R^{\alpha}(1-G_t(\hat{a}_t))^{\alpha-2}d_aG_t(\hat{a}_t)d_{\tau_t}(\hat{a}_t)$$
$$\Rightarrow d_{\tau_t}(\hat{a}_t) = \frac{\hat{a}_t(1+r)}{(1+r)(1-\tau_t)+\alpha(1-\alpha)R^{\alpha}(1-G_t(\hat{a}_t))^{\alpha-2}d_aG_t(\hat{a}_t)} > 0.$$

Note that:

$$d_{\tau_t}(w_t) = (1-\alpha)\alpha k_t^{\alpha-1}d_{\tau_t}(k_t), \tag{39}$$

$$d_{\tau_t}(p_t) = -\alpha(1-\alpha)k_t^{\alpha-2}d_{\tau_t}(k_t). \tag{40}$$

Since $k_t = R(1-G_t(\hat{a}_t))$, then $d_{\tau_t}(k_t) = -Rg_t(\hat{a}_t)d_{\tau_t}(\hat{a}_t) < 0$. Thus, $d_{\tau_t}(w_t) < 0$ and $d_{\tau_t}(p_t) > 0$, which concludes the proof. ■

Proposition 2 *The preferred tax rate function $\tau^*(a_t, G_t)$ is as follows:*

$$\tau^*(a_t, G_t) = \begin{cases} \underline{\tau} & \text{if } a_t \leq \hat{a}(\underline{\tau}, G_t) \\ \Psi(a, G_t) & \text{if } a_t \in (\hat{a}(\underline{\tau}, G_t), \tilde{a}(G_t)) \\ \underline{\tau} & \text{if } a \geq \tilde{a}(G_t) \end{cases}$$

where $\Psi(a, G_t) \in [\underline{\tau}, 1)$ is a continuous and strictly increasing function in a implicitly defined by

$$a = \hat{a}(\Psi(a, G_t), G_t).$$

The threshold, $\tilde{a}(G_t) \in (\hat{a}(\underline{\tau}, G_t), +\infty)$ is given by

$$d_\tau(y_t^E(\tilde{a}(G_t), \tau)) = 0.$$

Proof: I start by studying the political preferences of entrepreneurs. In what follows I omit time subscripts to simplify notation. Differentiation of $y^E(a, \tau)$ in terms of τ gives,

$$d_\tau(y^E(a, \tau)) = d_\tau(p)R - ra + d_\tau(w)\bar{l}. \tag{41}$$

Using equations (39) and (40) leads to:

$$\begin{aligned} d_\tau(y^E(a, \tau)) &= -\alpha(1-\alpha)k^{\alpha-2}Rd_\tau(k) - ra + \alpha(1-\alpha)k^{\alpha-1}d_\tau(k) \cdot \bar{l}, \\ \Rightarrow d_\tau(y^E(a, \tau)) &= -\underbrace{\alpha(1-\alpha)d_\tau(k)k^{\alpha-2}(R - k\bar{l})}_{>0} - ra. \end{aligned} \tag{42}$$

Note that the sign of (42) is ambiguous and depends on a. Take for instance $a = 0$, then $d_\tau(y^E(a, \tau)) > 0$. Suppose now that wealth is large enough $a \to +\infty$, then $d_\tau(y^E(a, \tau)) < 0$. Since $d_\tau(y^E(a, \tau))$ is continuous, the intermediate value theorem implies that there exists some cutoff $\tilde{a}(G_t) > 0$ such that $d_\tau(y^E(a, \tau)) > 0$ if $a < \tilde{a}(G_t)$ and $d_\tau(y^E(a, \tau)) < 0$ if $a > \tilde{a}(G_t)$.

Now consider workers:

$$d_\tau y^E(a,\tau) = -ra + d_\tau(w) < 0. \tag{43}$$

Therefore workers are worse off when the tax rate increases.

In what follows I consider the effect of τ on occupational choice. I study the solution to (7) as function of τ. Since $\hat{a}(\tau, G)$ is increasing in τ, the minimum possible threshold to obtain credit is $\hat{a}(\underline{\tau}, G)$. Thus agents with wealth $a < \hat{a}(\underline{\tau}, G)$ cannot become entrepreneurs no matter the tax rate. Since agents with $a < \hat{a}(\underline{\tau}, G)$ are workers, they are worse off when the tax rate increases. Hence, their preferred tax rate is $\underline{\tau}$, i.e. $\tau^*(a,G) = \underline{\tau}$ if $a < \hat{a}(\underline{\tau})$. Additionally, since $\tilde{a}(G)$ defines the last entrepreneur that is better off when the tax rate increases, it must be that $\tilde{a}(G) \in (\hat{a}(\underline{\tau}, G), +\infty)$.

Secondly, suppose that $a \in [\hat{a}(\underline{\tau}, G), \tilde{a}(G))$. Assumption 1 implies that $y^E(a,\tau) \geq y^W(a,\tau), \forall \tau \in [\underline{\tau}, 1)$. Moreover, $y^E(a,\tau)$ is increasing in τ for those agents. Therefore, individuals with wealth $a \in [\hat{a}(\underline{\tau}, G), \tilde{a}(G))$ will choose a τ such that they can become entrepreneurs, i.e $\tau^*(a,G)$ is such that $a \geq \hat{a}(\tau^*(a,G), G)$.

The problem that agents with wealth $a \in [\hat{a}(\underline{\tau}, G), \tilde{a}(G))$ solve is:

$$\tau^*(a,G) = \arg\max_{\tau \in [\underline{\tau}, 1)} y^E(a,\tau)$$
$$s.t. \;\; a \geq \hat{a}(\tau, G).$$

Since $d_\tau(y^E(a,\tau)) > 0$ and $d_\tau(\hat{a}(\tau,G)) > 0$, the constraint is binding. Thus, if $a \in [\hat{a}(\underline{\tau}, G), \tilde{a}(G))$, then $\tau^*(a,G)$ is given by a continuous function denoted by $\Psi(a,G)$ that satisfies:

$$a = \hat{a}(\Psi(a,G), G). \tag{44}$$

Moreover, differentiation of (44), leads to $d_a\Psi(a,G) = \frac{1}{d_\tau \hat{a}(\Psi(a,G),G)} > 0$. Thus $\Psi(a,G)$ is strictly increasing in a.

Finally, agents with wealth $a \geq \tilde{a}(G)$ can become entrepreneurs, but prefer the minimum tax, i.e. $\tau^*(a,G) = \underline{\tau}$. This concludes the proof. ■

Lemma 4 *Given a cumulative wealth distribution G_t, the equilibrium tax rate τ_t that solves* (22) *satisfies:*

$$1 - G_t(\hat{a}(\tau_t, G_t)) = \Theta,$$

where $\Theta \equiv \left(\frac{\alpha R^\alpha[(1-\alpha)\bar{l}+\alpha]}{rI}\right)^{\frac{1}{1-\alpha}}$.

Proof: The political objective function is given by

$$\begin{aligned} W(\tau, G_t) &= w_t\bar{l} + rA_t + (p_r R - rI)[1 - G_t(\hat{a}(\tau, G_t)], \\ &= (1-\alpha)k_t^\alpha \bar{l} + rA_t + (\alpha k_t^{\alpha-1}R - rI)[1 - G_t(\hat{a}(\tau, G_t)], \\ &= k_t^\alpha[(1-\alpha)\bar{l} + \alpha] + rA_t - \frac{rI}{R}k_t. \end{aligned} \tag{45}$$

Taking the FOC in (45):

$$\begin{aligned} &\alpha k_t^{\alpha-1}[(1-\alpha)\bar{l} + \alpha]d_\tau(k_t) - \frac{rI}{R}d_\tau(k_t) = 0, \\ &\Leftrightarrow \eta_t^{1-\alpha} = \left(\frac{\alpha R^\alpha[(1-\alpha)\bar{l}+\alpha]}{rI}\right)^{\frac{1}{1-\alpha}}, \end{aligned} \tag{46}$$

where I have used that $d_\tau(k_t) = -Rd_aG(\hat{a}_t) \cdot d_\tau \hat{a}_t < 0$ and that $k_t = R \cdot \eta_t$. Using that $\eta_t = 1 - G_t(\hat{a}_t)$ leads to (24). ■

Proposition 3 *Assume that the initial wealth distribution G_0 is such that the tax rate that solves* (24) *satisfies: $\tau_0 \neq \frac{r-\rho}{r}$. If $\tau_0 < \frac{r-\rho}{r}$, then i) $d_tG_t(a) \leq 0, \forall a$, ii) $d_t\tau_t \geq 0$, iii) $d_t\hat{a}_t \geq 0$, and iv) $d_tA_t \geq 0$. Otherwise, if $\tau_0 > \frac{r-\rho}{r}$ the signs of i) to iv) are reversed.*

Proof: Differentiating condition (37) in terms of t leads to:

$$d_t(\hat{a}_t)(1+r)(1-\tau_t) - \hat{a}_t(1+r)d_t(\tau_t) = -\alpha(1-\alpha)R^\alpha\eta_t^{\alpha-2}(d_tG_t(\hat{a}_t) + d_aG_t(\hat{a}_t)d_t(\hat{a}_t)).$$

Defining $\chi_t \equiv \alpha(1-\alpha)R^{\alpha}\eta_t^{\alpha-2}$ and solving for $d_t(\hat{a}_t)$:

$$d_t(\hat{a}_t) = \frac{\hat{a}_t(1+r)d_t(\tau_t) - \chi_t d_t G_t(\hat{a}_t)}{(1+r)(1-\tau_t) + \chi_t d_a G_t(\hat{a}_t)} \tag{47}$$

Now differentiate condition (24) in terms of t to obtain:

$$d_t G_t(\hat{a}_t) + d_a G_t(\hat{a}_t) d_t(\hat{a}_t) = 0. \tag{48}$$

Replacing (47) in (48):

$$\begin{aligned} d_t G_t(\hat{a}_t) + \frac{d_a G_t(\hat{a}_t)\hat{a}_t(1+r)d_t(\tau_t)}{(1+r)(1-\tau_t) + \chi_t d_a G_t(\hat{a}_t)} - \frac{d_a G_t(\hat{a}_t)\chi_t d_t G_t(\hat{a}_t)}{(1+r)(1-\tau_t) + \chi_t d_a G_t(\hat{a}_t)} &= 0 \\ \Rightarrow d_t G_t(\hat{a}_t) \underbrace{\left[1 - \frac{\chi_t d_a G_t(\hat{a}_t)}{(1+r)(1-\tau_t) + \chi_t d_a G_t(\hat{a}_t)}\right]}_{\geq 0} &= \frac{-d_a G_t(\hat{a}_t)\hat{a}_t(1+r)d_t(\tau_t)}{(1+r)(1-\tau_t) + \chi_t d_a G_t(\hat{a}_t)} \end{aligned}$$

Rearranging terms and solving for $d_t(\tau_t)$ gives:

$$d_t(\tau_t) = -\frac{d_t G_t(\hat{a}_t)}{d_a G_t(\hat{a}_t)} \frac{1-\tau_t}{\hat{a}_t}. \tag{49}$$

Thus, $\text{sign}\{d_t(\tau_t)\} = \text{sign}\{-d_t G_t(\hat{a}_t)\}$. Note that proposition 1 applies for any sequence of tax rates that satisfies either that $\tau_t < \frac{r-\rho}{r}, \forall t$ or $\tau_t > \frac{r-\rho}{r}, \forall t$.

Suppose that G_0 is such that $\tau_0 < \frac{r-\rho}{r}$. Then $d_t G_t(\hat{a}_t) \leq 0$, which implies that $d_t(\tau_t) \geq 0$. Otherwise, if $\tau_0 < \frac{r-\rho}{r}$ then $d_t G_t(\hat{a}_t) \geq 0$ and thus, $d_t(\tau_t) \leq 0$. This shows items i) and ii).

To prove item iii) use the results of items i) and ii) in equation (47). Item iv) comes from the fact that the path of tax rates implies that $\theta_t \geq 0$. This concludes the proof. ■

Lemma 5 *Consider some stationary distribution $G \in \mathcal{G}$.*

1. *Take $G_0 \in \mathcal{G}_{FOSD}(G)$ then: i) $d_t G(a) \geq 0, \forall a$, ii) $d_t(\tau_t) \leq 0$, iii) $d_t \hat{a}_t \leq 0$, and iv) $d_t A_t \leq 0$.*

2. *Take $G_0 \in \mathcal{G}_{FOSD_{rev}}(G)$ then the signs of i) to iv) are reversed.*

Proof: Consider an economy that starts in some steady state distribution $G_0 \in \mathcal{G}$ at $t = 0$. At period $t = \Delta$, there is an unanticipated shock that shifts the initial distribution towards that a distribution that FOSD G_0. That is, $G_\Delta \in \mathcal{G}_{FOSD}(G_0)$.

Considering a time interval of length Δ, condition (49) reads as follows:

$$\tau_\Delta - \tau_0 = -\frac{G_\Delta(\hat{a}_0) - G_0(\hat{a}_0)}{G_0(\hat{a}_0 + \Delta) - G_0(\hat{a}_0)} \frac{1 - \tau_\infty}{\hat{a}_\infty} \tag{50}$$

The denominator is positive, because $d_a G(a) \geq 0$. Since G_Δ FOSD G_0 the numerator is negative. Thus, $\tau_\Delta > \tau_0$. From proposition 3, I conclude that $d_t(\tau_t) \leq 0$. The results are reversed when $G_\Delta \in \mathcal{G}_{FOSD_{rev}}(G_0)$. This concludes the proof. ■

Proposition 4 *Consider some stationary distribution $G \in \mathcal{G}$ with mean A. Take $G_0 \in \mathcal{G}_{MPS}(G)$, then:*

1. *If $A < \hat{a}_\infty$, then: i) $d_t G(a) \geq 0$, ii) $d_t(\tau_t) \leq 0$, iii) $d_t \hat{a}_t \leq 0$, and iv) $d_t A_t \leq 0$.*
2. *If $A > \hat{a}_\infty$, then the signs of i) to iv) are reversed.*

where $\hat{a}_\infty = \frac{rI}{\rho}\left(1 - \frac{r}{(1+r)[(1-\alpha)\bar{l}+\alpha]}\right)$. The transition dynamics are reversed if $G_0 \in \mathcal{G}_{MPS_{rev}}(G)$.

Proof: Using condition (24) in equation (37), the stationary minimum collateral to obtain a loan, $\hat{a}_\infty$ is given by

$$\begin{aligned}
\hat{a}_\infty &= \left(I - \frac{\alpha R^\alpha \Theta^{\alpha-1}}{1+r}\right) \frac{1}{1-\tau_\infty}, \\
&= \left(1 - \frac{r}{(1+r)[(1-\alpha)\bar{l}+\alpha]}\right) \frac{rI}{1-\tau_\infty}.
\end{aligned}$$

Using that $\tau_\infty = \frac{r-\rho}{r}$ leads to the expression in the proposition.

Consider an economy that starts with the stationary wealth distribution $G_0 \in \mathcal{G}$ at $t = 0$. In period $t = \Delta$ there is an unanticipated shock that applies an MPS on the initial distribution. Take some MPS distribution $G' \in \mathcal{G}_{MPS}(G_0)$. Then define the wealth distribution in period $t = \Delta$, G_Δ as follows:

$$G_\Delta = \lambda G' + (1-\lambda) G_0, \quad \lambda \in [0, 1] \tag{51}$$

Thus the set of distributions generated by G_Δ as λ increases is a sequence of MPSs of the initial distribution G_0, i.e. $G_\Delta \in \mathcal{G}_{MPS}(G_0)$. Differentiating of condition (37) in terms of λ around the steady-state and evaluating at $\lambda = 0$ leads to:

$$d_\lambda(\hat{a}_\infty)\big|_{\lambda=0} = \frac{\hat{a}_\infty(1+r)d_\lambda(\tau_\infty)\big|_{\lambda=0} - \chi_t d_\lambda G_\Delta(\hat{a_\infty})\big|_{\lambda=0}}{(1+r)(1-\tau_\infty) + \chi_t d_a G_\Delta(\hat{a}_\infty)\big|_{\lambda=0}}. \tag{52}$$

Differentiate (24) with respect to λ at $\lambda = 0$ to obtain:

$$d_\lambda G_\Delta(\hat{a}_\infty)\big|_{\lambda=0} + d_a G_\Delta(\hat{a}_\infty) d_\lambda(\hat{a}_\infty)\big|_{\lambda=0} = 0. \tag{53}$$

Replacing (52) in (8.1) and solving for $d_\lambda(\tau_\infty)\big|_{\lambda=0}$ gives:

$$d_\lambda(\tau_\infty)\big|_{\lambda=0} = -\frac{d_\lambda G_\Delta(\hat{a}_\infty)\big|_{\lambda=0}}{d_a G_\Delta(\hat{a}_\infty)\big|_{\lambda=0}} \frac{1-\tau_\infty}{\hat{a}_\infty} = -\frac{G'(\hat{a}_\infty) - G_0(\hat{a}_\infty)}{d_a G_0(\hat{a}_\infty)} \frac{1-\tau_\infty}{\hat{a}_\infty}, \tag{54}$$

where I have used equation (51). Since $d_a G_0(\hat{a}_\infty) \geq 0$, then: $\operatorname{sign} d_\lambda(\tau_\infty)\big|_{\lambda=0} = \operatorname{sign} G_0(\hat{a}_\infty) - G'(\hat{a}_\infty)$. By definition 1, if $A < \hat{a}_\infty$ then $G_0(\hat{a}_\infty) - G'(\hat{a}_\infty) \geq 0$ because $G' \in \mathcal{G}_{MPS}(G_0)$. Therefore, $\tau_\Delta \geq \tau_\infty$. Moreover, equation (52) reads as:

$$d_\lambda(\hat{a}_\infty)\big|_{\lambda=0} = \frac{\hat{a}_\infty(1+r)d_\lambda(\tau_\infty)\big|_{\lambda=0} + \chi_t(G_0(\hat{a}_\infty) - G'(\hat{a}_\infty))}{(1+r)(1-\tau_\infty) + \chi_t d_a G_0(\hat{a}_\infty)}.$$

Thus, $d_\lambda(\hat{a}_\infty)\big|_{\lambda=0} \geq 0$ if $A < \hat{a}_\infty$, which implies that $\hat{a}_\Delta = \hat{a}(\tau_\Delta, G_\Delta) \geq a_\infty$, i.e. the economy still satisfies that $A < \hat{a}_\Delta$ after an MPS shock.

By proposition 3, it must be that $d_t\tau_t \leq 0$ when $A < \hat{a}_\infty$. Otherwise, $A > \hat{a}_\infty$ then $G_0(\hat{a}_\infty) - G'(\hat{a}_\infty) \leq 0$ and thus, $\tau_\Delta \leq \tau_\infty$, which implies that $d_t\tau_t \geq 0$. This concludes the proof. ■

8.2 Additional figures

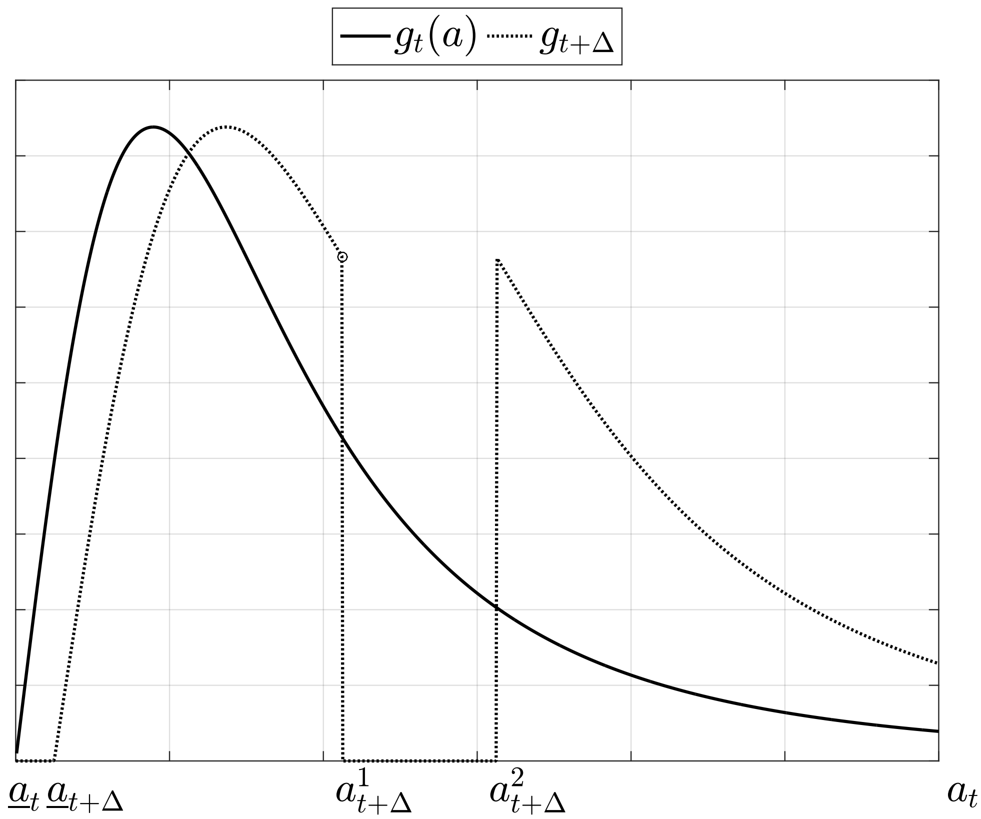


Figure 6: FOSD: $g_t(a)$ and $g_{t+\Delta}(a)$

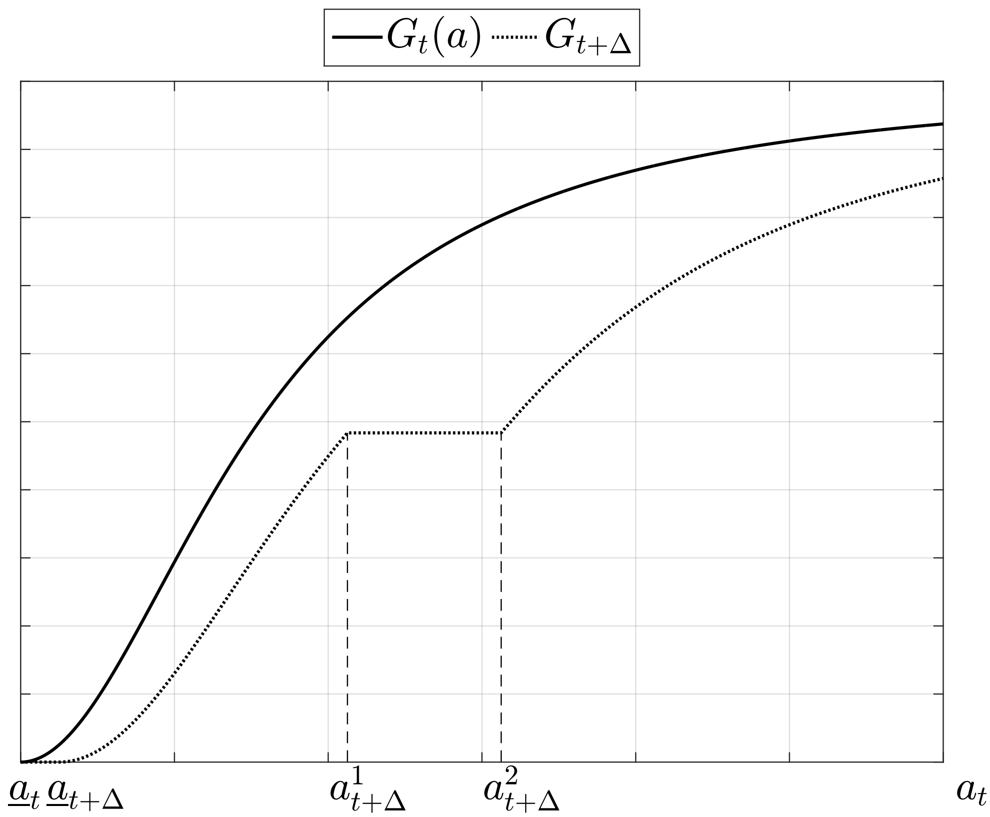


Figure 7: FOSD: $G_t(a)$ and $G_{t+\Delta}(a)$