

Credit Default Options on Stocks

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Abstract—This project studies the valuation of credit default options on stocks by using their equivalence to European and American cash-or-nothing put options. We extend the binomial tree approach by implementing five discrete-time models: Cox–Ross–Rubinstein (CRR), Tian, Jarrow–Rudd, CRR with drift adjustment, and Leisen–Reimer. For European binary options we calibrate the number of time steps required for convergence against the Black–Scholes closed-form solution, and then use these step sizes to price the corresponding American binary options. The numerical experiments allow us to compare convergence speed, stability, and the impact of the “shark-teeth” effect across models, and to discuss their practical implications for credit risk protection on individual stocks.

I. PRESENTATION

Credit default options on stocks provide a simple way to obtain protection against large downward moves in the value of an individual firm. When the credit event is defined as the stock price falling below a given barrier at maturity, these contracts can be modeled as cash-or-nothing binary put options. This equivalence allows us to study credit protection within the standard option-pricing framework and to compare continuous-time valuations based on the Black–Scholes formula with discrete-time approximations based on binomial trees.

The objective of this project is to implement and compare several binomial models for pricing such binary options: the classical Cox–Ross–Rubinstein (CRR) tree, the Tian and Jarrow–Rudd trees, a drift-adjusted CRR specification, and the Leisen–Reimer tree. For European binary puts we calibrate, for each model and parameter scenario, the minimum number of time steps required so that the binomial price converges to the Black–Scholes benchmark within a prescribed error tolerance. We then use these calibrated step sizes to value the corresponding American binary options and analyze the impact of early exercise. The numerical results allow us to compare convergence speed, stability, and the presence of the so-called “shark-teeth” effect across models, and to discuss the practical implications of model choice for the design and valuation of credit default options on individual stocks.

II. THEORY

A. American and European options

In derivative pricing, the fundamental difference between European and American options lies in their exercise rights. A European option may only be exercised at its maturity date T , making its valuation dependent solely on the terminal distribution of the underlying asset under the risk-neutral measure. In contrast, an American option grants the holder the right to exercise at any time up to maturity. Because of this early-exercise feature, European options often admit closed-form solutions in continuous-time models such as Black–Scholes, whereas American options generally do not, and typically require discrete-time numerical methods such as binomial trees for their valuation [4].

B. Binary options

Binary options are derivatives whose payoffs take only two possible values, typically a fixed cash amount or zero. The most common structures are Cash-or-Nothing and Asset-or-Nothing options. This project focuses on Cash-or-Nothing binary put options, which pay a fixed amount if the underlying asset price at maturity falls below a predefined strike K , and pay nothing otherwise. The payoff graph for a binary Cash-or-Nothing put option can be seen in Figure 1 and the function is formally defined as follows.

$$f_T^{\text{Put CN}} = f(S_T, T) = \begin{cases} C, & \text{if } S_T \leq K, \\ 0, & \text{if } S_T > K. \end{cases}$$

C. Black-Scholes form for a Cash-or-Nothing Put

Under the Black–Scholes framework, the fair price for a European Cash-or-Nothing option is given by:

$$f_t = Ce^{-rT} \mathcal{N}(-d_2)$$

Where:

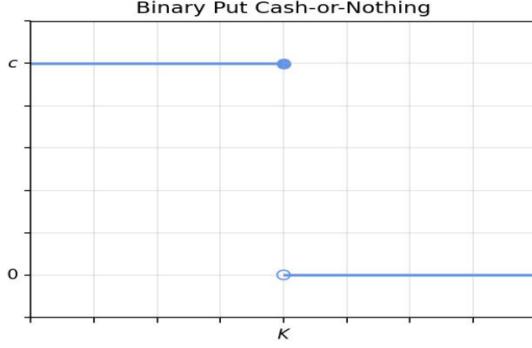


Fig. 1. Payoff plot for a binary Cash-or-Nothing put option.

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.$$

- S_0 : Current price of the underlying asset
- K : Strike price of the binary option
- T : Time to maturity (in years)
- r : Continuously compounded risk-free interest rate
- σ : Volatility of the underlying asset (annualized)
- $\mathcal{N}(\cdot)$: Cumulative distribution function of the standard normal distribution
- d_2 : Term defining the risk-neutral probability that $S_T < K$
- V_0 : Present value of the European Cash-or-Nothing put option

D. Credit default options

Credit default options are derivatives that pay a fixed amount when a credit-related event occurs, typically interpreted as the default of a firm or a severe deterioration in the value of an underlying asset. In credit-risk this is modeled as the event where the firm's asset value falls below a critical threshold at maturity. This setup makes credit default options mathematically equivalent to binary put options on the firm value, since the payoff depends exclusively on whether the asset ends below a default barrier.

In practice, this equivalence allows analysts to represent credit protection using binary options, where the “default event” corresponds to the underlying asset price S_T falling below a strike K . Under the risk-neutral measure, the price of such credit default options becomes the discounted probability of barrier breach, which aligns directly with the Black–Scholes valuation of European binary puts or with its discrete-time approximation through binomial trees.

E. CRR model

The Cox-Ross-Rubinstein (CRR) Binomial Model provides a discrete-time approximation to the stochastic evolution of asset prices and serves as the foundation for valuing both European and American derivatives. Under this framework, the underlying asset follows a recombining tree where, over each time interval $\Delta t = T/N$ the price either moves up by a

factor u or down by a factor d . The binomial tree structure can be seen on Figure 2.

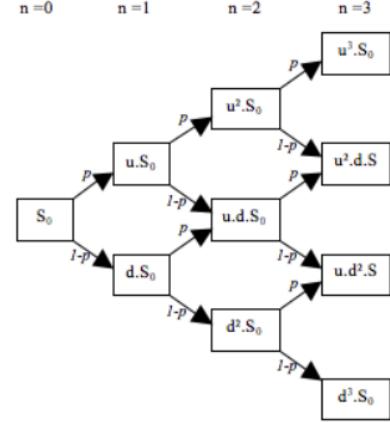


Fig. 2. Example of a binomial tree model with four time steps.

Using this structure, option values are computed backward from maturity by discounting the risk-neutral expected value at each node. Although CRR converges well for smooth payoffs, its performance deteriorates for discontinuous payoffs such as binary options. The discrete placement of terminal nodes relative to the strike creates oscillatory patterns commonly referred to as the “shark-teeth” effect. This makes CRR a useful baseline for benchmarking, but not the most efficient model for binary options. The model is defined as follows.

$$f_0 = e^{-rtdt} (pf_u + (1-p)f_d)$$

Where:

$$p = \frac{e^{rtdt} - d}{u - d}$$

$$u = e^{\sigma\sqrt{dt}}$$

$$d = \frac{1}{u}$$

F. Shark-teeth effect

The “shark-teeth” effect refers to the oscillatory convergence pattern often observed when binomial trees are used to price options with discontinuous payoffs, such as binary options. Because the payoff jumps at the strike, small changes in the placement of terminal nodes around this point can cause the binomial approximation to alternately overshoot and undershoot the true value as the number of steps increases. This produces a jagged, tooth-like error profile rather than smooth convergence to the Black–Scholes benchmark. The effect is most pronounced in classical models such as CRR and is significantly reduced in more sophisticated constructions like Leisen–Reimer, which better match the underlying distribution at maturity.

G. Tian model

The Tian model is an alternative binomial specification designed to match not only the mean and variance of the underlying asset's log-return, but also its third moment (skewness) over each time step [2]. By matching three moments of the continuous-time lognormal distribution, the Tian tree aims to provide a more accurate discrete approximation, particularly for options that are sensitive to the tails of the distribution. The three equations used by Tian are the following.

$$pu + (1 - p) = e^{rdt}$$

$$pu^2 + (1 - p)d^2 = (e^{rdt})^2 e^{\sigma^2 dt}$$

$$pu^3 + (1 - p)d^3 = (e^{rdt})^3 (e^{\sigma^2 dt})^3$$

Where:

$$p = \frac{e^{rdt} - d}{u - d}$$

$$u = 0.5e^{rdt}v \left((v + 1 + \sqrt{v^2 + 2v - 3}) \right)$$

$$d = 0.5e^{rdt}v \left(v + 1 - \sqrt{v^2 + 2v - 3} \right)$$

$$v = e^{\sigma^2 dt}$$

H. Jarrow–Rudd model

The Jarrow–Rudd model is another alternative binomial tree that is constructed in the logarithmic price space and is often referred to as the equal-probability model [2]. In its simplest form, the up and down moves in log-price are symmetric around the drift term, and the risk-neutral probabilities satisfy

$$p = \frac{1}{2}, \quad 1 - p = \frac{1}{2}.$$

Since there is an equal probability of the asset price rising or falling, this leads to:

$$p = \frac{1}{2}$$

$$u = e^{(r - \sigma^2/2)dt + \sigma\sqrt{dt}}$$

$$d = e^{(r - \sigma^2/2)dt - \sigma\sqrt{dt}}$$

Because the Jarrow–Rudd tree is centered in log-price space, it can exhibit different convergence properties from CRR, especially for out-of-the-money options and for payoffs sensitive to the behavior of the underlying in the tails. In the context of binary options, this may translate into a different pattern of shark-teeth oscillations when compared with the CRR and Tian models.

I. CRR with drift adjustment

The drift-adjusted Cox–Ross–Rubinstein (CRR) model extends the classical binomial tree by incorporating a drift parameter η into the up and down factors. Instead of assuming symmetric movements around the risk-neutral growth rate, the model allows the underlying asset price to be shifted upward or downward according to

$$u = e^{\eta\Delta t + \sigma\sqrt{\Delta t}}, \quad d = e^{\eta\Delta t - \sigma\sqrt{\Delta t}}, \quad p = \frac{e^{r\Delta t} - d}{u - d}.$$

When $\eta = 0$, the model collapses exactly to the standard CRR specification [2], preserving variance matching and tree recombination. Introducing a non-zero drift can shift the distribution of terminal nodes, which may be beneficial when pricing discontinuous or asymmetric payoffs.

J. Leisen–Reimer Model

The Leisen–Reimer (LR) model was developed to improve the rate and smoothness of convergence of binomial trees toward the Black–Scholes price. While classical binomial models such as CRR, Tian, or Jarrow–Rudd converge to the correct continuous-time limit as $\Delta t \rightarrow 0$, their convergence for discontinuous payoffs is often oscillatory and slow. The LR model addresses this issue by constructing the tree probabilities from a discrete inversion approximation of the cumulative normal distribution, ensuring a closer match to the terminal distribution of the underlying.

The Leisen–Reimer parameters are defined as

$$\bar{p} = h^{-1}(d_1), \quad p = h^{-1}(d_2),$$

$$u = \frac{e^{r\Delta t} \bar{p}}{p}, \quad d = \frac{e^{r\Delta t} - pu}{1 - p},$$

where d_1 and d_2 are the standard Black–Scholes terms and $h^{-1}(\cdot)$ is a discrete approximation to the inverse cumulative distribution function of a standard normal random variable.

One approximation proposed by Leisen and Reimer for the inversion operator is

$$h^{-1}(z) = 0.5 + \text{sgn}(z) \left[0.25 \right]$$

$$-0.25 \exp \left(- \frac{z}{n + 1/3 + 0.1/(n+1)} \left(n + \frac{1}{6} \right) \right]^{1/2}$$

where n is the number of time steps in the tree (including times 0 and T), which must be odd for the LR construction. This formulation creates a binomial tree whose probabilities more closely match the continuous distribution, leading to faster and smoother convergence.

III. RESEARCH & METHODOLOGY

This section describes the numerical design used to compare the convergence properties of the binomial models presented in Section II and to evaluate their performance for pricing credit default options modeled as cash-or-nothing binary puts. All experiments are implemented in Python using a common set of parameter scenarios and a unified pricing interface for the five trees.

A. Numerical set-up

We consider a stock paying no dividends and model credit default protection as a cash-or-nothing binary put written on the stock price. The option is characterized by the initial stock price S_0 , strike K , maturity T , risk-free rate r , volatility σ , and payoff amount C . To test the models under different conditions, we define four benchmark scenarios that combine low and high strikes with low and high volatility. The specific parameter values used in the experiments are summarized in Table I. In all cases the Black–Scholes price $f_0 = Ce^{-rT}\mathcal{N}(-d_2)$ serves as the analytical benchmark for the European binary put [1].

TABLE I
SCENARIO PARAMETERS

| Variable | Esc 1 | Esc 2 | Esc 3 | Esc 4 |
|----------|-------|-------|-------|-------|
| S_0 | 100 | 100 | 100 | 100 |
| K | 50 | 150 | 50 | 150 |
| r | 5% | 5% | 5% | 5% |
| σ | 60% | 60% | 5% | 5% |
| T | 2 | 2 | 2 | 2 |
| t | 0 | 0 | 0 | 0 |
| Cash | 100 | 100 | 100 | 100 |

For each of the five binomial trees—CRR, Tian, Jarrow–Rudd, drift-adjusted CRR and Leisen–Reimer—we implement a generic pricing function that receives $(S_0, K, r, \sigma, T, C, N)$ and returns the option value computed by backward induction on a recombining tree, using the model-specific parameters (u, d, p) described in Section II.

B. Convergence study for European binary options

To assess convergence, we study how the binomial price of a European cash-or-nothing put approaches the Black–Scholes benchmark as the number of time steps N increases. For each model and each scenario we perform the following procedure:

- 1) Fix the parameters $(S_0, K, r, \sigma, T, C)$ corresponding to the scenario under consideration.
- 2) Compute the Black–Scholes price f_0^{BS} of the binary put.
- 3) For a sequence of tree sizes $N = N_{\min}, N_{\min} + \Delta N, \dots, N_{\max}$ (with N restricted to odd integers in the Leisen–Reimer case), compute the corresponding binomial price $f_0^{\text{Tree}}(N)$.
- 4) For each N evaluate the absolute error

$$\text{err}(N) = |f_0^{\text{Tree}}(N) - f_0^{\text{BS}}|.$$

- 5) Record the smallest value N^* such that the mean of the errors over the last 100 values of N falls below the tolerance $\varepsilon = 0.001$.

The sequence $\text{err}(N)$ is plotted as a function of N for each model and each scenario. These convergence plots highlight both the speed at which the binomial price approaches the Black–Scholes benchmark and the presence of oscillatory “shark-teeth” patterns caused by the discrete alignment of terminal nodes relative to the strike K . The collection of values N^* across models and scenarios is later summarized in a table to facilitate a direct comparison of efficiency.

C. Pricing American binary options

Once the required number of steps N^* has been identified for the European binary put in a given scenario, we use the same tree size to compute the price of the corresponding American cash-or-nothing put. For each model we employ a standard backward-induction algorithm:

- 1) Construct the recombining stock–price tree with N^* steps using the model-specific parameters (u, d, p) .
- 2) At maturity, set the option value at each terminal node equal to the binary payoff C if $S_T \leq K$ and 0 otherwise.
- 3) For $n = N^* - 1, \dots, 0$, compute at each node the continuation value as the discounted risk-neutral expectation of the two successor nodes, and then take the maximum between this continuation value and the immediate-exercise payoff:

$$f_n = \max \left\{ C_{\{S_n \leq K\}}, e^{-r\Delta t} (pf_{n+1}^{(u)} + (1-p)f_{n+1}^{(d)}) \right\}.$$

- 4) The value at the root node f_0 is reported as the American binary price for that model and scenario.

By keeping the tree size equal to N^* , we ensure that the comparison across models for American options is performed at a level of discretization that already delivers satisfactory accuracy for the European case. This allows us to focus on differences arising from early-exercise features and from the structure of each tree rather than from arbitrary choices of N .

D. Implementation details

All numerical experiments are implemented in Python using Jupyter Notebook. For reproducibility, the same code structure is used across models: each tree is encapsulated in a pricing function for European and American binary puts, and a separate driver script performs the parameter sweeps over N , generates the convergence plots, and records the calibrated values N^* .

IV. DISCUSSION AND RESULTS

This section summarizes the main numerical findings of the convergence study and the pricing of European and American cash-or-nothing binary puts under the tested models.

A. Convergence results for European options

The convergence results for the European cash-or-nothing put reveal clear and systematic differences across the binomial models. Figures 3 and 4 (Scenario 1) illustrate the behavior of the classical CRR and Leisen–Reimer models respectively, as the number of steps N increases.

The left panel shows that the binomial price does converge toward the Black–Scholes benchmark, but the convergence is not smooth. Instead, the sequence exhibits the characteristic “shark–teeth” pattern: the price alternately overshoots and undershoots the true value as N grows, producing a jagged oscillation whose amplitude decays slowly. This effect is reflected in the right panel, where the absolute error displays repeated spikes on a logarithmic scale.

These oscillations arise from the discontinuous payoff of the binary option and from the shifting alignment of terminal nodes relative to the strike. The same qualitative behavior was observed for the Tian, Jarrow–Rudd, and drift-adjusted CRR models, with only minor differences in amplitude and rate of decay across scenarios.

In contrast, the Leisen–Reimer model exhibits the smoothest and most stable convergence among all methods tested. Figure 4 shows that the LR price approaches the Black–Scholes value monotonically, without the oscillatory behavior seen in CRR and the other models. The corresponding error curve decays over N , remaining well below the oscillatory movements presented on the other models. This improvement is due to the LR construction of probabilities via the h^{-1} transformation, which matches the terminal distribution of the underlying more accurately and thus mitigates node–strike misalignment.

As a result, LR consistently requires fewer steps to reach the desired tolerance level, particularly in high–volatility scenarios where the shark–teeth effect is most severe. The Minimum numer of steps N^* for each model and scenario to converge can be seen in Table II and III. Overall, the CRR tree usually requires the largest N^* , while the alternative trees reach the error tolerance with fewer steps. The Leisen–Reimer model tends to be among the most efficient, especially in high–volatility or deep out–of–the–money scenarios.

TABLE II
MINIMUM NUMBER OF STEPS N^* TO CONVERGENCE FOR EACH MODEL
AND SCENARIO WITH $N=20,000$

| Model | Scen. 1 | Scen. 2 | Scen. 3 | Scen. 4 |
|----------------|--------------|--------------|---------|---------|
| CRR | No conv. (p) | No conv. (p) | 500 | 500 |
| Tian | No conv. (p) | No conv. | 500 | 500 |
| Jarrow–Rudd | No conv. (p) | No conv. | 500 | 500 |
| CRR with drift | No conv. (p) | No conv. (p) | 600 | 550 |
| Leisen–Reimer | No conv. (p) | No conv. (p) | 500 | 500 |

Note: (p) indicate cases where the model shows signs of potential convergence beyond $N = 20,000$, but the tolerance threshold was not reached within the tested range.

TABLE III
EUROPEAN CASH-OR-NOTHING PUT PRICES

| Model | Scen. 1 | Scen. 2 | Scen. 3 | Scen. 4 |
|----------------|---------|---------|---------|---------|
| CRR | 27.7028 | 70.7905 | 0.0000 | 90.4833 |
| Tian | 27.4031 | 70.9161 | 0.0000 | 90.4832 |
| Jarrow–Rudd | 27.4040 | 70.9157 | 0.0000 | 90.4831 |
| CRR with drift | 27.8088 | 71.0865 | 0.0000 | 90.4833 |
| Leisen–Reimer | 27.3613 | 71.0862 | 0.0000 | 90.4833 |
| Black–Scholes | 27.5850 | 70.8990 | 0.0000 | 90.4831 |

B. American binary options

Using the calibrated step sizes N^* from the European study, we priced American cash–or–nothing puts with each tree. The resulting root–node values are summarized in Table IV.

TABLE IV
AMERICAN CASH-OR-NOTHING PUT PRICES

| Model | Scen. 1 | Scen. 2 | Scen. 3 | Scen. 4 |
|----------------|---------|----------|---------|----------|
| CRR | 49.5409 | 100.0000 | 0.0000 | 100.0000 |
| Tian | 49.5369 | 100.0000 | 0.0000 | 100.0000 |
| Jarrow–Rudd | 49.5315 | 100.0000 | 0.0000 | 100.0000 |
| CRR with drift | 49.5366 | 100.0000 | 0.0000 | 100.0000 |
| Leisen–Reimer | 49.5316 | 100.0000 | 0.0000 | 100.0000 |

The comparison between the European and American prices shows that early–exercise flexibility has a significant impact only in scenarios where the credit event is reasonably likely prior to maturity. In Scenario 1, the American prices are around 49.5 across all models, whereas the European counterparts remain near 27.6. This gap reflects the value of being able to trigger the fixed cash payoff as soon as the underlying crosses the default threshold, rather than waiting until T .

In Scenarios 2 and 4, where the strike is high and the probability of ending below K is substantial throughout the life of the option, all American models reach the maximum payoff of $C = 100$, while the European prices remain strictly below this level (approximately 70.9 and 90.5, respectively) due to discounting and the fact that the risk–neutral probability of default at maturity is still less than one.

By contrast, Scenario 3 yields a price of zero for both European and American versions across all models, indicating that with a low strike and very low volatility the likelihood of a credit event is essentially negligible, rendering the option worthless even when early exercise is permitted.

The convergence behavior of the American binary put further highlights the structural differences among the binomial models. As shown in Figure 5, the Tian tree produces a smooth and monotonic convergence toward the limiting price, with no oscillatory behavior. This pattern is consistent across all alternative models tested—Tian, Jarrow–Rudd, CRR with drift, and Leisen–Reimer—none of which exhibit the shark–teeth effect in the American setting.

Alternatively, the classical CRR model (Figure 6) displays clear oscillations reminding of those observed in the European case. These fluctuations stem from the discrete interaction between the early–exercise boundary and the binomial tree, making CRR the only model in our study that retains the shark–teeth pattern when pricing American cash–or–nothing puts.

C. Key findings

The numerical experiments reveal several clear patterns. First, low–volatility scenarios converge substantially faster, since the narrower distribution of terminal stock prices reduces the sensitivity of the binary payoff to node placement. Second, the Leisen–Reimer model consistently delivers smooth and

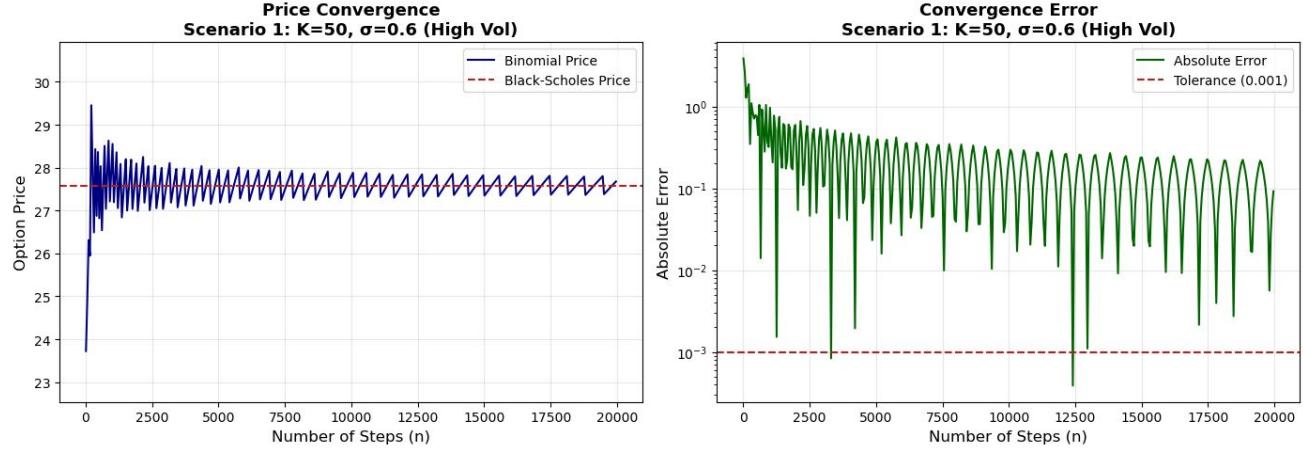


Fig. 3. Convergence results for Scenario 1 using the CRR model.

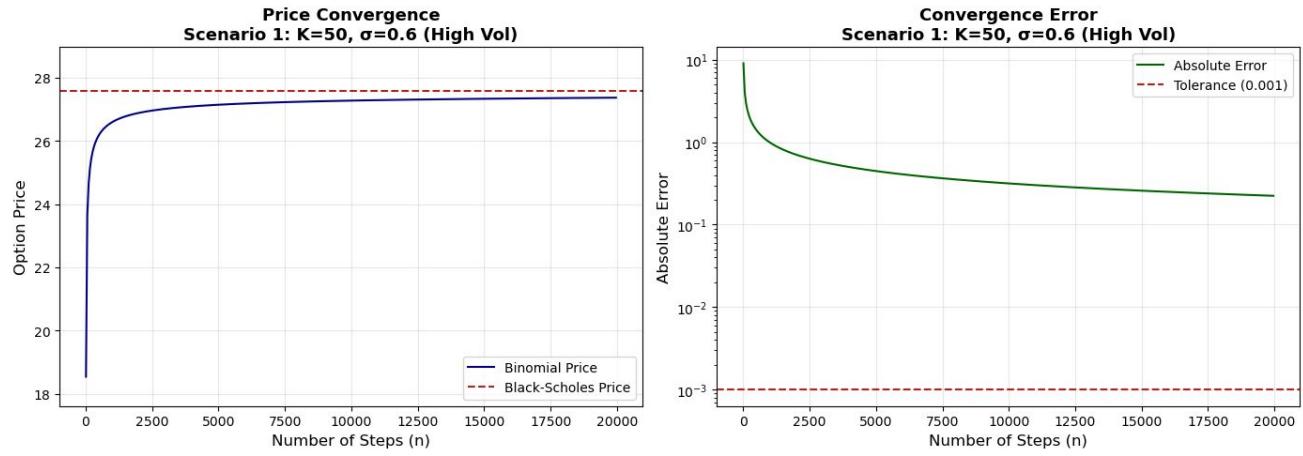


Fig. 4. Convergence results for Scenario 1 using the Leisen-Reimer model.

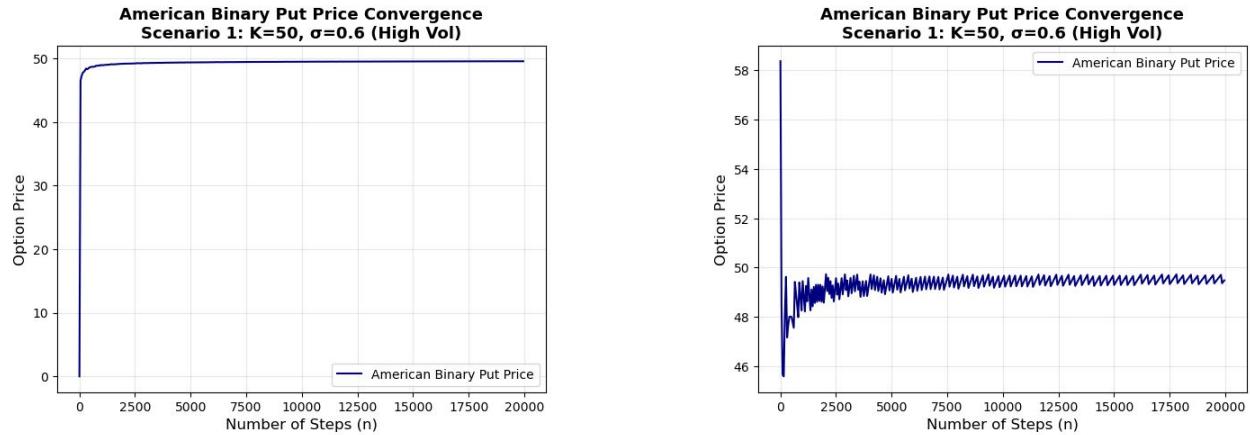


Fig. 5. American binary put price convergence (scenario 1) with Tian model.

monotonic convergence, effectively eliminating the shark-teeth oscillations present in CRR. This behavior follows from its use of the $h^{-1}(z)$, which aligns the binomial probabilities

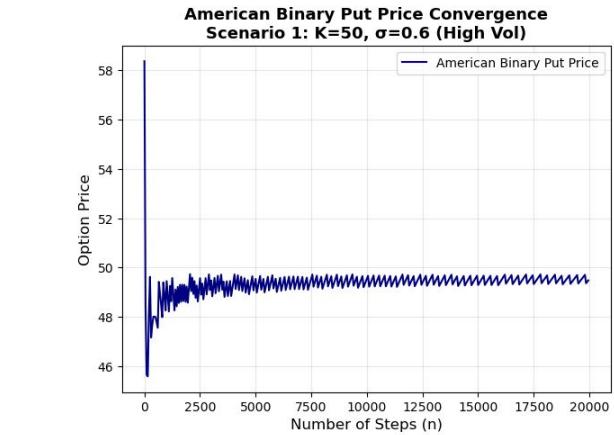


Fig. 6. American binary put price convergence (scenario 1) with CRR model.

with the continuous normal distribution more accurately.

For American options, the possibility of early exercise reduces the situations where the holder prefers to wait rather

than exercise early. Consequently, the American price quickly approaches either zero or the cash amount, explaining the sharp contrast with the European values in several scenarios. Finally, among the models that struggled to meet the convergence tolerance within the maximum tree depth, Leisen–Reimer showed the strongest convergence potential, achieving near-zero error already at $N = 20,001$.

V. CONCLUSION

This paper analyzed the valuation of credit default options on stocks by exploiting their equivalence to cash-or-nothing binary put options. Within this framework we implemented five binomial trees—CRR, Tian, Jarrow–Rudd, drift-adjusted CRR and Leisen–Reimer—and compared their performance by examining convergence toward the Black–Scholes benchmark for European contracts and then using the calibrated step sizes to price the corresponding American binary options.

The convergence experiments for European options confirm that the classical CRR tree provides a natural baseline but suffers from pronounced non-monotone “shark–teeth” behavior when applied to discontinuous payoffs. Tian, Jarrow–Rudd and CRR with drift show the same qualitative pattern, with some reduction in the amplitude of the oscillations but without eliminating them. In the low–volatility scenarios (3 and 4) all models are able to reach the prescribed error tolerance with a relatively small number of steps (N^* in the range of 500–600), and the resulting prices are around the Black–Scholes values reported in Table III. In the high–volatility scenarios (1 and 2), none of the trees attains the tolerance before $N = 20,000$, but the Leisen–Reimer model exhibits the smoothest and most regular decay of the error and achieves the smallest discrepancy with the Black–Scholes benchmark at the maximum depth, confirming its superior convergence properties for digital payoffs.

The comparison between European and American prices highlights the impact of early exercise on credit protection. In Scenario 1, the American prices are around 49.5 across models, while the European values remain close to 27.6, reflecting the value of triggering the payoff as soon as the default barrier is reached rather than waiting until maturity. In Scenarios 2 and 4, where the strike is high and the risk–neutral probability of default is significant throughout the life of the contract, American prices reach the maximum cash amount $C = 100$, whereas the European prices remain strictly below this upper bound due to discounting and the fact that default at T is not certain.

In Scenario 3, both European and American options are essentially worthless for all models, indicating that with low volatility and a low strike the credit event is extremely unlikely. Overall, once N^* is calibrated from the European study, differences across trees in the American setting are small, since all models are evaluated at a discretization that already delivers highly accurate prices.

Finally, the convergence behavior of American options reveals an additional dimension of model performance. All alternative trees—Tian, Jarrow–Rudd, CRR with drift and

Leisen–Reimer—display smooth, monotonic convergence for the American binary put, while the CRR model is the only one that retains visible shark–teeth oscillations when early exercise is allowed. Taken together, these results suggest that for stock–based credit default options it is preferable to employ binomial schemes specifically designed for digital payoffs, such as Leisen–Reimer, especially in high–volatility or deep out–of–the–money settings where convergence is most challenging. At the same time, the strong dependence of prices on volatility, moneyness and the possibility of early exercise underlines the importance of carefully selecting both the lattice model and the parameter scenarios when using binomial trees for credit–risk applications.

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