



# Hands-on Bioeconomy Measurement Workshop ICABR 2024

- Input-Output Analysis and the Measurement of Bioeconomy Value Added

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# Learning Objectives

- Understanding IO Matrix
- Analyzing the different (static, dynamic) measurement methods of Bioeconomy

# What is Bioeconomy?

## Bioeconomy Strategy 2018
## EU Green Deal

Through these policies Europe aims to solve

- Social inequality
- Climate change

Providing a better future


”The bioeconomy covers all **sectors and systems that rely on biological resources** (animals, plants, micro-organisms and derived biomass, including organic waste), their functions and principles”
(Definition by the European Commission)

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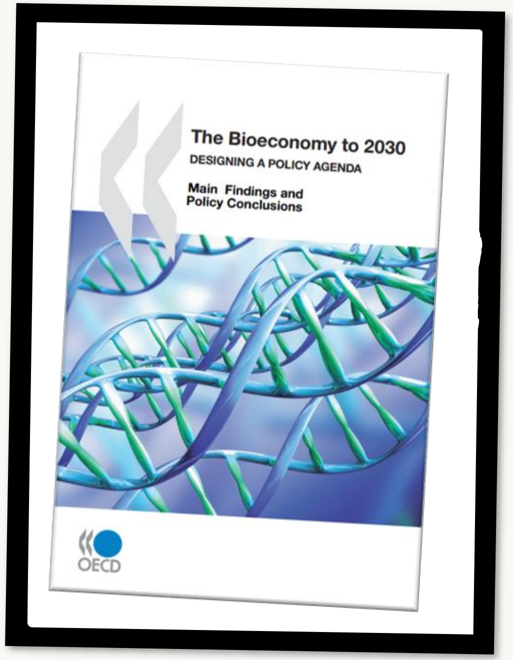



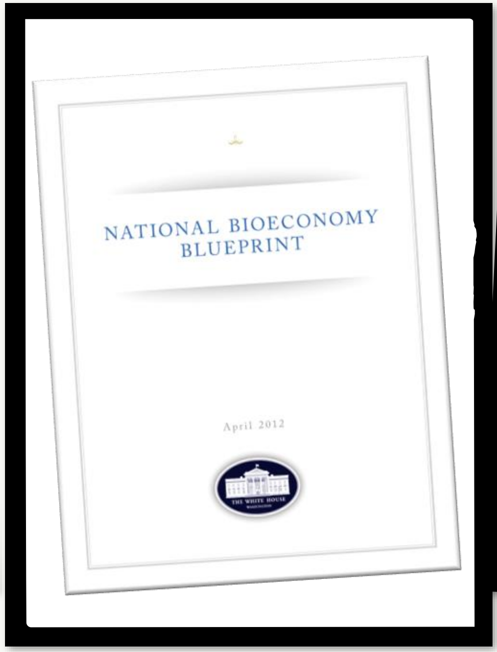



# ***Problems of measuring bioeconomy***

- **Lack of standardized definitions and frameworks:** there is a lack of universally accepted definitions and frameworks.
- **Data availability and quality:** data collection and reporting systems may be fragmented, inconsistent, or unavailable across different regions and sectors.

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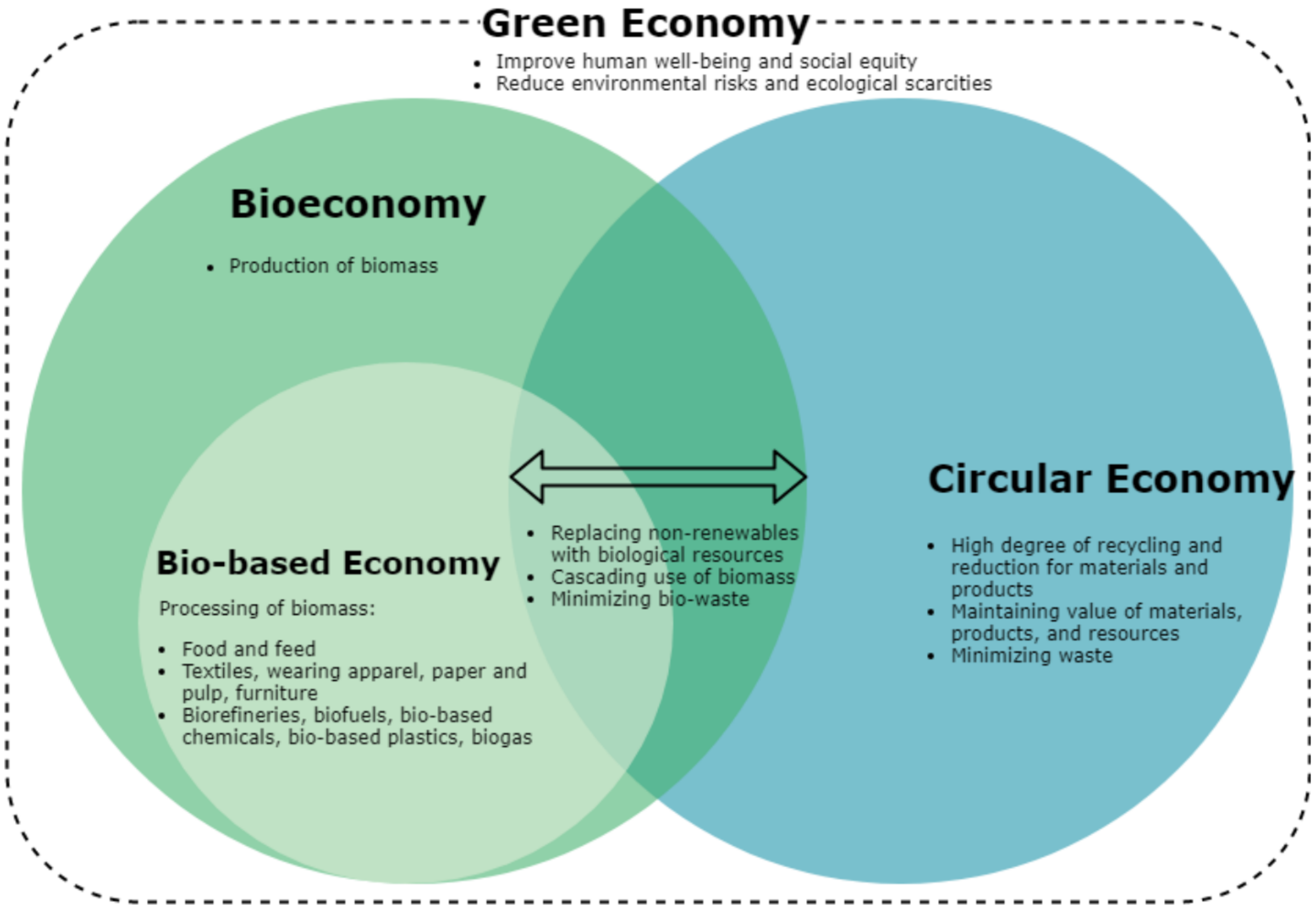
# Problems of measuring bioeconomy

**SCOPE AND BOUNDARIES:** DETERMINING THE SCOPE AND BOUNDARIES OF THE BIOECONOMY IS CHALLENGING.

**INTERCONNECTEDNESS AND INDIRECT EFFECTS:** MEASURING THE DIRECT AND INDIRECT EFFECTS OF BIOECONOMY ACTIVITIES ON OTHER SECTORS CAN BE COMPLEX AND REQUIRE SOPHISTICATED MODELING TECHNIQUES.

**DYNAMIC NATURE AND EVOLVING TECHNOLOGIES:** THE BIOECONOMY IS A RAPIDLY EVOLVING FIELD, WITH NEW TECHNOLOGIES AND INNOVATIONS CONTINUALLY EMERGING.




Source: Kardung et al. 2021

# Static Models

- The “output-based” approach quantifies the value added generated by an industry in proportion to the bioeconomy nature of the outputs it produces that is proportionally to the biomass content of tangible outputs and to the bioeconomy relevance of intangible outputs.

- The “input-based” approach quantifies the value added generated by an industry in proportion to the inputs it uses in the form of biological renewable resources.

- The “weighted Input-Output” approach seeks providing a middle ground quantification of the bioeconomy value added, taking into account the parameters quantified by the output-based and the input-based approaches

- The “Upstream-Downstream” approach measures the bioeconomy value added within the industries by using input flows to downstream and output flows to upstream between industries.

# Downstream-Upstream Model

**Cingiz et al. 2021, 2023**

Suppose there are two types of industries; full bioeconomy denoted with index $i$, and the partly bioeconomy with index $j$.

Suppose the time index is $t$, and assuming there are n sectors, $O_i^t$ stands for output of the full bioeconomy industry $i$, $I_i^t$ stands for inputs to the full bioeconomy industry $i$, $I_{ii}^t$ stands for inputs from the full bioeconomy industry $i$ to the full bioeconomy industry $i$, $I_{ij}^t$ stands for inputs from the full bioeconomy industry $i$ to the partly bioeconomy industry $j$.

$$V_j^t = O_j^t - \sum_{k=1}^{n} I_{kj}^t$$

Assuming simple production function with perfectly substitutable inputs, we have for each sector $i$

$$O_i^t = \alpha_i^t \sum_j^n I_{ji}^t = \alpha_i^t I_i^t$$

If α is known, then we can calculate the bioeconomic part of sector $i$. (Heijman 2016)

# Downstream-Upstream Model

**Downstream effect:** It is calculated by measuring the input flow from full bio-economy industry $i$ to other industries $j$. It is the proportional value added of partly bioeconomy industry, the proportionality comes from the ratio between "the input flow from fully bioeconomic sectors to partly bioeconomy sector" and "the total inputs of partly bioeconomy sector".

$$D_j^t = \frac{\sum_i \left(I_{ij}^t\right)}{I_j^t} \times V_j^t = \beta_j^t V_j^t$$

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# Downstream-Upstream Model

**Upstream effect:** It is the effect that we see in the direction from other industries to fully bioeconomy industries. In order to avoid double counting, we will deduct the downstream value added of partially bioeconomic industry. We do this by multiplying the value added of industry $j$ at time $t$ by 1−β_j^t. The magnitude of the effect is calculated by measuring the output flow from other industries to bioeconomy industry as an input. The percentage is the ratio between “the output flow to the full bioeconomic industries” and “the outputs of partly or non-bioeconomy industries”.

$$U_j^t = \frac{\sum_i I_{ji}^t}{\sum_k I_{jk}^t + F(j,t) + E(j,t)} \times (1 - \beta_j^t) V_j^t .$$

# Downstream-Upstream Model

**Bioeconomy Value Added:** The summation of value added of downstream and upstream effects and the value added of fully bioeconomic industries that is

$$\sum_j (D_j^t + U_j^t) + \sum_i V_i^t$$

**Hence the share of Bioeconomy VA is**

$$\frac{\sum_j (D_j^t + U_j^t) + \sum_i V_i^t}{\sum_i V_i^t + \sum_j V_j^t}$$

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# Downstream-Upstream Model

**Theorem:** For any time $t$, for any sector $j$, the bioeconomic part of sector $j$ can be calculated by both $\alpha_j^t$ and $\beta_j^t$ .

**Proof:**

# Downstream-Upstream Model

**Theorem:** For any time $t$, for any sector $j$, the bioeconomic part of sector $j$ can be calculated by both $\alpha_j^t$ and $\beta_j^t$ .

**Proof:** Assume that for time $t$ and sector $j$, both $\alpha_j^t$ and $\beta_j^t$ exists and greater than 0. It is enough to show that

$$(\alpha_j^t - 1) \sum_{i \in A} \left(I_{ij}^t\right) = \frac{\sum_{i \in A} \left(I_{ij}^t\right)}{I_j^t} \times V_j^t = \beta_j^t V_j^t \qquad (1)$$

where $A$ is the set of fully bioeconomy industries. Since $\alpha_j^t$ and $\beta_j^t$ are greater than 0, we have $\sum_{i \in A} \left(I_{ij}^t\right) \neq 0$ and $I_j^t \neq 0$. Then we cancel $\sum_{j \in A} \left(I_{ji}^t\right)$ both sides and multiply by $I_j^t$

$$(\alpha_j^t - 1) = \frac{V_j^t}{I_j^t}$$

$$\alpha_j^t I_j^t - I_j^t = V_j^t$$

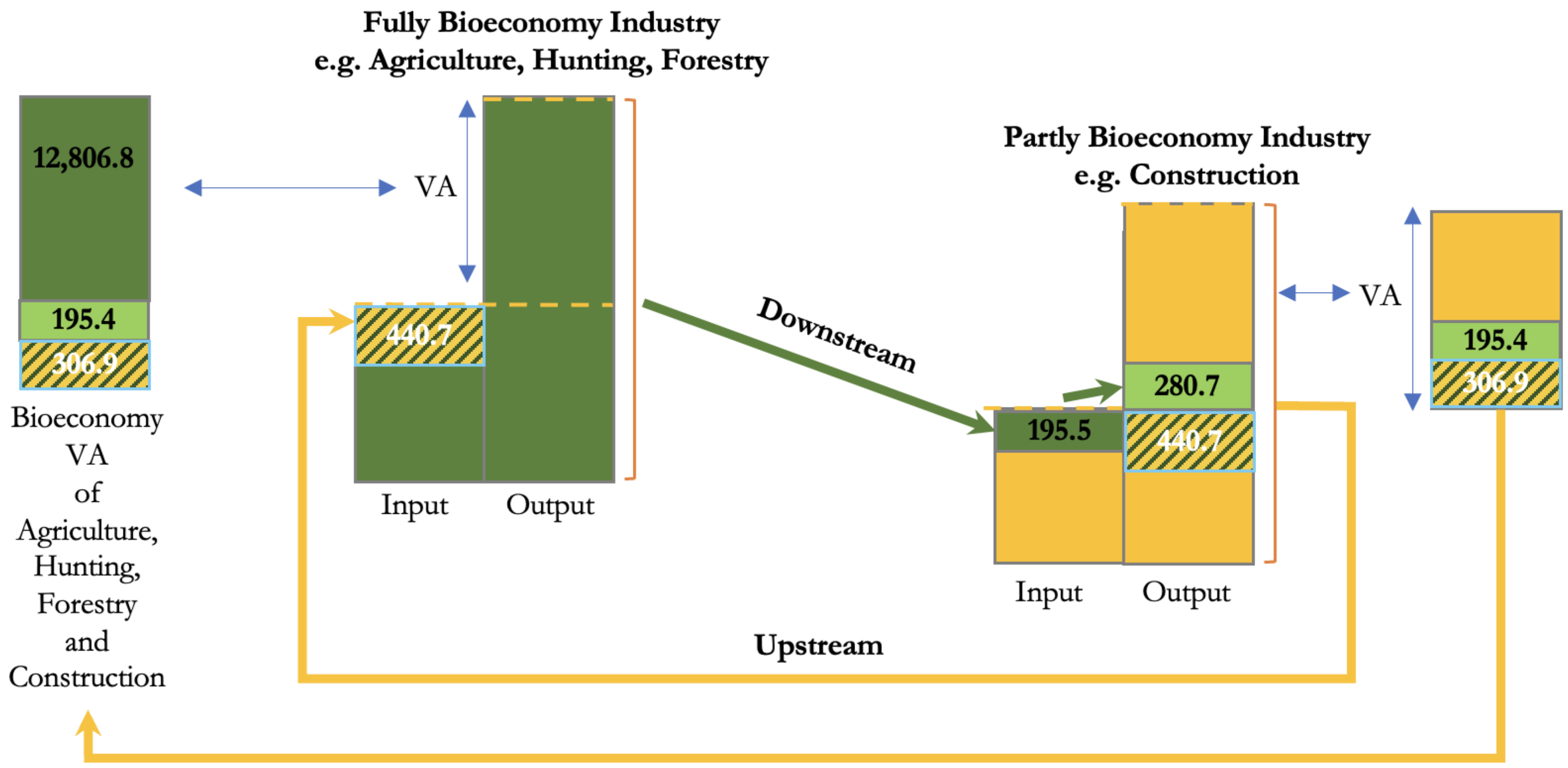
By the definition of $\alpha_j^t$, we conclude that

$$O_j^t - I_j^t = \alpha_j^t I_j^t - I_j^t = V_j^t$$

Hence equation (1) satisfies. □

# Downstream-Upstream Model

**Figure: Upstream, Downstream Input-Output Flow of Netherlands 2018.**




**Bioeconomy VA of two industries = Fully Bioeconomy VA + Downstream VA + Upstream VA**

*Table 2: Sectors of the bioeconomy included in previous studies and in the BioMonitor project*

| NACE | | Fumagalli and Trenti(2014) | SAT-BBE (2015) | Efken et al. (2016) | European Commission (2018a) | Piotrowski et al. (2018) | Ronzon et al. (2017) | BioMonitor project |
|---|---|---|---|---|---|---|---|---|
| A01 | Crop and animal production, hunting and related service activities | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| A02 | Forestry and logging | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| A03 | Fishing and aquaculture | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| C10 | Manufacture of food | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓✓ |
| C11 | Manufacture of beverages | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓✓ |
| C12 | Manufacture of tobacco | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓✓ |
| C13 | Manufacture of textiles | X | ✓ | ✓ | ✓ | ✓ | ✓ | ✓✓ |
| C14 | Manufacture of wearing apparel | X | ✓ | ✓ | ✓ | ✓ | ✓ | ✓✓ |
| C15 | Manufacture of leather and related products | X | ✓ | ✓ | ✓ | ✓ | ✓ | ✓✓ |
| C16 | Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓✓ |
| C17 | Manufacture of paper and paper products | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓✓ |
| C19 | Manufacture of coke and refined petroleum products | X | ✓ | X | X | X | X | ✓✓ |
| C20 | Manufacture of chemicals and chemical products | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓✓ |
| C21 | Manufacture of basic pharmaceutical products and pharmaceutical preparations | X | X | ✓ | ✓ | ✓ | ✓ | ✓✓ |
| C22 | Manufacture of rubber and plastic products | X | ✓ | X | ✓ | ✓ | ✓ | ✓✓ |
| C2365 | Manufacture of fibre cement | X | X | X | X | X | X | ✓✓ |
| C31 | Manufacture of furniture | X | ✓ | X | ✓ | ✓ | ✓ | ✓✓ |
| C7211 | Research and experimental development on biotechnology | X | X | X | X | X | X | ✓✓ |
| D35 | Electricity, gas, steam and air conditioning supply | X | ✓ | X | ✓ | ✓ | X | ✓✓ |
| *D3511* | *Production of electricity* | X | ✓ | X | X | X | ✓ | ✓✓ |
| *E36* | *Water collection, treatment and supply* | X | X | X | X | X | X | ✓ |
| *E37* | *Sewerage* | X | X | X | X | X | X | ✓ |
| *E38* | *Waste collection, treatment and disposal activities; materials recovery* | X | X | X | X | X | X | ✓ |
| *E39* | *Remediation activities and other waste management services* | X | X | X | X | X | X | ✓ |
| F41 | Construction of buildings | X | ✓ | X | X | X | X | ✓ |
| F42 | Civil engineering | X | ✓ | X | X | X | X | ✓ |
| G46 | Wholesale trade, except of motor vehicles and motorcycles | X | X | ✓ | X | X | X | ✓ |
| G47 | Retail trade, except of motor vehicles and motorcycles | X | X | ✓ | X | X | X | ✓ |
| H | Transportation and storage | X | X | X | X | X | X | ✓ |
| I55 | Accommodation | X | X | ✓ | X | X | X | ✓ |
| I56 | Food and beverage service activities | X | X | ✓ | X | X | X | ✓ |
| R9104 | Botanical and zoological gardens and nature reserves activities | X | X | X | X | X | X | ✓ |
| | ✓ = Included, ✓✓ = Focus | | | | | | | |

# Industries

- 5 Fully bioeconomy sectors (ISIC Rev.4 codes):

01T02 (Agriculture, hunting, forestry),

03 (Fishing and aquaculture),

10T12 (Food products, beverages and tobacco)

16 (Wood and products of wood and cork)

17T18 (Paper products and printing)

- Partly bioeconomy sectors: Biomonitor scope or all other sectors.

biomonitor

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# Exercise

**Question:** Please calculate the bioeconomy VA generated by the two sectors? (Note: Input flow from Construction (F) to all other industries is equal to 16,008.1.)

| | A1-3 | F | Final Demand | Exports | Total Output |
|---|---|---|---|---|---|
| A1-3 | 4,302.6 | 194.9 | 19,847.4 | 13,247.2 | 32,637.9 |
| F | 172.3 | 6,315.1 | 77,865.2 | 669.1 | 93,290.8 |
| Imports | 8,402.4 | 592.1 | | | |
| Total Input | 20,207.9 | 61,716.8 | | | |
| Value Added | 12,429.7 | 31,573.7 | | | |
| Total Output | 32,637.6 | 93,290.5 | | | |

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# Exercise

Now we can calculate the downstream and upstream effects (all values are in million Euros.)

$$D_{construction}^{2015} = \frac{\sum_i (I_{ij}^t)}{I_{construction}^{2015}} \times V_{construction}^{2015} = \frac{194.9}{61{,}716.8} \times 31{,}573.7 = 99.7 \qquad \textbf{(8)}$$

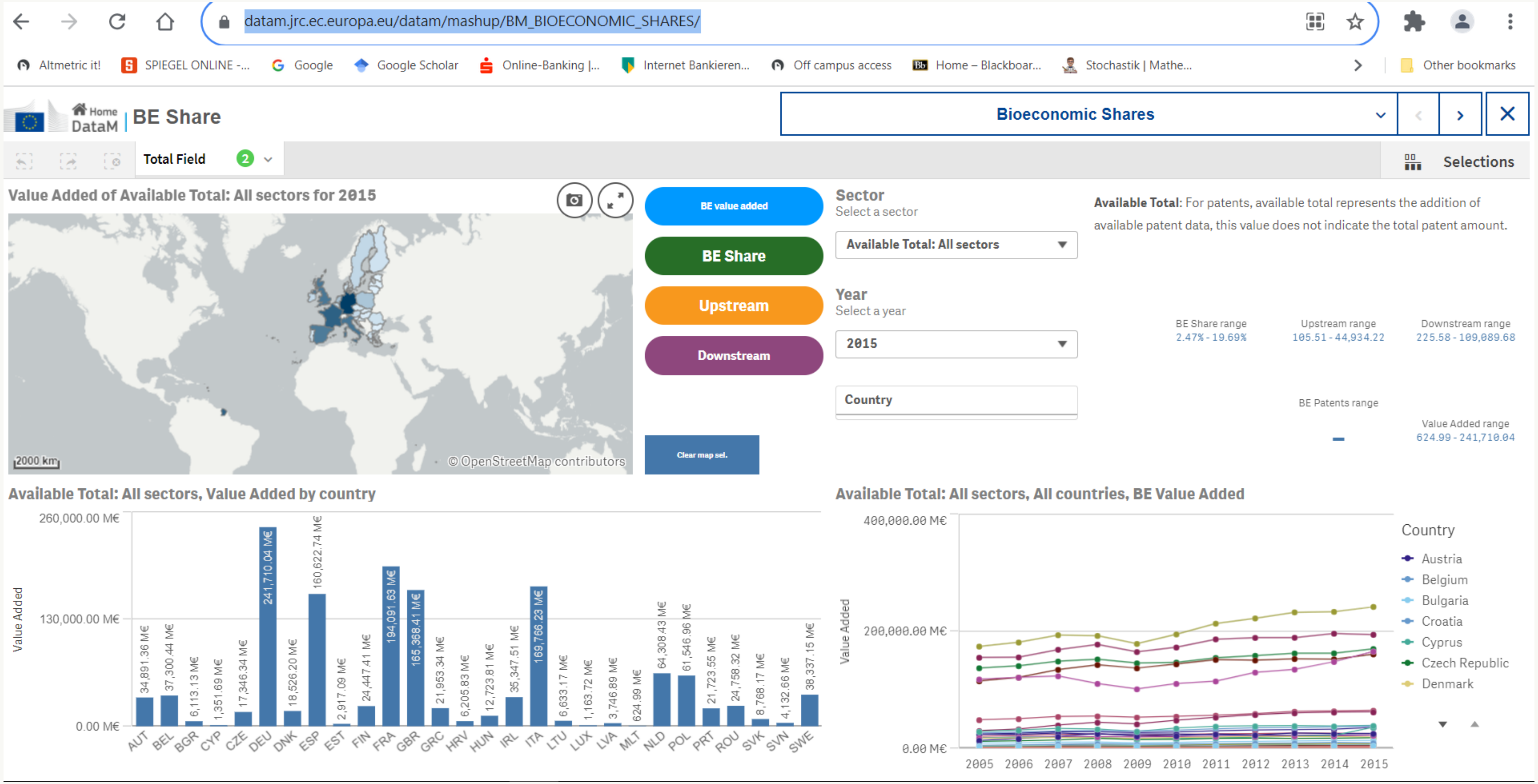
Assume there are fixed binary ratios between value added, output and input. Now we deduct the calculated downstream effect from the value added of construction which gives us $31{,}573.7 \times (1 - \frac{194.9}{61{,}716.8}) = 31473.99$. Then the upstream calculation is as follows

$$U_{construction}^{2015} = \frac{I_{construction,S1}^{2015}}{\sum_k I_{construction,k}^{2015} + F(construction,2015) + E(construction,2016)} \times$$

$$(1 - \frac{194.9}{61{,}716.8}) V_{construction}^{2015} = \frac{172.3}{16{,}008.1 + 77{,}865.2 + 669.1} \times 31473.99 = 57.36. \qquad \textbf{(9)}$$

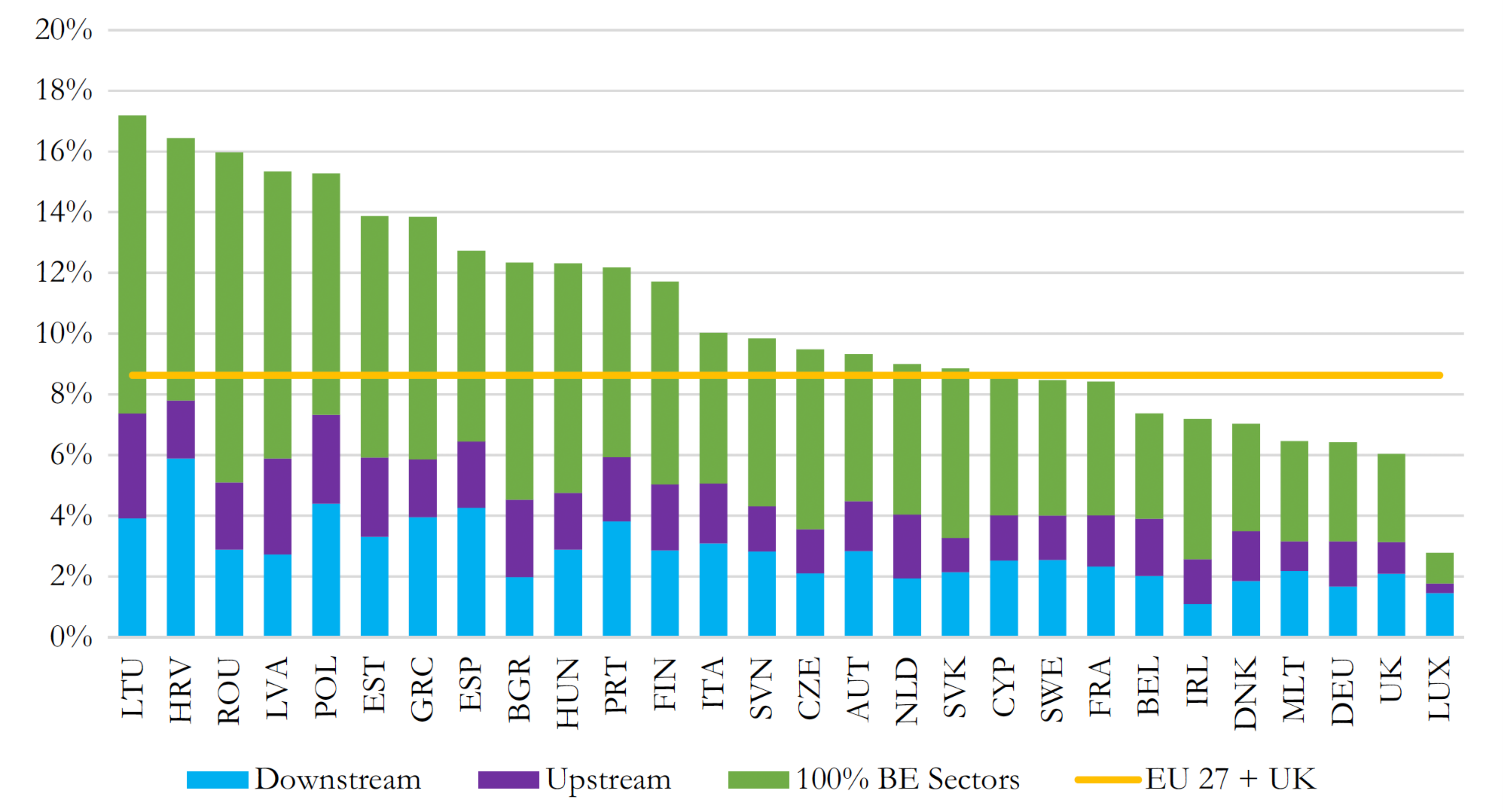
# Bioeconomy Dashboard

https://datam.jrc.ec.europa.eu/datam/mashup/BM_BIOECONOMIC_SHARES/



# Results

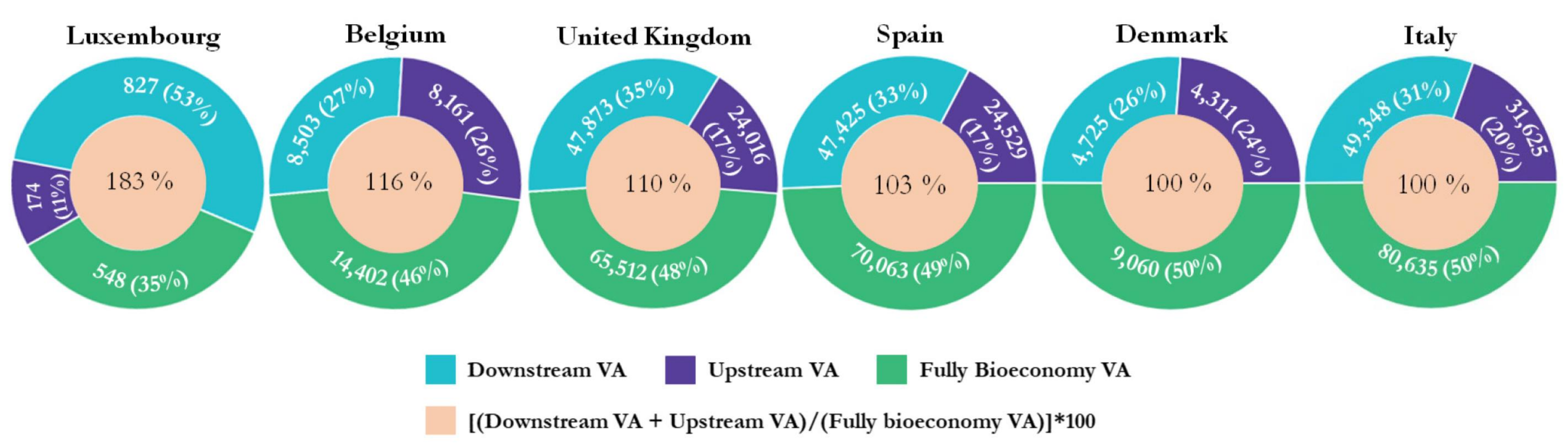
*Figure 2: EU Member States bioeconomy shares in value added, 2016-2018 average*




Source: Cingiz et al. 2023

# Results

Figure 3: Countries where downstream and upstream VA constitutes at least 50% of the total bioeconomy, 2018

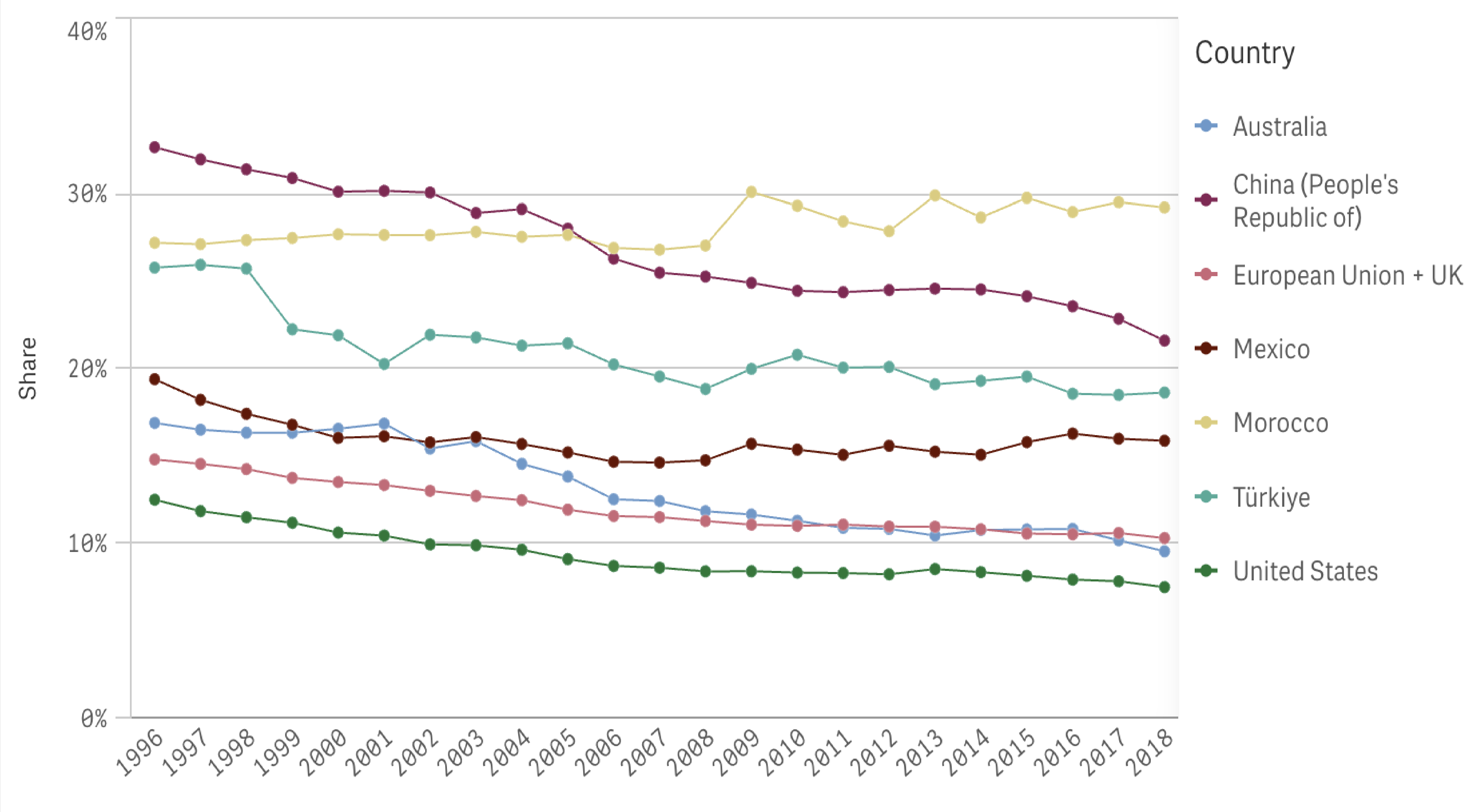



The centre indicator is the ratio of bioeconomy downstream and upstream VAs to fully bioeconomy industries VA in percentage. The figures in the ring represent the value of each component in million Euros, with their relative contribution to total bioeconomy VA in parentheses.

Source: Cingiz et al. 2023

# Results

Figure 3: Bioeconomy shares around the world 1996-2018



# Dynamic Model:
# Input Output (IO) Matrix Description

- It shows inter-industry linkages within an economy
- Input-Output flows between industries
- It makes clear the inter dependencies between industries
- Each column represents the monetary of inputs to each sector
- Each row represents the monetary value of each sectors output.

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# IO Matrix Assumptions

- Perfect equilibrium
- Homogenous output
- Constant returns to scale
- Fixed factor combination

# IO Matrix Model

**Table 2.1** Single-region input–output table with macro-economic totals

| | Industry 1 | Industry $j$ | Industry $I$ | Local final demand exports | | | Total |
|---|---|---|---|---|---|---|---|
| | *1st quadrant* | | | *2nd quadrant* | | | |
| Industry 1 | [redacted] | … | [redacted] | [redacted] | … | [redacted] | $x_1$ |
| Industry $i$ | … | [redacted] | … | … | [redacted] | … | $x_i$ |
| Industry $I$ | [redacted] | … | [redacted] | [redacted] | … | [redacted] | $x_I$ |
| | *3rd quadrant* | | | *4th quadrant* | | | |
| Imports | … | … | … | … | … | … | $M$ |
| Value added | … | [redacted] | … | … | [redacted] | … | $Y$ |
| Total | $x_1$ | $x_j$ | $x_I$ | $C$ $I$ | $G$ | $E$ | |

Source:Oosterhaven, Jan. "Rethinking Input-Output Analysis."

A Spatial Perspective: Springer International Publishing (2019).

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# IO Matrix Model

Following the notations in Miller and Blair (2009);

- let $z\_ij$ denote the input from sector $i$ to $j$,
- $f\_i$ denote the final demand of sector $i$ including imports and exports,
- $v\_i$ denote the value added of sector $i$,
- $x\_i$ denote the output of sector $i$.

All transactions are measured in monetary terms, and all variables are members of the set of real numbers. The following shows the structure of an input-output table.

| | Sector 1 | … | Sector n | Final Demand | Total |
|---|---|---|---|---|---|
| Sector 1 | $z_{11}$ | | $z_{1n}$ | $f_1$ | $x_1$ |
| … | … | … | … | … | … |
| Sector n | $z_{n1}$ | … | $z_{nn}$ | $f_n$ | $x_n$ |
| Value Added | $v_1$ | … | $v_n$ | | |
| Total | $x_1$ | … | $x_n$ | | |

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# IO Matrix Model

$$Z = [z_{ij}] = \begin{bmatrix} z_{11} & \cdots & z_{1n} \\ \vdots & z_{ij} & \vdots \\ z_{n1} & \cdots & z_{nn} \end{bmatrix}, \ f = [f_i] = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix}$$

$$V = [v_i] = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}, \ X = [x_i] = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}.$$

Let $A = [a_{ij}] = \begin{bmatrix} z_{11}/x_1 & \cdots & z_{1n}/x_n \\ \vdots & z_{ij}/x_j & \vdots \\ z_{n1}/x_1 & \cdots & z_{nn}/x_n \end{bmatrix}$ denote the Leontief direct input coefficient matrix, then the Leontief input inverse matrix is $L = (I - A)^{-1}$, where $I$ is the identity matrix, and $X = Lf = (I - A)^{-1}f$.

# IO Matrix Model

Note that $A$ matrix row sums has the following property:

$$\forall i \; and \; \forall j, \;\; z_{1j}/x_j + \; z_{2j}/x_j + \cdots + z_{ij}/x_j + \cdots + \; z_{n1}/x_j \leq 1$$

We can interpret $X$ as the final demand (external demand) and $(I-A)^{-1}f$ as the amount that the sectors must produce. If we increase the final demand with 1 unit extra of industry 1 output, then we have

$$X^{new} = (I-A)^{-1}(f + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}) = \; (I-A)^{-1}f + \; (I-A)^{-1} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Note that $(I-A)^{-1} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ contains the first column of $(I-A)^{-1}$ which shows us

that the first column in $(I-A)^{-1}$ indicates how the production must change in each of the Sectors if the final demand in the first Sector changes by +1.

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# IO Matrix Model

Remember the geometric taylor series of $\frac{1}{1-x}$, so we can write the following equation:

$$(I - A)^{-1} = I + A + A^2 + \ldots + A^n + \ldots$$

We wish to produce final demand. To do so would suggest perhaps $X = f$. However, this does not consider the fact that we need to produce not just $f$ but also enough to feed the internal demand, so perhaps $X = f + Af$. But we also need to feed the internal demand to feed that internal demand, this is $A(Af) = A^2 f$, and so on, so really:

$$X = f + Af + A^2 f + \ldots + A^n f + \ldots = (I - A)^{-1} f$$

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# Matrix Inverse

**Definition:** If $A$ is an $n \times n$ matrix then the $(i, j)$-cofactor of $A$ is

$$C_{ij} = (-1)^{i+j} \det(A_{ij})$$

where $A_{ij}$ is the matrix $A$ with row $i$ and column $j$ removed.

**Theorem:** Let $A$ be an invertible $n \times n$ matrix, then

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

where $\operatorname{adj}(A)$ is the adjugate of $A$ (attention: subscript order):

$$\operatorname{adj}(A) = \begin{pmatrix} C_{11} & C_{21} & \cdots & C_{n1} \\ C_{12} & C_{22} & \cdots & C_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ C_{1n} & C_{2n} & \cdots & C_{nn} \end{pmatrix}$$

# Matrix Inverse

The $(i, j)$ entry of $A^{-1}$ is:

$$(A^{-1})_{ij} = \frac{1}{\det(A)} C_{ji}$$

$$= \frac{1}{\det(A)} (-1)^{i+j} \det(A_{ji})$$

$$= \frac{(-1)^{i+j} \det(A_{ji})}{\det(A)}$$

where $A_{ji}$ is the matrix $A$ with row $j$ and column $i$ removed.

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} +\det(A_{11}) & -\det(A_{21}) & +\det(A_{31}) & \cdots \\ -\det(A_{12}) & +\det(A_{22}) & -\det(A_{32}) & \cdots \\ +\det(A_{13}) & -\det(A_{23}) & +\det(A_{33}) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

# Leontief Matrix Example

Consider the consumption matrix for three Sector

$$M = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.4 & 0.1 & 0.2 \\ 0.3 & 0.3 & 0.1 \end{bmatrix}$$

- Suppose the external demand for Sector 3 change by +2. How must production in Sector 1 change?
- HW: Suppose the external demand for Sector 2 change by +1. How must production in Sector 3 change?
- HW: Suppose the external demand for Sector 1 change by +3. How must production in Sector 2 change?

# Leontief Matrix Example

## Step 1: Calculate $I - M$

Given the consumption matrix $M$:

$$M = \begin{pmatrix} 0.1 & 0.2 & 0.1 \\ 0.4 & 0.1 & 0.2 \\ 0.3 & 0.3 & 0.1 \end{pmatrix}$$

The identity matrix $I$ is:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So, $I - M$ is:

$$I - M = \begin{pmatrix} 0.9 & -0.2 & -0.1 \\ -0.4 & 0.9 & -0.2 \\ -0.3 & -0.3 & 0.9 \end{pmatrix}$$

# Leontief Matrix Example

## Step 2: Find the Determinant of $I - M$

The determinant of a $3 \times 3$ matrix $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ is calculated as:

$$\det(A) = a(ei - fh) - b(di - fg) + c(dh - eg)$$

For our matrix $I - M$:

```
det(I - M) = 0.9[(0.9 * 0.9) - (-0.2 * -0.3)] - (-0.2)[(-0.4 * 0.9)
             -(-0.2 * -0.3)] + (-0.1)[(-0.4 * -0.3) - (0.9 * -0.3)]
           = 0.9[0.81 - 0.06] - (-0.2)[-0.36 - 0.06] + (-0.1)[0.12 + 0.27]
           = 0.9[0.75] - (-0.2)[-0.42] + (-0.1)[0.39]
           = 0.675 - 0.084 - 0.039
           = 0.552
```

# Leontief Matrix Example

## Step 3: Calculate the Cofactor Matrix

To find the cofactor matrix, we need to calculate the cofactor for each element $a_{ij}$ of the matrix $I - M$. The cofactor $C_{ij}$ is given by:

$$C_{ij} = (-1)^{i+j} \det(M_{ij})$$

where $M_{ij}$ is the minor of $a_{ij}$, i.e., the matrix obtained by deleting the $i$-th row and $j$-th column.

Let's compute the cofactors one by one:

$$C_{11} = \det \begin{pmatrix} 0.9 & -0.2 \\ -0.3 & 0.9 \end{pmatrix} = 0.9 \cdot 0.9 - (-0.2) \cdot (-0.3) = 0.81 - 0.06 = 0.75$$

$$C_{12} = -\det \begin{pmatrix} -0.4 & -0.2 \\ -0.3 & 0.9 \end{pmatrix} = -(-0.4 \cdot 0.9 - (-0.2) \cdot (-0.3)) = -(-0.36 - 0.06) = 0.42$$

$$C_{13} = \det \begin{pmatrix} -0.4 & 0.9 \\ -0.3 & -0.3 \end{pmatrix} = -0.4 \cdot (-0.3) - 0.9 \cdot (-0.3) = 0.12 + 0.27 = 0.39$$

# Leontief Matrix Example

## Step 3 Cont.: Calculate the Cofactor Matrix

$$C_{21} = -\det\begin{pmatrix} -0.2 & -0.1 \\ -0.3 & 0.9 \end{pmatrix} = -(-0.2 \cdot 0.9 - (-0.1) \cdot (-0.3)) = -(-0.18 - 0.03) = 0.21$$

$$C_{22} = \det\begin{pmatrix} 0.9 & -0.1 \\ -0.3 & 0.9 \end{pmatrix} = 0.9 \cdot 0.9 - (-0.1) \cdot (-0.3) = 0.81 - 0.03 = 0.78$$

$$C_{23} = -\det\begin{pmatrix} 0.9 & -0.2 \\ -0.3 & -0.3 \end{pmatrix} = -(0.9 \cdot (-0.3) - (-0.2) \cdot (-0.3)) = -(-0.27 - 0.06) = 0.33$$

$$C_{31} = \det\begin{pmatrix} -0.2 & -0.1 \\ 0.9 & -0.2 \end{pmatrix} = -0.2 \cdot (-0.2) - (-0.1) \cdot 0.9 = 0.04 + 0.09 = 0.13$$

$$C_{32} = -\det\begin{pmatrix} 0.9 & -0.1 \\ -0.4 & -0.2 \end{pmatrix} = -(0.9 \cdot (-0.2) - (-0.1) \cdot (-0.4)) = -(-0.18 - 0.04) = 0.22$$

$$C_{33} = \det\begin{pmatrix} 0.9 & -0.2 \\ -0.4 & 0.9 \end{pmatrix} = 0.9 \cdot 0.9 - (-0.2) \cdot (-0.4) = 0.81 - 0.08 = 0.73$$

# Leontief Matrix Example

## Step 4: Compute the Adjugate Matrix

The cofactor matrix is:

$$C = \begin{pmatrix} 0.75 & 0.42 & 0.39 \\ 0.21 & 0.78 & 0.33 \\ 0.13 & 0.22 & 0.73 \end{pmatrix}$$

The adjugate (adjoint) matrix is the transpose of the cofactor matrix:

$$\text{adj}(I - M) = C^T = \begin{pmatrix} 0.75 & 0.21 & 0.13 \\ 0.42 & 0.78 & 0.22 \\ 0.39 & 0.33 & 0.73 \end{pmatrix}$$

# Leontief Matrix Example

## Step 5: Calculate the Inverse Using the Adjugate and Determinant

The inverse of $I - M$ is given by:

$$(I - M)^{-1} = \frac{1}{\det(I - M)} \operatorname{adj}(I - M)$$

Given:

$$\det(I - M) = 0.552$$

$$\operatorname{adj}(I - M) = \begin{pmatrix} 0.75 & 0.21 & 0.13 \\ 0.42 & 0.78 & 0.22 \\ 0.39 & 0.33 & 0.73 \end{pmatrix}$$

Calculate $\frac{1}{0.552} \approx 1.8116$

Multiply each element of the adjugate matrix by 1.8116

Result:

$$(I - M)^{-1} \approx \begin{pmatrix} 1.3587 & 0.3804 & 0.2355 \\ 0.7609 & 1.4130 & 0.3986 \\ 0.7072 & 0.5978 & 1.3225 \end{pmatrix}$$

# Leontief Matrix Example

## Step 6: Calculate the Production Vectors

The initial production vector $x$ is calculated as:

$$x = (I - M)^{-1} d$$

Given the inverse matrix $(I - M)^{-1}$:

$$(I - M)^{-1} = \begin{pmatrix} 1.3587 & 0.3804 & 0.2355 \\ 0.7609 & 1.4130 & 0.3986 \\ 0.7072 & 0.5978 & 1.3225 \end{pmatrix}$$

Perform the matrix multiplication:

$$x = \begin{pmatrix} 1.3587 & 0.3804 & 0.2355 \\ 0.7609 & 1.4130 & 0.3986 \\ 0.7072 & 0.5978 & 1.3225 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.9746 \\ 2.5725 \\ 2.6275 \end{pmatrix}$$

# Leontief Matrix Example

## Step 7: Change in Production for Sector 1

The new production vector $x'$ is calculated as:

$$x' = (I - M)^{-1} d'$$

Perform the matrix multiplication:

$$x' = \begin{pmatrix} 1.3587 & 0.3804 & 0.2355 \\ 0.7609 & 1.4130 & 0.3986 \\ 0.7072 & 0.5978 & 1.3225 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2.4456 \\ 3.3692 \\ 4.2748 \end{pmatrix}$$

The change in production for Sector 1 is:

$$\Delta x_1 = x_1' - x_1$$

$$\Delta x_1 = 2.4456 - 1.9746$$

$$\Delta x_1 = 0.471$$

# Measuring Bioeconomy

International Consortium on Applied Bioeconomy Research ICABR

## 2.1 The Hypothetical Extraction Method (HEM)

The basic principle of HEM is to quantify how much the total economy would decrease if a particular sector were removed from the input-output table (Miller & Blair 2009).

| | **Sector 1** | **…** | **Sector n** | **Final Demand** | **Total** |
|---|---|---|---|---|---|
| **Sector 1** | $z_{11}$ | | $z_{1n}$ | $f_1$ | $x_1$ |
| **…** | … | … | … | … | … |
| **Sector n** | $z_{n1}$ | … | $z_{nn}$ | $f_n$ | $x_n$ |
| **Value Added** | $v_1$ | … | $v_n$ | | |
| **Total** | $x_1$ | … | $x_n$ | | |

# The Hypothetical Extraction Method (HEM)

To identify how important some sectors, for example, the first k sectors are, now we partition A with the sector in the upper left (square) submatrix.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \xrightarrow{\text{Remove}} \bar{A} = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix}$$

$$\bar{L} = (I - \bar{A})^{-1}$$

$$\bar{X} = \bar{L}f$$

$$\Delta\bar{X} = e(X - \bar{X}) = e'(L - \bar{L})f \longrightarrow \Delta\bar{V} = \left[\frac{V}{X}\right]'(\bar{X} - X) = \left[\frac{V}{X}\right]'(L - \bar{L})f$$

# The Hypothetical Extraction Method (HEM)

**HEM to measure the bioeconomy value added**

We classify all industries into two categories, fully bioeconomy sectors (F sectors) and partly bioeconomy sectors (P sectors).

1. For F sectors, we think **Transactions between F to F sectors are bioeconomy transactions.**

1. For P sectors, we calculate the downstream effect, the blocks from F to P sectors, and the upstream effect, the blocks from P to F sectors. **Transactions between F to P sectors, and P to F sectors, are bioeconomy transactions.**

# The Hypothetical Extraction Method (HEM)

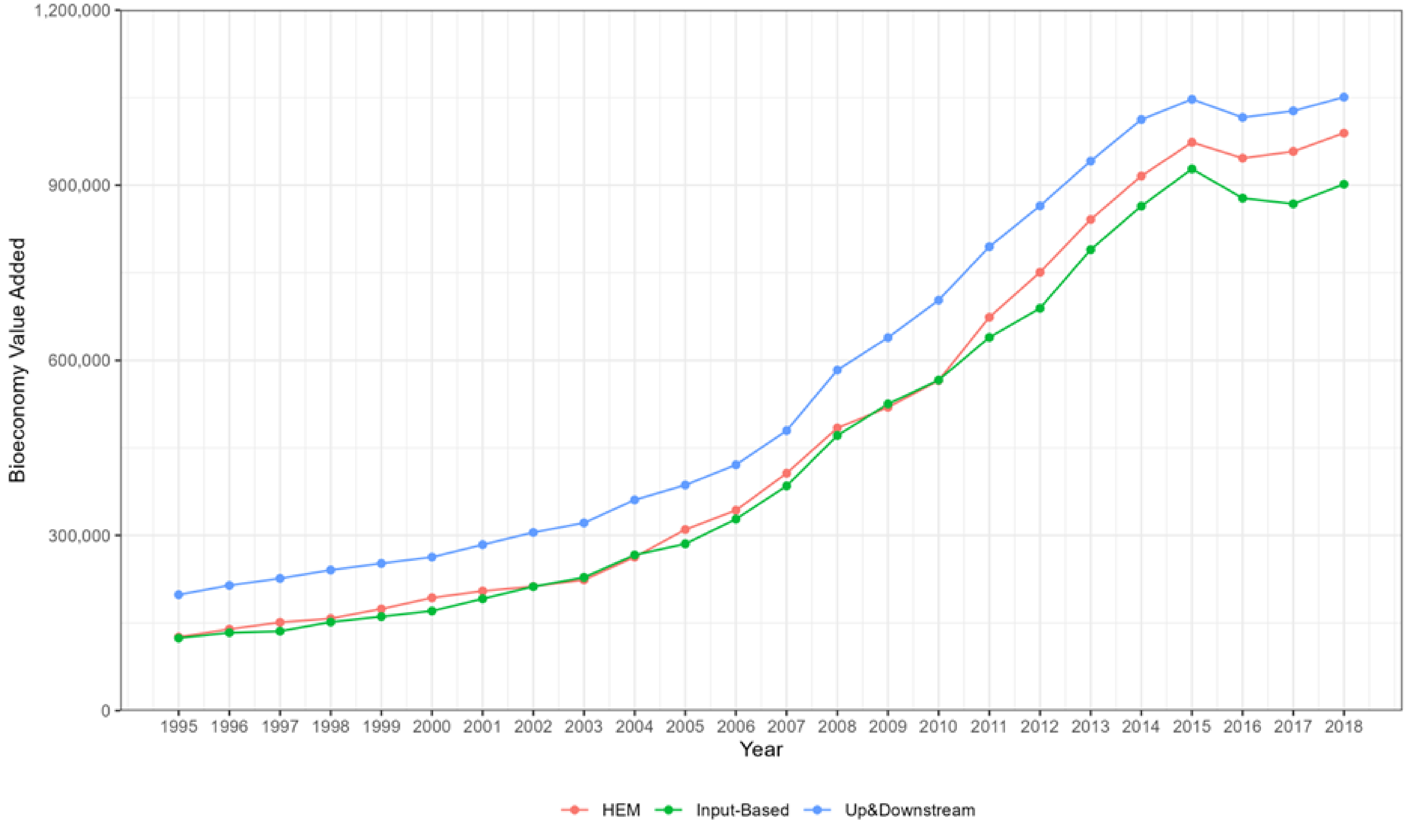
For example, assume industries 1, 3 and 5 are F Sectors, the rest are P Sectors. When extracting the blocks from F to F sectors, F to P sectors and P to F sectors in A, we could get

$$\bar{A} = [\overline{a_{ij}}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & a_{24} & 0 & a_{26} & \cdots & a_{2n} \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 & a_{46} & \cdots & a_{4n} \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & a_{62} & 0 & a_{64} & 0 & a_{66} & \cdots & a_{6n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & a_{n2} & 0 & a_{n4} & 0 & a_{n6} & \cdots & a_{nn} \end{bmatrix}$$

# The Hypothetical Extraction Method (HEM)

| | Original | After-Extraction |
|---|---|---|
| **Coefficient** | $A$ | $\bar{A}$ |
| **Inverse** | $L = (I - A)^{-1}$ | $\bar{L} = (I - \bar{A})^{-1}$ |
| **Output** | $X = Lf$ | $\bar{X} = \bar{L}f$ |
| **Bioeconomy·Output** | $e'(L - \bar{L})f$ | |
| **Bioeconomy·Value·Added** | $[\frac{V}{X}]'(L - \bar{L})f$ | |

# Results Compared




The Chinese bioeconomy value added measured by HEM, the input-based approach (Heijman 2016, Kuosmanen et al. 2020) and the up- and downstream (Cingiz et al. 2021) approach.

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