

Homework 9

Math/CSCI 166, Fall 2022

Assigned: Friday, November 18, 2022

Due: Friday, December 2, 2022 by 1:30pm on Gradescope

(1) **(Richardson Extrapolation Formulas)**

- (a) Suppose you have two values from the Trapezoidal Rule, $T(h)$ and $T(h/3)$. Use Richardson extrapolation to combine these two values into a better one $R_1(h)$. What is the order of the error for your answer?
- (b) Perform Richardson extrapolation on $R_1(h)$ and $R_1(h/3)$ to obtain $R_2(h)$. What is the formula for $R_2(h)$? To compute $R_2(h)$, we need $T(h)$ for several values of h – what are these values of h ?
- (c) Write a program that computes $R_2(1)$ for the following integral:

$$\int_0^1 x e^{-x^2} dx$$

Within your code, you should write your own code for computing the trapezoidal rule. Turn in your code and the result. (The true answer is 0.31606027941....)

(2) **(Gaussian Quadrature)** In this problem, you will write code that uses 5-point Gaussian quadrature to integrate a function from -1 to 1.

- (a) First, write a code that finds the evaluation points and associated weights for the 5-point Gaussian quadrature. Here, $P_5(x) = x^5 - \frac{10}{9}x^3 + \frac{5}{21}x$. Turn in your code and the output of the points and weights.
- (b) Now write code that uses the results from part (a) to integrate any function on $[-1,1]$. You can hardcode in the results from part (a) if you prefer. Turn in your code.
- (c) Use your code to evaluate $\int_{-1}^1 x^8 + 42x^7 dx$ and verify you get the exact answer (since this is a polynomial of degree ≤ 9 (to roughly machine precision). (Answer is 2/9).
- (d) Use your code to evaluate $\int_{-1}^1 x^{10} + 42x^7 dx$. Should you get the exact answer here? (Answer is 2/11).
- (e) Use your code to evaluate $\int_0^1 x e^{-x^2} dx$ (after an appropriate change of variables). How does your answer compare to the results in Question (1)?