Homework 8
Math/CSCI 166, Fall 2022

Assigned: Friday, November 11, 2022
Due: Friday, November 18, 2022 by 1:30pm on Gradescope

(1) **(Nonlinear Systems of Equations)** Consider the nonlinear system

$$\begin{aligned} 2x_1 - 3x_2 + x_3 - 4 &= 0 \\ 2x_1 + x_2 - x_3 + 4 &= 0 \\ x_1^2 + x_2^2 + x_3^2 - 4 &= 0 \end{aligned}$$

(a) Write down the Jacobian matrix J of this system.
(b) How many flops does it take to evaluate this Jacobian?
(c) Write a program that implements Newton's method to solve this system. Your code should not explicitly compute the inverse of the Jacobian. Start from the initial guess $[15\ 15\ 15]^T$ and report the iterates as well as the error and residual for each iteration (in the infinity norm). Note that there are two solutions to this system $[-2/3\ \ -4/3\ \ 4/3]^T$ and $[-6/7\ \ -12/7\ \ 4/7]^T$ (which one does it converge to?). Turn in your code and your results.

(2) **(Numerical Differentiation Formula)**
(a) Derive the following difference approximation for the first derivative:

$$f'(x_0) \approx \frac{f(x_0 + 2h) - f(x_0 - h)}{3h}$$

You should manipulate Taylor series in a way similar to the examples in class.
(b) What is the error term associated with this formula?
(c) Numerically verify the order of approximation (in terms of h) using $f(x) = \ln x$ and $x_0 = 2$. That is, make a table for small values of h and see at what rate the error decreases.

(3) **(Preliminary Final Project Topic)** This is an easy part of your HW7 assignment grade. For credit on this problem, you should write down your choice of topic for your final project.

You are welcome to also write down any additional ideas about the topic, or any questions you may have. I will respond to everyone's "Preliminary Final Project Topic" from the previous homework assignment with feedback. If you are still looking for ideas, please send me an email or come talk to me.

This homework assignment is significantly shorter than previous homeworks. I highly recommend you start/continue working on your final project this week. A good place to start is the theory section. I recommend start looking for resources (internet resources or textbooks are both fine) where you can learn about your project topic. Even better, start writing down some of the things you've learned!