Homework 5
Math 166, Fall 2022

Assigned: Friday, October 14, 2022
Due: Wednesday, October 19, 2022 by 5:00pm on Gradescope

- In your submission, please label all problems (with answers boxed when appropriate) on Gradescope.
- Include printouts of all code (ideally with some comments).

(1) **(Newton's Method on Roots with Multiplicity)**

A function $f(x)$ has a root of multiplicity $m > 1$ at p if $f(x)$ can be written as

$$f(x) = (x-p)^m q(x) \qquad \text{where } \lim_{x \to p} q(x) \neq 0 \ . \tag{1}$$

For example, $f(x) = x^3 - 5x^2 + 8x - 4$ has a root at $p = 2$ with multiplicty $m = 2$ because it can be written as $f(x) = (x-2)^2(x-1)$.

Consider a function $f(x)$ with a root at p of multiplicity $m > 1$. Prove that Newton's method will converge only *linearly* ($\alpha = 1$) to this root, assuming we start the iterations close enough.

HINT: Substitute in the form of $f(x)$ in (1) into Newton's method $g(x) = x - f(x)/f'(x)$. Then compute $g'(p)$.

(2) **(Restoring Quadratic Convergence)**

Again, consider a function $f(x)$ with a root at p of multiplicity $m > 1$. Consider the modification to Newton's method of

$$\tilde{g}(x) = x - m\frac{f(x)}{f'(x)} \ . \tag{2}$$

Prove that this modified Newton method $\tilde{g}$ converges quadratically ($\alpha = 2$) to p. This should be similar to the previous problem.