Homework 6

Math 166, Fall 2022

Assigned: Wednesday, October 26, 2022
Due: Friday, November 4, 2022 by 1:30pm on Gradescope

- In your submission, please label all problems (with answers boxed when appropriate) on Gradescope.
- Include printouts of all code (ideally with some comments).

(1) **(Counting Flops I)**

(a) Write out pseudocode for running Newton's method on the function $f(x) = x^2 + x - 6$. Then count the total number of flops required to run your code for n iterations (your answer should depend on n).

(b) Consider the following pseudocode that takes as input an $n \times n$ matrix A and modifies its entries (in a useless manner):

```
for i=1,...,n
     a_ii = a_ii/(a_ii+2)
     for j=1,...,n-1
         a_ij = a_ij + a_{i,j+1}^2 - a_{i,j-1}
         for k=j,...,n
              a_jk = 2*(a_jk - a_ii/3)
```

Count the number of flops required to run the code as currently written.

(c) Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Count the number of flops to compute the multiplication AB.

(2) **(Counting Flops II)**

The inverse of an $n \times n$ matrix can be computed by performing Gauss-Jordan elimination on an $n \times 2n$ augmented matrix where the last n columns are the $n \times n$ identity matrix (review this if need be). Note that Gauss-Jordan elimination is Gaussian elimination with the additional step of eliminating all entries *above* the pivot as well. Below is pseudocode for this procedure. Note that the pseudocode has three parts (there are multiple ways to write out the pseudocode of course). The first chunk is for eliminating entries below each pivot, and the second chunk is for eliminating entries above each pivot. The third chunk puts ones along the diagonal so that we have the identity matrix.

Eliminating below each pivot:

```
for pass=1,..,n-1
    for row=pass+1,..,n
        m=-a_row,pass/a_pass,pass
        set a_row,pass=0
        for col=pass+1,...,2n
             a_row,col = a_row,col+m*a_pass,col
```

Eliminating above each pivot:

```
for pass=2,..,n
    for row=1,..,pass-1
        m=-a_row,pass/a_pass,pass
```

```
        set a_row,pass=0
        for col=pass+1,...,2n
             a_row,col = a_row,col+m*a_pass,col
```

Dividing by pivot:

```
    for row=1,..,n
        d=a_row,row
        set a_row,row=1
        for col=n+1,...,2n
             a_row,col = a_row,col/d
```

Show that if one applies this algorithm, then computation of the inverse requires $3n^3 - 2n^2$ operations.

(3) **(Pivoting)**

(a) Prove that the matrix

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

does not have an LU factorization.

(b) Does the system

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

have a unique solution for all $a, b \in \mathbb{R}$? Why?

(c) How can you modify the matrix in part (a) so that it has an LU factorization?

(4) **(LU Decomp)** Compute the LU decomposition for the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 1 & 3 & 2 \end{bmatrix} .$$

You should compute the U matrix and the E matrices by hand, and will need a calculator to compute L.