Newton's interpolating polynomials, of degree k are given by:

$$\sum_{n=0}^{k} \binom{s}{n} \Delta^n f_0$$

Where $\Delta^n f_j = \Delta^{n-1} f_{j+1} - \Delta^{n-1} f_j$, $\Delta^0 f_i = f_i$ Let k be 6:

$$\sum_{n=0}^{6} \binom{s}{n} \Delta^n f_0 = \binom{s}{0}\Delta^0 f_0 + \binom{s}{1}\Delta^1 f_0 + \binom{s}{2}\Delta^2 f_0 + \binom{s}{3}\Delta^3 f_0 + \binom{s}{4}\Delta^4 f_0 + \binom{s}{5}\Delta^5 f_0 + \binom{s}{6}\Delta^6 f_0$$

From the definitions, follows that:

$$\binom{s}{0}\Delta^0 f_0 = f_0$$
$$\binom{s}{1}\Delta^1 f_0 = s(f_1 - f_0)$$
$$\binom{s}{2}\Delta^2 f_0 = \frac{s(s-1)}{2}(f_2 - 2f_1 + f_0)$$
$$\binom{s}{3}\Delta^3 f_0 = \frac{s(s-1)(s-2)}{6}(f_3 - 3f_2 + 3f_1 - f_0)$$
$$\binom{s}{4}\Delta^4 f_0 = \frac{s(s-1)(s-2)(s-3)}{24}(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)$$
$$\binom{s}{5}\Delta^5 f_0 = \frac{s(s-1)(s-2)(s-3)(s-4)}{120}(f_5 - 5f_4 + 10f_3 - 10f_2 + 5f_1 - f_0)$$
$$\binom{s}{6}\Delta^6 f_0 = \frac{s(s-1)(s-2)(s-3)(s-4)(s-5)}{720}(f_6 - 6f_5 + 15f_4 - 21f_3 + 16f_2 - 6f_1 + f_0)$$

The second order derivative of the sixth-degree Newton interpolating polynomial is, then:

$$\frac{1}{h^2}[\Delta^2 f_0 + (s-1)\Delta f_0 + \frac{1}{24}(12s^2 - 34s - 22)\Delta^4 f_0 + \frac{1}{12}(2s^3 - 12s^2 + 21s - 10)\Delta^5 f_0 +$$
$$+ \frac{1}{360}(15s^4 - 150s^3 - 510s^2 - 675s + 274)\Delta^6 f_0$$

Let us generalize f_0 to f_i, for any given sampling point i:

$$\frac{1}{h^2}[\Delta^2 f_i + (s-1)\Delta f_i + \frac{1}{24}(12s^2 - 34s - 22)\Delta^4 f_i + \frac{1}{12}(2s^3 - 12s^2 + 21s - 10)\Delta^5 f_i +$$
$$+ \frac{1}{360}(15s^4 - 150s^3 - 510s^2 - 675s + 274)\Delta^6 f_i$$

To get Backward type formula, we set $s = k$, for Forward, $s = 0$, and, central, $s = \frac{k}{2}$.

7-points Forward formula

$$\frac{1}{(h^2)180}(812f_i - 3132f_{i+1} + 5265f_{i+2} - 5080f_{i+3} + 2970f_{i+4} - 972f_{i+5} + 137f_{i+6})$$

7-points Backward formula

$$\frac{1}{(h^2)180}(812f_i - 3132f_{i-1} + 5265f_{i-2} - 5080f_{i-3} + 2970f_{i-4} - 972f_{i-5} + 137f_{i-6})$$

7-points Central formula

$$\frac{1}{(h^2)180}(2f_{i+3} - 27f_{i+2} + 270f_{i+1} - 490f_i + 270f_{i-1} - 27f_{i-2} + 2f_{i-3})$$