A Lagrange polynomial of degree n is described as:

$$P_n(x) = \frac{1}{2^n n!} \frac{\mathrm{d}^n}{\mathrm{d}x^n}[(x^2 - 1)^n]$$

Having $n = 4$, we get:

$$\begin{aligned} P_4(x) &= \frac{1}{2^4 4!} \frac{\mathrm{d}^4}{\mathrm{d}x^4}[(x^2 - 1)^4] \\ &= \frac{1}{(16)(24)} \frac{\mathrm{d}^4}{\mathrm{d}x^4}[x^8 - 4x^6 + 6x^4 - 4x^2 + 1] \\ &= \frac{1}{(16)(24)}(1680x^4 - 1440x^2 + 144) \\ &= \frac{35}{8}x^4 - \frac{30}{8}x^2 + \frac{3}{8} \end{aligned}$$

Being $\{\sqrt{\frac{3}{7} - \frac{\sqrt{\frac{6}{5}}}{7}}, \sqrt{\frac{3}{7} + \frac{\sqrt{\frac{6}{5}}}{7}}, -\sqrt{\frac{3}{7} - \frac{\sqrt{\frac{6}{5}}}{7}}, -\sqrt{\frac{3}{7} + \frac{\sqrt{\frac{6}{5}}}{7}}\}$ the set composed by the roots of $P_4(x)$.

Let $x_1 = -\sqrt{\frac{3}{7} + \frac{\sqrt{\frac{6}{5}}}{7}}, x_2 = -\sqrt{\frac{3}{7} - \frac{\sqrt{\frac{6}{5}}}{7}}, x_3 = \sqrt{\frac{3}{7} - \frac{\sqrt{\frac{6}{5}}}{7}}, x_4 = \sqrt{\frac{3}{7} + \frac{\sqrt{\frac{6}{5}}}{7}}$.

We can evaluate the weights w_i of Legendre-Gauss Quadrature by applying the following formula:

$$w_i = \frac{2}{(1 - x_i^2)[P_n'(x_i)]^2}$$

We also know that $P_n'(x) = \frac{35}{2}x^3 - \frac{15}{2}x$.

Given this, we have that:

$$\begin{aligned} w_1 &= \frac{1}{36}(18 - \sqrt{30}) \\ w_2 &= \frac{1}{36}(18 + \sqrt{30}) \\ w_3 &= \frac{1}{36}(18 + \sqrt{30}) \\ w_4 &= \frac{1}{36}(18 - \sqrt{30}) \end{aligned}$$