1. Comprobamos que $A = L \cdot U$.

$$A = \overbrace{\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{pmatrix}}^{L} \cdot \overbrace{\begin{pmatrix} 4 & -3 & 1 \\ 0 & 4 & -3 \\ 0 & 0 & 2 \end{pmatrix}}^{U} = \begin{pmatrix} 8 & -6 & 2 \\ -4 & 11 & -7 \\ 4 & -7 & 6 \end{pmatrix} = A \checkmark$$

Coincide.

2. Resolvemos $Ax = b$, utilizando $A = LU$.

$$Ax = b \rightarrow LUx = b$$

$$Ly = b \qquad Ux = y.$$

Resolvemos el sistema $Ly = b$.

$$\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 28 \\ -40 \\ 33 \end{pmatrix} \Rightarrow \begin{cases} 2y_1 = 28 \\ -y_1 + 2y_2 = -40 \\ y_1 - y_2 + y_3 = 33 \end{cases}$$

$$\left.\begin{aligned} &2y_1 = 28 \rightarrow y_1 = 14 \\ &-14 + 2y_2 = -40 \rightarrow y_2 = -\frac{26}{2} = -13 \\ &14 + 13 + y_3 = 33 \rightarrow y_3 = 6 \end{aligned}\right]$$

Forward substitution

$$y = \begin{bmatrix} 14 \\ -13 \\ 6 \end{bmatrix}$$

Resolvemos el sistema $Ux = y$

$$\begin{pmatrix} 4 & -3 & 1 \\ 0 & 4 & -3 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 14 \\ -13 \\ 6 \end{pmatrix} \Rightarrow \begin{cases} 4x_1 - 3x_2 + x_3 = 14 \\ 4x_2 - 3x_3 = -13 \\ 2x_3 = 6 \end{cases}$$

$$\left.\begin{aligned} &4x_1 + 3 + 3 = 14 \rightarrow x_1 = \frac{8}{4} = 2 \\ &4x_2 - 9 = -13 \rightarrow x_2 = -1 \\ &2x_3 = 6 \rightarrow x_3 = 3 \end{aligned}\right]$$

Backward substitution $\Rightarrow \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$