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The computation aspects of the equivalent-layer technique: review and perspective

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ABSTRACT

Equivalent-layer technique is a powerful tool for processing potential-field data in the space domain. However, the greatest hindrance for using the equivalent-layer technique is its high computational cost for processing massive data sets. The large amount of computer memory usage to store the full sensitivity matrix combined with the computational time required for matrix-vector multiplications and to solve the resulting linear system, are the main drawbacks that made unfeasible the use of the equivalent-layer technique for a long time. More recently, the advances in computational power propelled the development of methods to overcome the heavy computational cost associated with the equivalent-layer technique. We present a comprehensive review of the computation aspects concerning the equivalent-layer technique addressing how previous works have been dealt with the computational cost of this technique. Historically, the high computational cost of the equivalent-layer technique has been overcome by using a variety of strategies such as: moving data-window scheme, equivalent data concept, wavelet compression, quadtree discretization, reparametrization of the equivalent layer by a piecewise-polynomial function, iterative scheme without solving a system of linear equations and the convolutional equivalent layer using the concept of block-Toeplitz Toeplitz-block (BTTB) matrices. We compute the number of floating-point operations of some of these strategies adopted in the equivalent layer technique to show their effectiveness in reducing the computational demand. Numerically, we also address the stability of some of these strategies used in the equivalent layer technique by comparing with the stability via the classic equivalent-layer technique with the zeroth-order Tikhonov regularization.

Keywords: equivalent layer, gravimetry, fast algorithms, computational cost, stability analysis

1 INTRODUCTION

2 METHODOLOGY

3 RESULTS

3.1 Floating-point operations calculation

To measure the computational effort of the different algorithms to solve the equivalent layer linear system, a non-hardware dependent method can be useful because allow us to do direct comparison between them. Counting the floating-point operations (*flops*), i.e., additions, subtractions, multiplications and divisions is a good way to quantify the amount of work of a given algorithm (Golub and Loan, 2013). For example, the number of *flops* necessary to multiply two vectors $\mathbb{R}^N$ is $2N$. A commom matrix-vector multiplication with dimension $\mathbb{R}^{N \times N}$ and $\mathbb{R}^N$, respectively, is $2N^2$ and a multiplication of two matrices $\mathbb{R}^{N \times N}$ is $2N^3$.

3.1.1 Normal equations using Cholesky algorithm

3.1.2 Windowed method (Leão and Silva, 1989)

3.1.3 PEL method (Oliveira Jr. et al., 2013)

3.1.4 Conjugate gradient least square (CGLS)

3.1.5 Wavelet compression method with CGLS (Li and Oldenburg, 2010)

3.1.6 Convolutional equivalent layer for gravity data (Takahashi et al., 2020)

3.1.7 Convolutional equivalent layer for magnetic data (Takahashi et al., 2022)

3.1.8 Deconvolution method

3.2 Stability analysis

4 DISCUSSION AND CONCLUSION

CONFLICT OF INTEREST STATEMENT

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

AUTHOR CONTRIBUTIONS

The Author Contributions section is mandatory for all articles, including articles by sole authors. If an appropriate statement is not provided on submission, a standard one will be inserted during the production process. The Author Contributions statement must describe the contributions of individual authors referred to by their initials and, in doing so, all authors agree to be accountable for the content of the work. Please see here for full authorship criteria.

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SUPPLEMENTAL DATA

Supplementary Material should be uploaded separately on submission, if there are Supplementary Figures, please include the caption in the same file as the figure. LaTeX Supplementary Material templates can be found in the Frontiers LaTeX folder.

DATA AVAILABILITY STATEMENT

The datasets [GENERATED/ANALYZED] for this study can be found in the [NAME OF REPOSITORY] [LINK].

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FIGURE CAPTIONS

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Figure 2a. This is Subfigure 1.

Figure 2b. This is Subfigure 2.

Figure 2. Enter the caption for your subfigure here. **(A)** This is the caption for Subfigure 1. **(B)** This is the caption for Subfigure 2.