3D Magnetic modelling of ellipsoidal bodies

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Abstract.

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1 Introduction

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2 Methodology

2.1 Geometrical parameters defining ellipsoids

Let (x, y, z) be a point referred to a Cartesian coordinate system with axes x, y and z pointing to, respectively, North, East and down. For convenience, we denominate this coordinate system as *main coordinate system*. Let us consider an ellipsoidal body with centre at the point (x_c, y_c, z_c), semi-axes defined by positive constants a, b, c, where $a > b > c$, and orientation defined by three angles α, β, and γ. The points (x, y, z) located on the surface of this ellipsoidal body satisfy the following equation:

$$(\mathbf{r} - \mathbf{r}_c)^T \mathbf{A} (\mathbf{r} - \mathbf{r}_c) = 1 \, , \tag{1}$$

where $\mathbf{r} = \begin{bmatrix} x & y & z \end{bmatrix}^\top$, $\mathbf{r}_c = \begin{bmatrix} x_c & y_c & z_c \end{bmatrix}^\top$, $\mathbf{A}$ is a positive definite matrix given by

$$\mathbf{A} = \mathbf{V} \begin{bmatrix} a^{-2} & 0 & 0 \\ 0 & b^{-2} & 0 \\ 0 & 0 & c^{-2} \end{bmatrix} \mathbf{V}^\top \, , \tag{2}$$

and $\mathbf{V}$ is an orthogonal matrix whose columns are defined by unit vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

The vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ depend on the orientation angles α, β, γ and are defined as follows (Clark et al., 1986):

$$\mathbf{v}_1 = \begin{bmatrix} -\cos\alpha \, \cos\delta \\ -\sin\alpha \, \cos\delta \\ -\sin\delta \end{bmatrix} , \tag{3}$$

$$\mathbf{v}_2 = \begin{bmatrix} \cos\alpha\, \cos\gamma\, \sin\delta + \sin\alpha\, \sin\gamma \\ \sin\alpha\, \cos\gamma\, \sin\delta - \cos\alpha\, \sin\gamma \\ -\cos\gamma\, \cos\delta \end{bmatrix} , \tag{4}$$

$$\mathbf{v}_3 = \begin{bmatrix} \sin\alpha\, \cos\gamma - \cos\alpha\, \sin\gamma\, \sin\delta \\ -\cos\alpha\, \cos\gamma - \sin\alpha\, \sin\gamma\, \sin\delta \\ \sin\gamma\, \cos\delta \end{bmatrix} . \tag{5}$$

For triaxial ellipsoids (i.e., $a > b > c$), the orthogonal matrix $\mathbf{V}$ (equation 2) is calculated by using equations 3, 4, and 5 as follows:

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix} . \tag{6}$$

Similarly, the matrix $\mathbf{V}$ (equation 2) for prolate ellipsoids (i.e., $a > b = c$) is calculated according to equation 6 by using equations 3, 4, and 5, but with $\gamma = 0°$ (Emerson et al., 1985). Finally, the matrix $\mathbf{V}$ (equation 2) for oblate ellipsoids (i.e., $a < b = c$) is calculated by using equations 3, 4, and 5, with $\gamma = 0°$, as follows (Emerson et al., 1985):

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}_2 & \mathbf{v}_1 & -\mathbf{v}_3 \end{bmatrix} . \tag{7}$$

The orientation of the semi-axes a, b, and c are defined by the first, second, and third columns of the matrix $\mathbf{V}$ given by equation 6, in the case of a triaxial or prolate ellipsoid, or the matrix $\mathbf{V}$ given by equation 7, in the case of an oblate ellipsoid.

The magnetic modelling of an ellipsoidal body is commonly performed in a particular Cartesian coordinate system that is aligned with the body semi-axes and has the origin coincident with the body centre. For convenience, we denominate this particular coordinate system as *local coordinate system*. The relationship between the Cartesian coordinates $(\tilde{x}, \tilde{y}, \tilde{z})$ of a point in a local coordinate system and the Cartesian coordinates (x, y, z) of the same point in the main system is given by:

$$\tilde{\mathbf{r}} = \mathbf{V}^\top (\mathbf{r} - \mathbf{r}_c) , \tag{8}$$

where $\tilde{\mathbf{r}} = [\ \tilde{x} \quad \tilde{y} \quad \tilde{z}\ ]^\top$, $\mathbf{r}$ and $\mathbf{r}_c$ are defined in equation 1 and the matrix $\mathbf{V}$ is defined according to equations 6 or 7, depending on the ellipsoid type.

A last but not least set of parameters defining the geometry of an ellipsoid are their focal points. The focal points are defined on the axes of the local coordinate system as follows:

$$(\tilde{x}, \tilde{y}, \tilde{z}) = \begin{cases} \left(\pm\sqrt{a^2 - b^2}, 0, 0\right) \\ \left(\pm\sqrt{a^2 - c^2}, 0, 0\right) \\ \left(0, \pm\sqrt{b^2 - c^2}, 0\right) \end{cases} . \tag{9}$$

All ellipsoids sharing a common set of focal points form a family of confocal ellipsoids.

2.2 Auxiliary variable λ

The methods commonly applied in the magnetic modelling of ellipsoidal bodies make use of an auxiliary variable λ, which is a real number.

2.3 Magnetic induction

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3 Conclusions

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Appendix A

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Author contributions. TEXT

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References

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