Mathematical Formulation of the Robust Taxation Model

This Supplementary Material presents the formal structure of the taxation model developed under conditions of ambiguity and partial observability, as introduced in Sections 3 and 4 of the main manuscript. It provides the complete mathematical specification of the framework, including notation, equilibrium definitions, and the optimization problem underlying the design of the distributionally robust tax schedule.

The formulation establishes the theoretical foundations for the simulation and empirical analyses discussed in the article, ensuring methodological transparency, internal consistency, and reproducibility. It focuses exclusively on the analytical structure of the model; the construction and calibration of the synthetic datasets used for simulation are documented separately in the accompanying data repository, which is available upon reasonable request to the corresponding author.

1 Agents and Preferences

Let $i \in \mathcal{I}$ index individuals in the economy. Each individual is characterized by a latent income $y_i \in R_+$, which is not directly observable by the planner. Instead, the planner observes a signal vector $z_i \in \mathcal{Z} \subset R^d$ containing imperfect information, such as region, education, sector, or neighborhood.

Each individual has a utility function over post-tax income and effort:

$$u_i(y_i - T(z_i), e_i), \tag{1}$$

where $T(z_i)$ denotes the tax liability based on observed signal z_i, and e_i denotes effort, which depends on incentives and individual characteristics. We assume a reduced-form representation in which effort is implicit in the income-generation process and integrated into the planner's uncertainty.

2 Planner's Ambiguity and Objective

The planner does not know the true income distribution $P = \{P_i\}_{i\in\mathcal{I}}$, but considers a set of plausible distributions $\mathcal{P}$, defined as a Wasserstein ball of radius δ around a nominal estimate $\hat{P}$. This captures epistemic uncertainty from partial observability, survey error, or informal earnings.

The planner maximizes the minimum expected utility across all distributions in $\mathcal{P}$, that is:

$$\max_{T\in\mathcal{T}} \min_{P\in\mathcal{P}} \sum_{i\in\mathcal{I}} E_{y_i\sim P_i}\left[u_i\left(y_i - T(z_i), e_i^*(y_i, T)\right)\right] \tag{2}$$

subject to implementability and feasibility constraints on the tax function *T*. Here, $e_i^*(y_i, T)$ denotes the optimal behavioral response of individual *i* given their income and the tax schedule.

3 GNN-Augmented Tax Schedule

Instead of estimating $T(z_i)$ directly from features, we use a learned embedding function $\phi: \mathcal{G} \to R^k$, where $\mathcal{G}$ is an economic interaction graph, and $\phi(z_i)$ is computed using a Graph Neural Network (GNN). The GNN maps local and global topological properties of individual *i* (e.g., sectoral ties, credit exposure, employer relationships) into a latent representation.

The tax function is parameterized as:

$$\mathrm{T}(z_i) = \tau(\phi(z_i)) = \alpha + \beta^\top \phi(z_i) \tag{3}$$

with $\alpha \in R$, $\beta \in R^k$ calibrated using a robust optimization criterion over synthetic or empirical training data. The function $\tau(\cdot)$ can be linear or nonlinear (e.g., spline or sigmoid), but is constrained to satisfy monotonicity and progressivity conditions:

$$\frac{dT}{d\phi} \geq 0, \quad T(0) = 0 \tag{4}$$

4 Benchmark Tax Rule

For comparison, the benchmark tax rule is:

$$T_{\text{bench}}(z_i) = \gamma + \delta_1 \cdot \text{edu}_i + \delta_2 \cdot \text{region}_i + \delta_3 \cdot \text{sector}_i \tag{5}$$

where parameters are estimated from observed correlations in a noisier, manually coded regime—emulating common presumptive or scoring-based tax systems used in many low-capacity tax administrations.

5 Robust Optimization and Implementation

Let $\mathcal{U}(T,P) = \sum_i E_{y_i \sim P_i}\left[u_i\left(y_i - T(z_i)\right)\right]$ be the total utility. The robust taxation problem becomes:

$$\max_{T\in\mathcal{T}} \min_{P\in\mathcal{P}} \mathcal{U}\,(T,P).\mathrm{T} \in T \tag{6}$$

The inner minimization over $\mathcal{P}$ is approximated using Wasserstein duality (Nguyen et al, 2023), yielding tractable reformulations under mild regularity. The optimization is solved via gradient-based methods using stochastic samples from the ambiguity set and synthetic income data constructed as described in Section 5.

The resulting tax function $T^*(z)$ is then evaluated out-of-sample in the simulated economies, with welfare, equity, and robustness indicators computed across multiple draws from $P \sim \mathcal{P}$.