$$C_{1111} = 2\left(\alpha + \frac{\beta\,\varepsilon'_{11}}{\sqrt{J_2}}\right)^2 + \frac{2\sqrt{\psi_d^*}\,\beta}{\sqrt{J_2}}\left[-\frac{\varepsilon'_{11}}{\sqrt{J_2}}\frac{\varepsilon_{11}}{\sqrt{J_2}} + \frac{2}{3}\right] \qquad (\text{sympy})$$

Pela dedução analítica,

$$C_{1111} = \alpha^2\,\delta_{11}\delta_{11} + \beta^2\left(\frac{\varepsilon'_{11}}{\sqrt{J_2}}\right)\left(\frac{\varepsilon'_{11}}{\sqrt{J_2}}\right) + \alpha\beta\left[\delta_{11}\frac{\varepsilon'_{11}}{\sqrt{J_2}} + \frac{\varepsilon'_{11}}{\sqrt{J_2}}\delta_{11}\right] + $$

$$+ \beta\sqrt{\psi_d^*}\left[J_{1111} - \frac{2}{\sqrt{}}\right.$$

$$\partial_\varepsilon\left[\frac{\partial_\varepsilon J_2}{\sqrt{J_2}}\right] = \frac{\sqrt{J_2}\,\partial_\varepsilon\partial_\varepsilon J_2 - \partial_\varepsilon J_2\,\partial_\varepsilon\sqrt{J_2}}{J_2}$$

Identidade simétrica desviadora de 4ª Ordem:

$$\underset{\sim}{J} \overset{\text{def}}{=} \underset{\sim}{\mathbb{I}} - \frac{1}{3}\underset{\sim}{1}\otimes\underset{\sim}{1} = \frac{1}{2}\left(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk}\right) - \frac{1}{3}\delta_{ij}\delta_{k\ell} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & & & \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & & \mathbb{O} & \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & & & \\ & & & 1 & & \\ & \mathbb{O} & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

Dado $\quad \psi_d^*(\underset{\sim}{\varepsilon}) = \left(\alpha I_1(\underset{\sim}{\varepsilon}) + \beta\sqrt{J_2(\underset{\sim}{\varepsilon})}\right)^2,$

$I_1(\varepsilon) = Tr(\varepsilon) = \varepsilon_{kk}$

$J_2(\varepsilon) = \frac{1}{2}Tr(\varepsilon' \cdot \varepsilon')$ $\longleftarrow$ verifique a definição usada pelo artigo

Calcular $\quad \partial_{\underset{\sim}{\varepsilon}}\psi_d^* = \sigma_d^* \quad$ e $\quad \partial_{\underset{\sim}{\varepsilon}}\partial_{\underset{\sim}{\varepsilon}}\psi_d^* = \mathbb{D}_d$