MATHEMATICAL MODEL

SYMBOL	DESCRIPTION
T	**Number of Teams. $|T| = 36$**
P	**Matches to play. $|P| = 8$**
F	**Rankings. $|F| = 4$**
N	**Nationality**
f_t	**Ranking of the team $t \, \epsilon \, T, f_t \, \epsilon \, F$**
n_t	**Nationality of the team $t \in T, n_t \in N$**
s_t	**City of the team $t \in T$**
lat_t	Latitude of the team $s_t, t \in T$
lon_t	Longitude of the team $s_t, t \in T$
$u_{i,p}$	$\in (0,1)$, unavailability variable
b_t	$\in (0,1)$, popolarity variable

SYMBOL	DESCRIPTION
$x_{\mathrm{i,j},p}$	$\begin{cases} 1, & \text{if team } i \text{ plays home against} \\ & \text{team } j \text{ in the } p \text{ slot } (i \neq j) \\ 0, & \text{otherwise} \end{cases}$
$h_{i,p}$	$\begin{cases} 1, & \text{if team } i \text{ plays home in slot } p \\ 0, & \text{otherwise (away)} \end{cases}$

Boolean variables for the soft constraint count:

$$d^{S1}{}_{i,p} \, , \, d^{S2,h}{}_{i,p} \, , \, d^{S2,a}{}_{i,p} \, , \, d^{S3}{}_{i,f} \, , \, d^{S5}{}_{i,n} \, , \, d^{S6}{}_{p} \, , \, d^{S7}{}_{i}$$

Int. variable for the distance soft constraint count:

$$d^{S4}$$

$$(\, i, j \in T \, ; \, p \in P \, ; \, f \in F \, ; \, n \in N \,)$$

MATHEMATICAL MODEL

HARD CONSTRAINTS

	TEXT	MATH
H1	Each team plays against each other AT MOST one time (***cut round robin***)	$$\sum_{p\in P}\left(x_{i,j,p}+x_{j,i,p}\right)\leq 1 \quad \forall\, i,j\in T, i<j$$
H2	Each team plays one match per slot (***time constrained***)	$$\sum_{j\in T\setminus\{i\}}\left(x_{i,j,p}+x_{j,i,p}\right)=1 \quad \forall\, i\in T, p\in P$$
H3	**Half of the matches at home** (and half away)	$$\sum_{j\in T\setminus\{i\}}x_{i,j,p}=h_{i,p} \qquad \forall\, i\in T, p\in P$$ $$\sum_{j\in T\setminus\{i\}}x_{j,i,p}=1-h_{i,p} \qquad \forall\, i\in T, p\in P$$ $$\sum_{p\in P}h_{i,p}=\frac{|P|}{2} \qquad \forall\, i\in T$$
H4	Teams with the **same city** don't play at home in the same slot	$$h_{i,p}+h_{j,p}\leq 1 \quad \forall i,j\in T, \forall p\in P\,|\, i<j, s_i=s_j$$

MATHEMATICAL MODEL

HARD CONSTRAINTS

	TEXT	MATH
H5	**2 opponent teams per ranking** (for each team and ranking)	$$\sum_{p \in P} \sum_{j \in T \setminus \{i\} \mid f_j = f} (x_{i,j,p} + x_{j,i,p}) = \frac{|P|}{|F|} \qquad \forall i \in T, \forall f \in F$$
H6	**1 match home and 1 away** against teams of the same ranking	$$\sum_{p \in P} \sum_{j \in T \setminus \{i\} \mid f_j = f} x_{i,j,p} = \sum_{p \in P} \sum_{j \in T \setminus \{i\} \mid f_j = f} x_{j,i,p} \qquad \forall i \in T, \forall f \in F$$
H7	**No national derby**	$$\sum_{p \in P} (x_{i,j,p} + x_{j,i,p}) = 0 \qquad \forall\, i,j \in T \mid i < j, n_i = n_j$$
H8	Maximum **two home or away matches in a row** (for each team)	$$h_{i,p} + h_{i,p+1} + h_{i,p+2} \leq 2 \qquad \forall\, i \in T, p \in \{1, \dots, |P|-2\}$$ $$(1 - h_{i,p}) + (1 - h_{i,p+1}) + (1 - h_{i,p+2}) \leq 2$$
H9	First two and last two matches, for each team, must be played in **alternation home/away**	$$h_{i,1} + h_{i,2} = 1 \qquad \forall\, i \in T$$ $$h_{i,|P|-1} + h_{i,|P|} = 1 \qquad \forall\, i \in T$$
H10	Maximum two opponent teams of the **same nationality** (for each team)	$$\sum_{p \in P} \sum_{j \in T \setminus \{i\} \mid n_j = n} (x_{i,j,p} + x_{j,i,p}) \leq 2 \qquad \forall i \in T, \forall n \in N$$

MATHEMATICAL MODEL

SOFT CONSTRAINTS

	TEXT	MATH
S1	**Unavailability** of the teams (at home) must be respected	$h_{i,p} - d^{S1}{}_{i,p} \leq 0 \qquad \forall\, i \in T, p \in P \mid u_{i,p} = 1$
S2	*Break constraint*: alternation home/away	$(h_{i,p-1} + h_{i,p}) - d^{S2,h}{}_{i,p} \leq 1 \qquad \forall\, i \in T, p \in P\backslash\{1\}$ $(1 - h_{i,p-1} + 1 - h_{i,p}) - d^{S2,a}{}_{i,p} \leq 1 \qquad \forall\, i \in T, p \in P\backslash\{1\}$
S3	**Simmetrical calendar**, according to the rankings	$\displaystyle\sum_{p\in\{1,\dots,\lvert P\rvert/2\}} \sum_{j\in T \mid f_j = f} (x_{i,j,p} + x_{j,i,p}) + d^{S3}{}_{i,f} \geq 1 \qquad \forall i \in T, \forall f \in F$
S4	**Total distance** must be as short as possible (*)	$dist_{tot} - M * d^{S4} \leq 0.9 * dist_{avg} * \lvert P\rvert * \lvert T\rvert / 2$
S5	Variability, as regards **different nations**	$\displaystyle\sum_{p\in P} \sum_{j\in T\backslash\{i\} \mid n_j = n} (x_{i,j,p} + x_{j,i,p}) - d^{S5}{}_{i,n} \leq 1 \qquad \forall i \in T, \forall n \in N$
S6	At least one **big match** per slot	$\displaystyle\sum_{i\in T \mid b_i = 1} \sum_{j\in T \mid i<j, b_j = 1} (x_{i,j,p} + x_{j,i,p}) + d^{S6}{}_{p} \geq 1 \qquad \forall p \in P$
S7	**Visibility** also for the «small teams»	$\displaystyle\sum_{p\in P} \sum_{j\in T\backslash\{i\}, b_j = 1} (x_{i,j,p} + x_{j,i,p}) + d^{S7}{}_{i} \geq 1 \qquad \forall i \in T \mid b_i = 0$

*distance: different auxiliary variables have been used. Next page

MATHEMATICAL MODEL

SOFT CONSTRAINTS: distance focus – S4

$$distance_{i,j} = \arccos(\sin lat_i * \sin lat_j + cos\, lat_i * cos\, lat_j * \cos(lon_j - lon_i)) * R \quad \forall i,j \in T$$

$$R = 6371 \text{ km}$$

Total distance during the tournament:

$$dist_{tot} = \sum_{p \in P} \sum_{i \in T} \sum_{j \in T \mid i < j} distance_{i,j} * \left(x_{i,j,p} + x_{j,i,p}\right)$$

Average distance between each team:

$$dist_{avg} = \sum_{i \in T} \sum_{j \in T \mid i \neq j} \frac{distance_{i,j}}{(n-1)(n-1)}$$

S4 constraint:

$$dist_{tot} - M * d^{S4} \leq 0.9 * dist_{avg} * |P| * |T| / 2$$

Where:
- M is the scale factor *(=1000 km)* for the d^{S4} count
- $|P| * |T| / 2$ total number of matches (144 for the UEFA CL 2024/25)
- $dist_{avg} * |P| * |T| / 2$ is the total average expected distance

MATHEMATICAL MODEL

OBJECTIVE FUNCTION

Minimization of the violations to the SOFT contraints

$$\textbf{min}\ w_1 \sum_{p \in P} \sum_{i \in T} d^{S1}_{i,p} + w_2 \sum_{p \in P} \sum_{i \in T} \left(d^{S2,h}_{i,p} + d^{S2,a}_{i,p} \right)$$

$$+ w_3 \sum_{f \in F} \sum_{i \in T} d^{S3}_{i,f} + w_4\, d^{S4} + w_5 \sum_{n \in N} \sum_{i \in T} d^{S5}_{i,n} + w_6 \sum_{p \in P} d^{S6}_{p} + w_7 \sum_{i \in T} d^{S7}_{i}$$

$d^{S1}_{i,p}, d^{S2,h}_{i,p}, d^{S2,a}_{i,p}, d^{S3}_{i,f},$
$d^{S4}, d^{S5}_{i,n}, d^{S6}_{p}, d^{S7}_{i}$

Auxiliary variables for the violations count

$w_1, w_2, w_3, w_4, w_5, w_6, w_7$

Weight of each component (subjective)

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