HW3, Nicholas DiGregorio, 1220871392

C3 Q22) Find Poles + Zeros of $H(z) = 1 + z^{-1} + z^{-2}$

$$H(z) = \frac{1 + z^{-1} + z^{-2}}{1} \frac{z^2}{z^2}$$

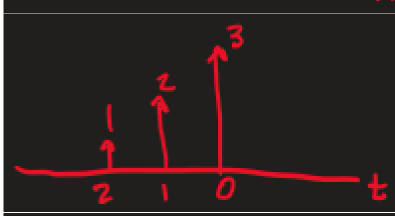
$$H(z) = \frac{z^2 + z + 1}{z^2}$$

$$\text{zeros} = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times 1}}{2} = \frac{-1 \pm j\sqrt{3}}{2}$$

Poles = 0, 0

C3 Q23) Given $H(z) = 3 + 2z^{-1} + z^{-2}$, determine and sketch the time domain signal corresponding to $H(z^{-1})$

$$H(z^{-1}) = 3 + 2z + z^2$$



C3 Q29) Give the transfer function of the digital filter whose impulse response is:

$$h(n) = 0.7^n u(n) + 0.7^{n-1} u(n-1)$$

$$\frac{1}{1 - 0.7z^{-1}} + \frac{z^{-1}}{1 - 0.7z^{-1}}$$

C4 Q21) Show that averaging filter has linear phase

$$h(n) = \frac{1}{L} \quad \text{for } n = 0, 1, 2, 3 \ldots, L-1$$

$\therefore$ $h(n) = h(L-1-n)$ for L being even or odd

$$\Rightarrow T = \frac{L+1}{2}$$

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C4 Q23) Design a causal linear phase FIR LPF of order 6 using the fourier series method for a cutoff frequency $\Omega_c = 0.4\pi$, give 3 designs corresponding to rectangular, triangular, and hamming windows

$$\Omega_c = 0.4\pi$$

$$h_d(n) = 0.4\,\text{sinc}(0.4\pi n) \quad n = \pm 1, \pm 2, \ldots$$

$$h_{LPF}(n) = [\,-0.0624 \quad 0.0935 \quad 0.3027 \quad 0.4 \quad 0.3027 \quad 0.0935 \quad -0.0624\,]$$

$-3 \le n \le 3$

Rectangle $\rightarrow$ $b_n = h_{LPF}(n-3)$

triangular $\rightarrow$ $w(n) = [\,1/4 \quad 1/2 \quad 3/4 \quad 1 \quad 3/4 \quad 1/2 \quad 1/4\,]$

$b_n = h_{LPF}(n-3) \times W(n-3)$

hamming $\rightarrow$ $w(n) = [\,0.08 \quad 0.31 \quad 0.77 \quad 1 \quad 0.77 \quad 0.31 \quad 0.08\,]$

$b_n = h_{LPF}(n-3) \times W(n-3)$

C4 Q24) Use transformations and repeat Q23 for a HPF with similar characteristics

$$h_{PF}(n) = \delta(n) - h_{LPF}(n) = \delta(n) - 0.4\,\text{sinc}(0.4\pi n) \quad -3 \le n \le 3$$

$$h(n) = \frac{1}{2\pi}\int_{-0.3\pi}^{0.3\pi} e^{j\Omega n}\, d\Omega = 0.3\,\text{sinc}(0.3\pi n)$$

$$h_{HP} = [\,0.9003 \quad 0.2095 \quad 0.2890 \quad 0.2890 \quad 0.2095 \quad 0.9003$$

rectangular $\rightarrow$ $b_n = h_{HP}(n-3)$

triangular $\rightarrow$ $W(n) = [\,1/4 \quad 1/2 \quad 3/4 \quad 1 \quad 3/4 \quad 1/2 \quad 1/4\,]$

$b_n = h_{HP}(n-3) \times W(n-3)$

hamming $\rightarrow$ $W(n) = [\,0.125 \quad 0.437 \quad 0.8268 \quad 0.8268 \quad 0.437 \quad 0.125\,]$

$b_n = h_{HP}(n-3) \times W_n(n-3)$

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05 Q(8) Determine the order and poles of a digital butterworth filter with the following specs:

$f_s = 8\text{ kHz}$

$\Omega_p = 1.2\text{ KHz}$

$\Omega_{st} = 1.8\text{ kHz}$

$\text{gain}_p = -1\text{dB}$

$\text{gain}_{st} = -30\text{dB}$

$$W_p = \frac{1.2}{8} \times 2\pi = .3\pi \rightarrow W_p = \tan\left(\frac{\Omega_p}{2}\right) = \tan\left(\frac{.3\pi}{2}\right) = .5095$$

$$W_{st} = \frac{1.8}{8} \times 2\pi = .45\pi \rightarrow W_{st} = \tan\left(\frac{\Omega_{st}}{2}\right) = \tan\left(\frac{.45\pi}{2}\right) = .854$$

$g_p = .794$

$g_{st} = 0.001$

$$G = \frac{\frac{1}{g_p} - 1}{\frac{1}{g_{st}} - 1} = \frac{\frac{1}{.794} - 1}{\frac{1}{0.001} - 1} = 2.597 \times 10^{-4}$$

$$M \geq \frac{\log G}{2\log\left(\frac{W_p}{W_{st}}\right)} = \frac{\log(2.597 \times 10^{-4})}{2\log\left(\frac{.5095}{.854}\right)} = 7.99$$

$$M = 8$$

$$\frac{1}{g_{st}} - 1 = \left(\frac{0.854}{W_c}\right)^{2\times 8}$$

$$1000 - 1 = \left(\frac{0.854}{W_c}\right)^{2\times 8}$$

$$\sqrt[16]{999} = \frac{0.854}{W_c}$$

$$W_c = \frac{0.854}{\sqrt[16]{999}}$$

$W_c = 0.5549$ radians

There are $2M = 16$ poles a $\frac{2\pi}{16}$ rad apart

$s_5 = -0.1082 + 0.5442i$ $z_5 = -0.4359 + 0.7140i$

$s_6 = -0.3082 - 0.4613i$ $z_6 = -0.3596 + 0.4794i$

$s_7 = -0.4136 - 0.3082i$ $z_7 = -0.3102 + 0.2764i$

$s_8 = -0.5442 - .1082i$ $z_8 = -0.2888 + 0.09034i$

$s_9 = -0.5442 + 0.1082i$ $z_9 = -0.288 - 0.09034i$

$s_{10} = -0.4136 + 0.3082i$ $z_{10} = -0.3102 - 0.2764i$

$s_{11} = -0.3082 + 0.4613i$ $z_{11} = -0.3596 - 0.4794i$

$s_{12} = -0.1082 + 0.5442i$ $z_{12} = -0.5439 - 0.7140i$

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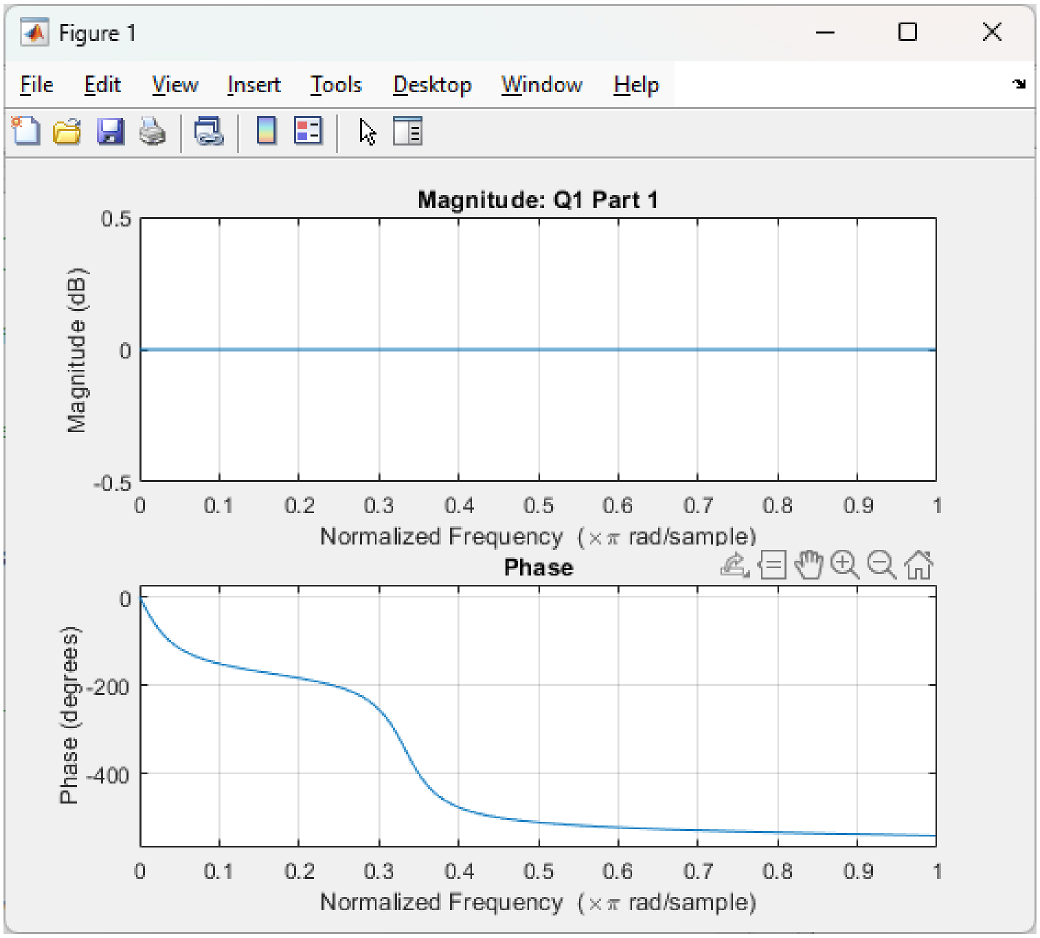
Matlab Part 1




```
Zeros
   1.1111 + 0.0000i
   0.5556 + 0.9623i
   0.5556 - 0.9623i

Poles
   0.9000 + 0.0000i
   0.4500 + 0.7794i
   0.4500 - 0.7794i

Gain
   -0.7290
```

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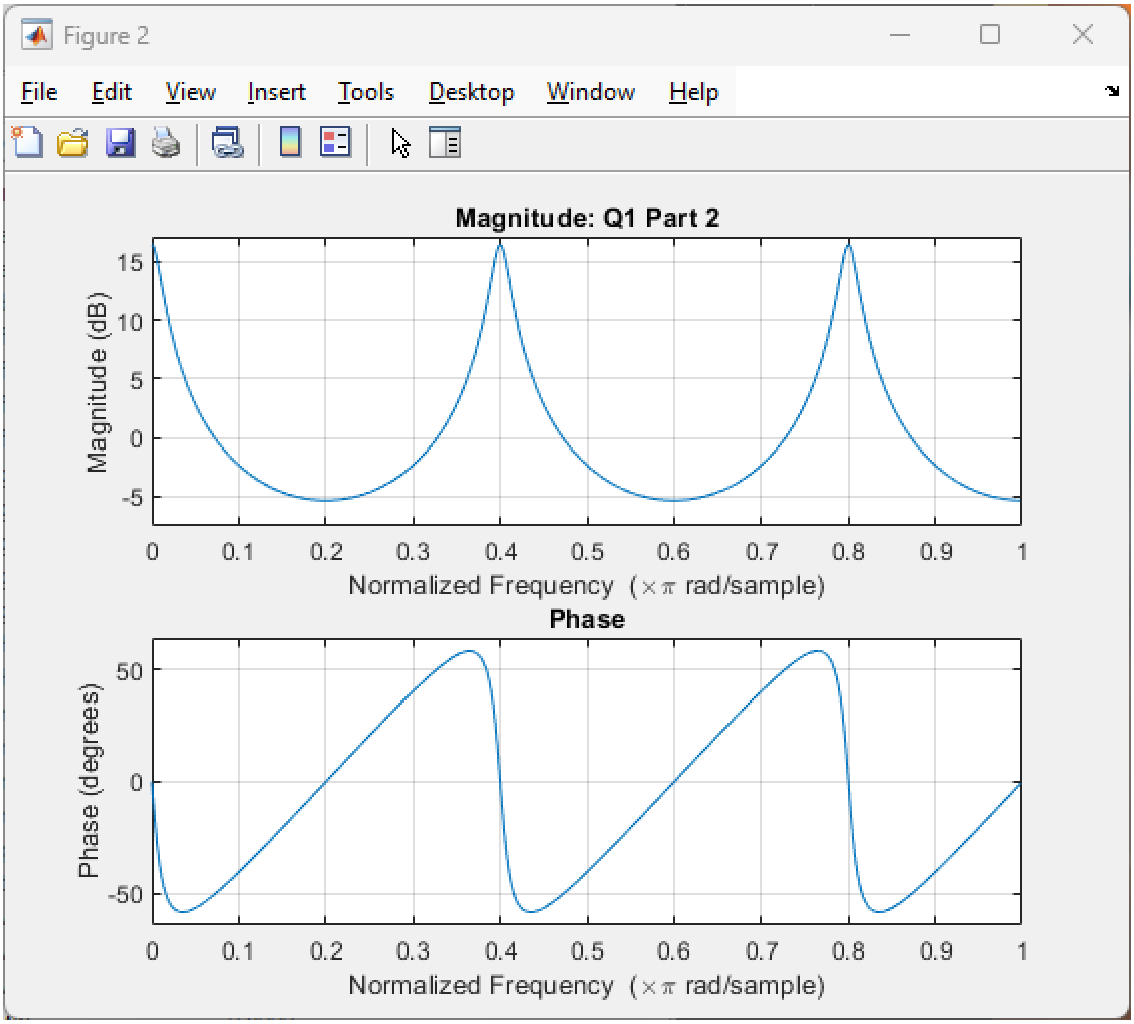


```
Zeros
Poles
  -0.9680 + 0.0000i
  -0.2991 + 0.9206i
  -0.2991 - 0.9206i
   0.7831 + 0.5690i
   0.7831 - 0.5690i

Gain
     1
```

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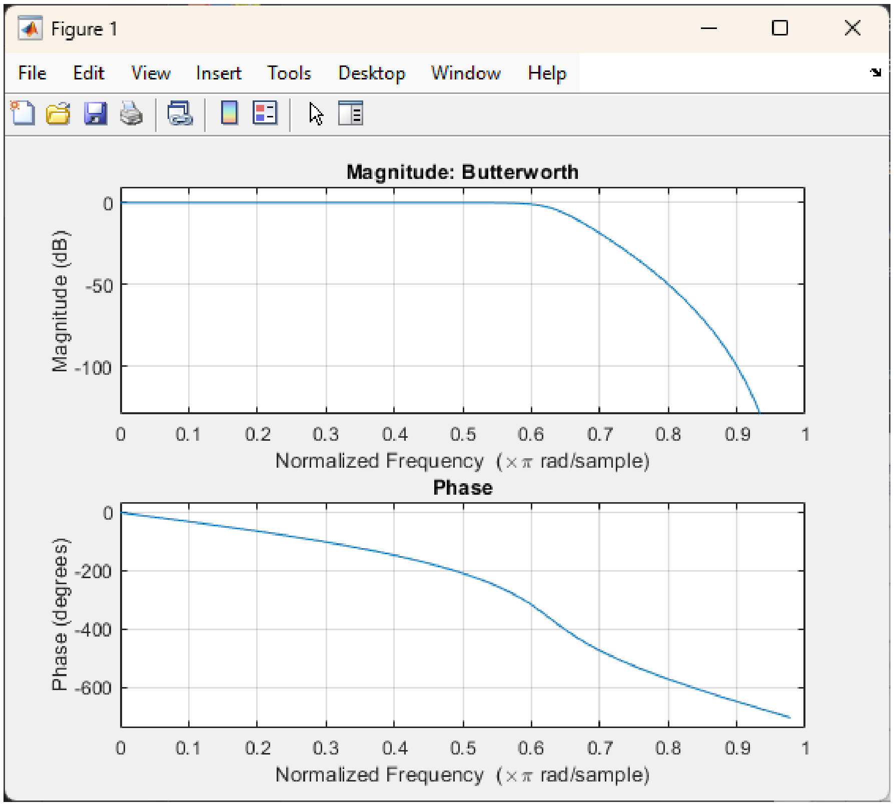
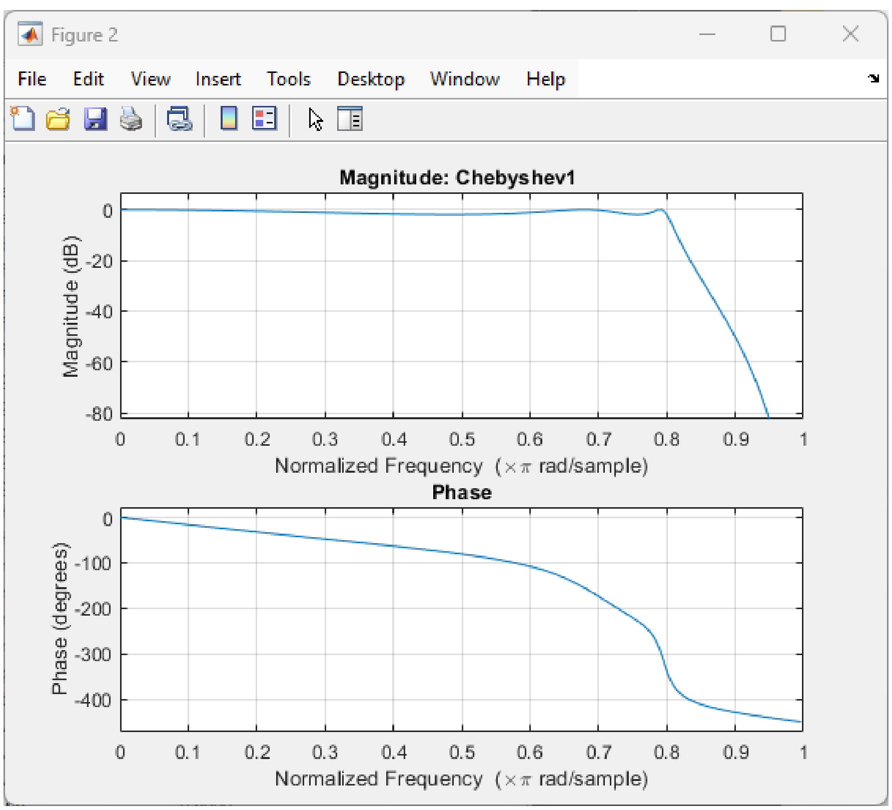
Part 2 of Matlab





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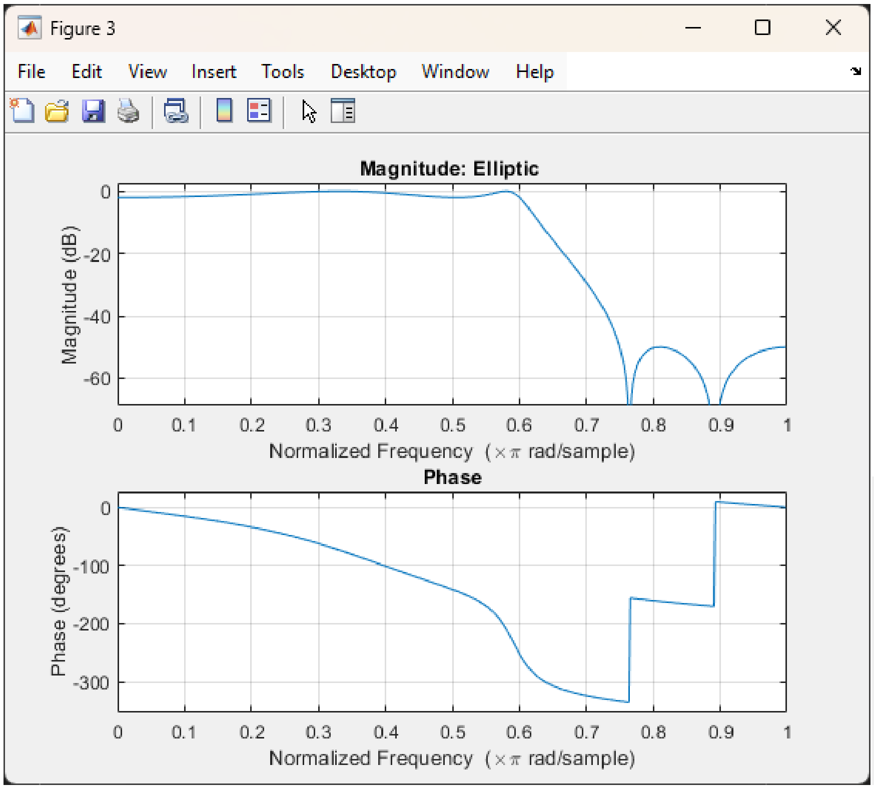


```
Order of Butter worth
     9

Order of chebyshev
     6

Order of Elliptic
     5
```

One can see that the elliptic filter does the best. It has the least order of coefficients and it also has the steepest roll off from the cutoff frequency. It has the worst phase shift of the 3 filters, however. The Chebyshev filter also has a steeper roll off but it is after the cutoff frequency by about .2pi rads. Its phase shift preforms the best out of the 3 filters. Lastly the Butterworth filter performs the worst. It has the most gradual roll off after the cut off frequency and it doesn’t perform that great on the phase shift. It does have the most linear phase shift of the 3 filters however. Lastly the Butterworth filter has the highest order as well.

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Matlab script for part 1 of the homework:

```
close all;

b =[-0.729 1.62 -1.8 1];
a =[1 -1.8 1.62 -0.729];

[z, p, g] = tf2zp(b, a);

disp("Zeros"); disp(z);
disp("Poles"); disp(p);
disp("Gain"); disp(g);

freqz(b, a);
title("Magnitude: Q1 Part 1")
figure;
b =[1];
a =[1 0 0 0 0 -0.85];

[z, p, g] = tf2zp(b, a);

disp("Zeros"); disp(z);
disp("Poles"); disp(p);
disp("Gain"); disp(g);

freqz(b, a);
title("Magnitude: Q1 Part 2")
```

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matlab script for part 2 of the homework:

```
close all;
fs  = 10000;
fp  = 3000;
fst = 4000;
wp  = (2)*(fp/fs);
wst = (2)*(fst/fs);
gp_db = 2;
gst_db = 50;
gp = 10^(gp_db/10);
gst = 10^(gst_db/10);
n = 256;
%%
[m, wn] = buttord(wp, wst, gp_db, gst_db);
[b, a]  = butter(m, wn);
freqz(b, a);
title("Magnitude: Butterworth");
disp("Order of Butter worth"); disp(max(length(b), length(a)));
%%
figure
[m, wn] = cheb2ord(wp, wst, gp_db, gst_db);
[b, a]  = cheby1(m, gp_db, wn);
freqz(b, a);
title("Magnitude: Chebyshev1 ");
disp("Order of chebyshev"); disp(max(length(b), length(a)));

%%
figure
[m, wn] = ellipord(wp, wst, gp_db, gst_db);
[b, a]  = ellip(m, gp_db, gst_db, wn);
freqz(b, a);
title("Magnitude: Elliptic ");
disp("Order of Elliptic"); disp(max(length(b), length(a)));
```