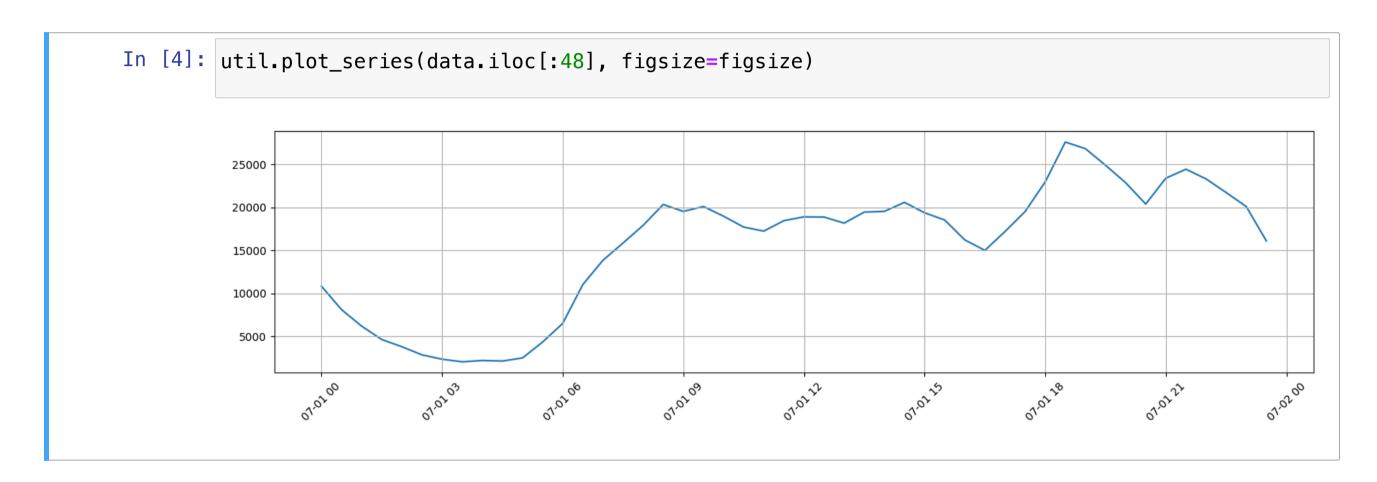


Let's have a closer look at our time series



- Nearby points tend to have similar values
- ...Meaning they are correlated

Determine the Correlation Interval

How can we study such correlation?

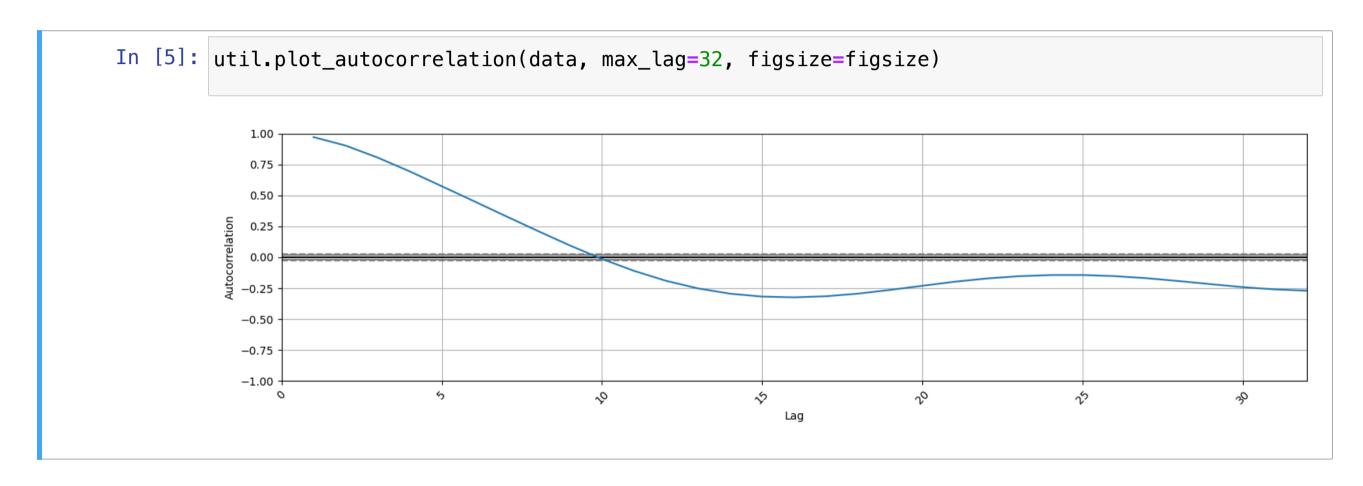
A useful tool: <u>autocorrelation</u> plots

- Consider a range of possible lags
- For each lag value *l*:
 - lacksquare Make a copy of the series and shift it by $m{l}$ time steps
 - Compute the <u>Pearson Correlation Coefficient</u> with the original series
- Plot the correlation coefficients over the lag values

Then we look at the resulting plot:

- Where the curve is far from zero, there is a significant correlation
- Where it gets close to zero, no significant correlation exists

Let's have a look at our plot



■ The correlation is strong up to 4-5 lags

These correlations are a source of information

- They could be exploited to improve our estimated probabilities
- ...But our models so far make no use of them

How can we take advantage of them?

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How can we take advantage of them?

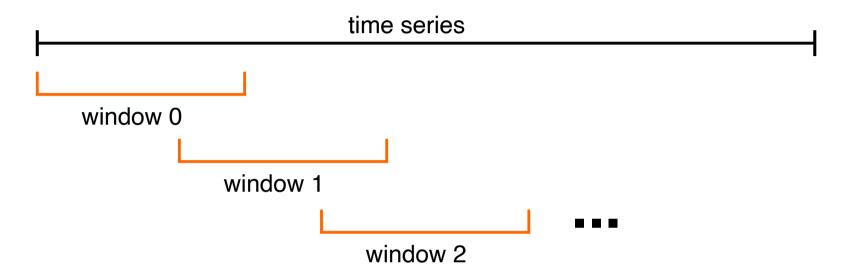
For example, rather then feeding our model with individual observations

We can use sequences of observations as input

- This is a very common approach in time series
- ...And in many cases it's a good idea

Sliding Window

A common approach consist in using a sliding window



- We choose a window length w, i.e. the length of each sub-sequence
- We place the "window" at the beginning of the series
- ...We extract the corresponding observations
- Then, we move the forward by a certain stride and we repeat

Sliding Window

The result is a table

Let m be the number of examples and w be the window length

	$\mathbf{S_0}$	\mathbf{S}_{1}	• • •	S_{W-1}
t_{w-1}	x_0	x_1	• • •	x_{w-1}
$t_{\rm w}$	x_1	x_2	• • •	x_w
t_{w+1}	$\overline{x_2}$	x_3	• • •	x_{w+1}
•	•	:	•	•
t_{m-1}	x_{m-w}	x_{m-w+1}	•	x_{m-1}

- The first window includes observations from x_0 to x_{w-1}
- The second from x_1 to x_w and so on
- t_i is the time window index (where it was applied)
- s_i is the position of an observation within a window

pandas provides a sliding window iterator

```
DataFrame.rolling(window, ...)
```

```
In [6]: wlen = 10
        for i, w in enumerate(data['value'].rolling(wlen)):
            print(w)
            if i == 2: break # We print the first three windows
        timestamp
        2014-07-01
                      10844
        Name: value, dtype: int64
        timestamp
        2014-07-01 00:00:00
                                10844
        2014-07-01 00:30:00
                                 8127
        Name: value, dtype: int64
        timestamp
        2014-07-01 00:00:00
                                10844
        2014-07-01 00:30:00
                                 8127
        2014-07-01 01:00:00
                                 6210
        Name: value, dtype: int64
```

Notice how the first windows are not full (shorter than wlen)

We can build our dataset using the rolling iterator

- We discard the first wlen-1 (incomplete) applications
- Then we store each window in a list, and we wrap everything in a DataFrame

```
In [7]: %%time
    rows = []
    for i, w in enumerate(data['value'].rolling(wlen)):
        if i >= wlen-1: rows.append(w.values)

wdata_index = data.index[wlen-1:]
    wdata = pd.DataFrame(index=wdata_index, columns=range(wlen), data=rows)

CPU times: user 170 ms, sys: 3.94 ms, total: 173 ms
    Wall time: 172 ms
```

- The values field allows access to the Series content as a numpy array
- We use it to discard the index
- ...Since the series for multiple iterations have inconsistent indexes

This method works, but it's a bit slow

- We are building our table by rows...
- ...But it is usually faster to do it by columns!
- After all, there are usually fewer columns than rows

Let us look again at our table:

	$\mathbf{S_0}$	\mathbf{S}_{1}	• • •	s_{w-1}
t_{w-1}	x_0	x_1	• • •	x_{w-1}
$t_{\rm w}$	x_1	x_2	• • •	x_w
t_{w+1}	x_2	x_3	• • •	x_{w+1}
•	•	•	•	•
t_{m-1}	x_{m-w}	x_{m-w+1}	•	x_{m-1}

We can build the columns by slicing the original DataFrame

```
In [8]: m = len(data)
        c0 = data.iloc[0:m-wlen+1] # first column
        c1 = data.iloc[1:m-wlen+1+1] # second column
        print(c0.iloc[0:3])
        print(c1.iloc[0:3])
                              value
        timestamp
        2014-07-01 00:00:00
                              10844
        2014-07-01 00:30:00
                               8127
        2014-07-01 01:00:00
                               6210
                              value
        timestamp
                               8127
        2014-07-01 00:30:00
        2014-07-01 01:00:00
                               6210
        2014-07-01 01:30:00
                               4656
```

• iloc in pandas allows to address a DataFrame by position

2014-07-01 04:30:00

2014-07-0105:00:00 8127

2014-07-01 05:30:00 6210

2014-07-01 06:00:00 4656

2014-07-01 06:30:00 3820

10844

6210

3820

2873

Now we collect all columns in a list and we stack them

8127 6210 4656 3820

3820

2369

2873

2064

2369

2221

2369 2064 2221 2158 2515 4364 6526 11039

2064

2873 2369 2064 2221 2158 2515 4364

4656

2873

4656 3820

2873 2369 2064 2221 2158

2221

2158 2515 4364 6526

2158

2515

We can wrap this approach in a function:

```
def sliding_window_1D(data, wlen):
    m = len(data)
    lc = [data.iloc[i:m-wlen+i+1] for i in range(0, wlen)]
    wdata = np.hstack(lc)
    wdata = pd.DataFrame(index=data.index[wlen-1:], data=wdata, columns=range(wlen))
    return wdata
```

```
In [10]: %%time
wdata = util.sliding_window_1D(data, wlen=wlen)

CPU times: user 1.16 ms, sys: 219 µs, total: 1.38 ms
Wall time: 1.11 ms
```

- This is available in the (updated)) nab module
- The function works for univariate data (but the approach is general)