Time Indexed Models

Exploiting Time

Let's consider how we dealt with time so far

- We learned an estimator for $f(t, x)$ and one for $f(t)$
- ...Which we used to compute $f(x \mid t) = f(t, x)/f(t)$

It worked well, but we had to introduce one additional dimension

What if we wanted to consider time and sequence input?

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What if we wanted to consider time and sequence input?

Let's consider a second approach to handle time

- This consists in learning many density estimators:
- Each estimator is specialized for a given time (e.g. 00:00, 00:30, 01:00...)

We can then choose which estimator to use based on the current time

Exploiting Time

Formally, what we have is a first ensemble model

In particular, we obtain our estimated probabilities by evaluating:

$$f_{g(t)}(x)$$

- Each f_i function is an estimator
- The $g(t)$ retrieves the correct f_i based (in our case) on the time value

We'll call this general idea a "selection ensemble"

In terms of properties:

- Each f_i estimator works with smaller amounts of data
- ...But the individual problems are easier!

Learning an Estimator for one Time Value

Let us make a test by learning an estimator for a single time value

First, we separate the training data

In [3]:
```
wdata_tr = wdata[wdata.index < train_end]
wdata_tr.head()
```

Out[3]:

	0	1	2	3	4	5	6	7	8	9
timestamp										
2014-07-01 04:30:00	0.357028	0.267573	0.204458	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050
2014-07-01 05:00:00	0.267573	0.204458	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	0.082804
2014-07-01 05:30:00	0.204458	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	0.082804	0.143680
2014-07-01 06:00:00	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	0.082804	0.143680	0.214862
2014-07-01 06:30:00	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	0.082804	0.143680	0.214862	0.363448

- We'll use the normalized version
- ...So as to simplify our guesses for bandwidth selection

Learning an Estimator for one Time Value

Let us make a test by learning an estimator for a single time value

Then, we focus on the values for a single time value

In [4]:
```
wdata_tr_test = wdata_tr.iloc[0::48] # 48 is the step
wdata_tr_test.head()
```

Out[4]:

	0	1	2	3	4	5	6	7	8	9
timestamp										
2014-07-01 04:30:00	0.357028	0.267573	0.204458	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050
2014-07-02 04:30:00	0.440194	0.327429	0.249267	0.194811	0.158694	0.119646	0.098541	0.083462	0.084615	0.081816
2014-07-03 04:30:00	0.416357	0.347743	0.277088	0.233694	0.191815	0.144306	0.107661	0.097060	0.103579	0.101307
2014-07-04 04:30:00	0.513318	0.473941	0.412702	0.373391	0.328581	0.276693	0.237053	0.216574	0.186251	0.147302
2014-07-05 04:30:00	0.578672	0.533006	0.475455	0.412702	0.362361	0.301287	0.263721	0.233629	0.210944	0.145557

Learning a 23:30 Estimator

Then we proceed as usual

We choose a bandwidth:

```
In [5]: grid = GridSearchCV(KernelDensity(kernel='gaussian'), {'bandwidth': np.linspace(0.01, 0.1, 
        grid.fit(wdata_tr_test)
        grid.best_params_

Out[5]: {'bandwidth': np.float64(0.019473684210526317)}
```

Then we store the bandwidth in a variable:

```
In [6]: h = grid.best_params_['bandwidth']
```

- For sake of simplicity, we'll use the same bandwidth for all estimators
- Even if we should re-calibrate h for each estimator in principle

Learning the Ensemble

Now, we need to repeat the process for every unique time value

```
In [7]: day_hours = data_tr.index.hour + data_tr.index.minute / 60
        day_hours = day_hours.unique()
        print(day_hours)

Index([ 0.0,  0.5,  1.0,  1.5,  2.0,  2.5,  3.0,  3.5,  4.0,  4.5,  5.0,  5.5,
        6.0,  6.5,  7.0,  7.5,  8.0,  8.5,  9.0,  9.5, 10.0, 10.5, 11.0, 11.5,
       12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.5,
       18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, 23.5],
      dtype='float64', name='timestamp')
```

- `unique` in pandas returns a `Series` with all unique values
- We do not care about how time is measured
- ...We only care about having 48 discrete steps

Learning the Ensemble

Finally, we can learn 48 specialized estimators

```
In [8]: kde = {}
        for hidx, hour in enumerate(day_hours):
            tmp_data = wdata_tr.iloc[hidx::48]
            kde[hour] = KernelDensity(kernel='gaussian', bandwidth=h)
            kde[hour].fit(tmp_data)
```

- For each unique time value, we separate a subset of the training data
- Then we build and learn a KDE estimator

We chose to store everything in a dictionary:

```
In [9]: print(str(kde)[:256], '...}')
```

```
{0.0: KernelDensity(bandwidth=np.float64(0.019473684210526317)), 0.5: KernelDensity(bandwi
dth=np.float64(0.019473684210526317)), 1.0: KernelDensity(bandwidth=np.float64(0.019473684
210526317)), 1.5: KernelDensity(bandwidth=np.float64(0.019473684210526317)), ...}
```

Generating the Signal

The we can generate the alarm signal

- In a practical implementation we should do this step by step
- ...But for an evaluation purpose it is easier to do it all at once

```
In [10]: ldens_list = []
         for hidx, hour in enumerate(day_hours):
             tmp_data = wdata.iloc[hidx::48]
             tmp_ldens = kde[hour].score_samples(tmp_data)
             tmp_ldens = pd.Series(index=tmp_data.index, data=tmp_ldens)
             ldens_list.append(tmp_ldens)
```

- For each unique time value, we separate a subset of the whole data
- Then we obtain the estimated (log) probabilities

The process is even faster than before

- ...Because each KDE estimator is trained a smaller dataset

Generating the Signal

All signals are stored in a list

- We need to concatenate them all in single `DataFrame`
- Then we can sort all rows by timestamp (it's the index)

```
In [11]: ldens = pd.concat(ldens_list, axis=0)
         ldens = ldens.sort_index()
         signal = -ldens
         signal.head()

Out[11]: timestamp
         2014-07-01 04:30:00   -27.059255
         2014-07-01 05:00:00   -27.505901
         2014-07-01 05:30:00   -27.741645
         2014-07-01 06:00:00   -27.925602
         2014-07-01 06:30:00   -27.657585
         dtype: float64
```

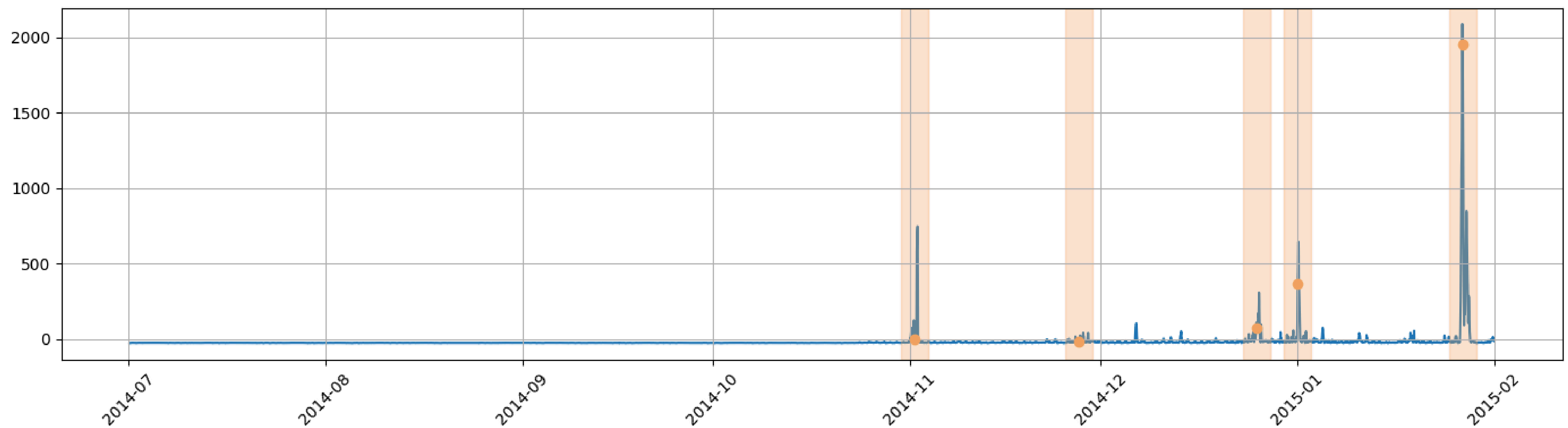
A suggestion: always do concatenations in a single step in `pandas`

It's way faster than appending `DataFrame` objects one by one

Generating the Signal

Now we can plot out signal:

In [12]:
```
util.plot_series(signal, labels=labels, windows=windows, figsize=figsize)
```



- It's very similar to that of the other time-based model
- ...But also a bit smoother, like that of the sequence-based model

Threshold Optimization and Evaluation

Now we can optimize the threshold and evaluate the results

In [13]:
```
signal_opt = signal[signal.index < val_end]
labels_opt = labels[labels < val_end]
windows_opt = windows[windows['end'] < val_end]
thr_range = np.linspace(10, 200, 100)

best_thr, best_cost = util.opt_thr(signal_opt, labels_opt, windows_opt, cmodel, thr_range)
print(f'Best threshold: {best_thr}, corresponding cost: {best_cost}')
```

```
Best threshold: 104.04040404040404, corresponding cost: 10
```

Let us see the cost on the whole dataset:

In [14]:
```
ctst = cmodel.cost(signal, labels, windows, best_thr)
print(f'Cost on the whole dataset {ctst}')
```

```
Cost on the whole dataset 10
```

This is the best result we have achieved so far!

What if we used this approach for the second period?