$$l(\theta \mid x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} \sum_{i=1}^{n} \left[\log \binom{m}{x_i} + x_i \log(\theta) + (m - x_i) \log(1-\theta) \right]$$

$$\frac{dl}{d\theta} = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \left(\frac{m - x_i}{1-\theta} \right) \right] = 0$$

Solving for $\theta \in \Theta = (0,1)$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = mn + \sum_{i=1}^{n} \frac{x_i}{1-\theta}$$

$$\boxed{\theta = \sum_{i=1}^{n} \frac{x_i}{n \cdot m}}$$

$$\boxed{\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n \cdot m}}$$

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1.) $f(x) = \frac{1}{\sqrt{2\pi\theta_2}} e^{\left(\frac{-(x_i-\theta_1)^2}{2\theta_2}\right)}$

$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\left(-\frac{(x_i-\theta_1)^2}{2\theta_2}\right)}$

Taking Log on both sides

$l(\theta_1, \theta_2) = \frac{n}{2}\log(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i - \theta_1)^2$

Differentiating w.r.t. θ_1 & θ_2

→ $\frac{dl}{d\theta_1} = \frac{1}{\theta_1}\sum_{i=1}^{n}(x_i - \theta_1) = 0$

$\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$

$\frac{dl}{d\theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2}\sum_{i=1}^{n}(x_i - \theta_1)^2 = 0$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}(x_i - \theta_1)^2$

$$\boxed{\begin{aligned} \text{MLE :- } \theta_1 &= \frac{1}{n}\sum_{i=1}^{n} x_i \\ \theta_2 &= \frac{1}{n}\sum_{i=1}^{n}(x_i - \theta_1)^2 \end{aligned}}$$

2) $L(\theta | x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{x_i}(1-\theta)^{n-x_i}$

Taking log on both sides