Transformations cont

Feb 8

(Affine)

transformations so far:

$$b_x = a_{11} x + a_{12} y$$
$$b_y = a_{21} x + a_{22} y$$

translations:

$$b_x = x + t_x$$
$$b_y = x + t_y$$

With operations

RSTRSSRSTRS

lets rewrite

$$b_x = \overbrace{a_{11}}^{1} x + \overbrace{a_{12}}^{0} y + t_x 1 = x + t_x$$

$$b_y = \underbrace{a_{21}}_{0} x + \underbrace{a_{22}}_{1} y + t_y 1 = y + t_y$$

$$\Rightarrow \begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

← not compatable w/ our rotation & scaling

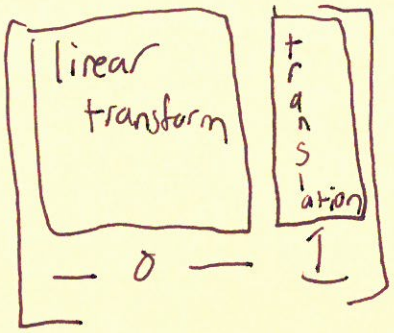
Maricies for rot and scaling

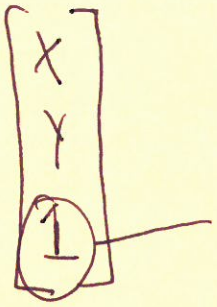
$$\begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11}x + m_{12}y \\ m_{21}x + m_{21}y \\ 1 \end{bmatrix}$$

matrix for translation

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

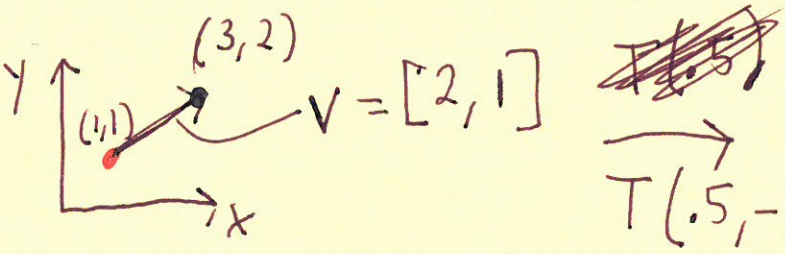
Affine transformation are linear transformations w/ translation

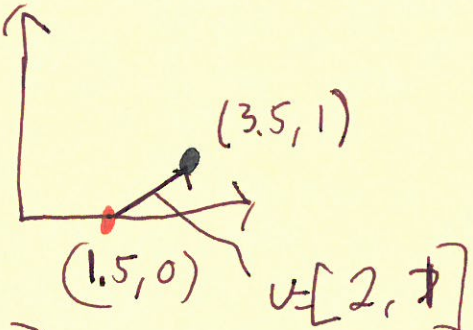




Homogeneous coordinate
usual as "w" coord

What about directions



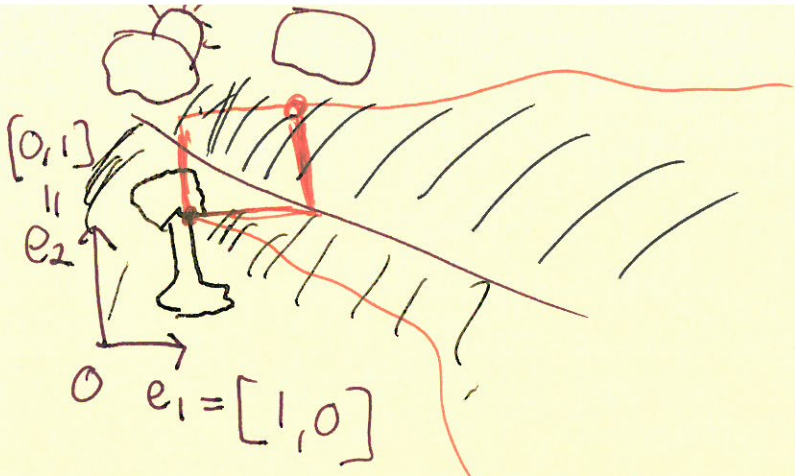


$$\begin{bmatrix} 1 & 0 & .5 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+.5 \\ 1+(-1) \\ 1 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 1 \end{bmatrix}$$

Wrong

With direction or difference

$$\begin{bmatrix} 1 & 0 & .5 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$



$h=(h_x, h_y)$

$=(l_x+3, l_y+2)$

lets convet our rect to

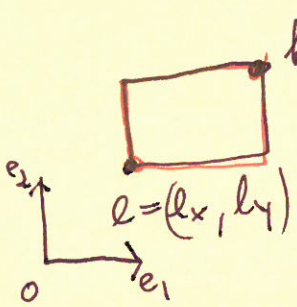
$[-1,1]\times[-1,1]$

or said another way box w/

min pt $(-1,-1)$

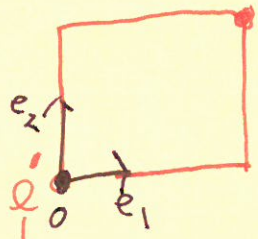
Max pt $(1,1)$

$l=(l_x, l_y)$



$\Downarrow T(-l_x, -l_y)$

$h'=(3,2)$



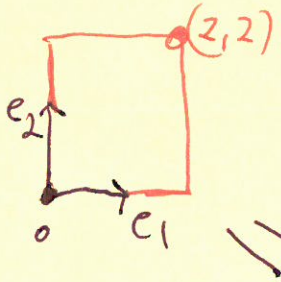
$\Downarrow S(\frac{3}{3}, 1)$

1) $S(\frac{1}{3}, \frac{1}{2})$

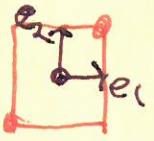
2) $S(\frac{2}{3}, 1)$

3) $S(15, 3)$

$3*x=2 \iff \frac{2}{3}=x$



$T(-1,-1)$



QED!