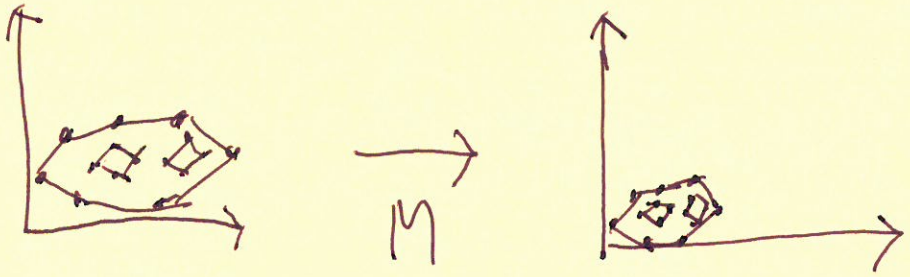
Transformations

Feb 6

Geometric transformations

- ✱ – rotation
- Th – translation
- ✱ – scaling
- Th – projection

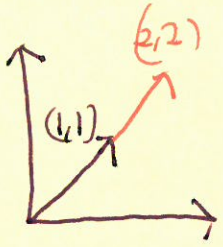


2D linear transforms

2 × 2 matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix}$$

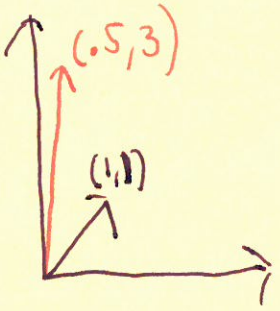
Scaling



$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

what are entries of this matrix

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

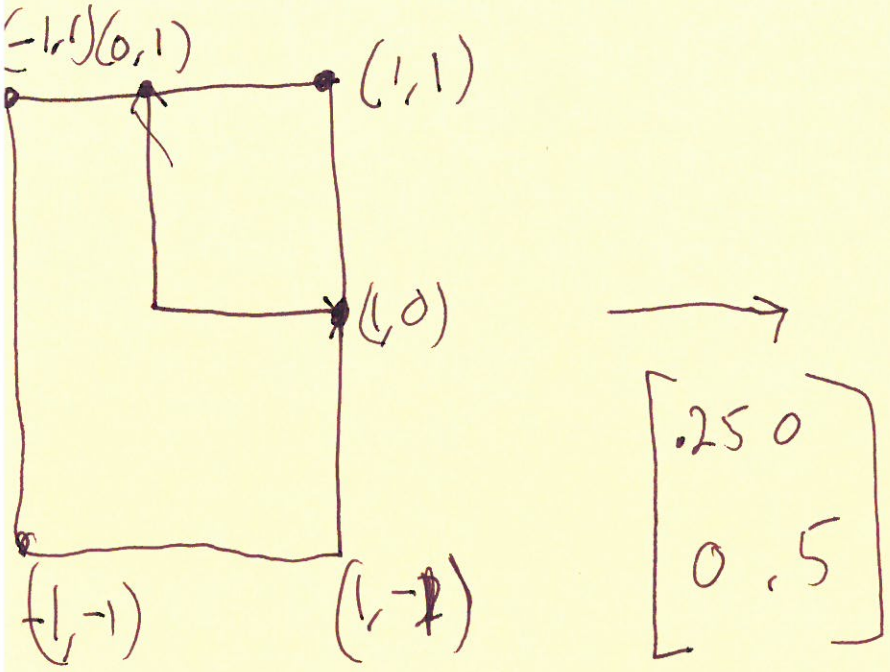


$$\begin{bmatrix} .5 \\ 3 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Scale along coordinate axis

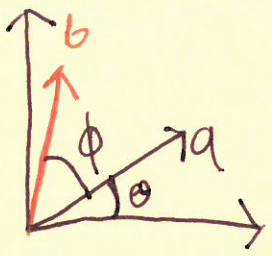
$$\text{scale}(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

Scaling on a "circle"



$$\begin{bmatrix} .25 & 0 \\ 0 & .5 \end{bmatrix}$$

Rotation



$$r = \|a\| = \sqrt{a_x^2 + a_y^2}$$

$$a_x = r\cos\theta$$
$$a_y = r\sin\theta$$

since

$$\|a\| = \|b\|$$

$$\Rightarrow \quad b_x = r\cos(\theta+\phi)$$
$$b_y = r\sin(\theta+\phi)$$

Trig identity

$$\Rightarrow \quad b_x = \overbrace{r\cos\theta}^{a_x}\cos\phi - \overbrace{r\sin\theta}^{a_y}\sin\phi$$

$$b_y = \underbrace{r\sin\theta}_{a_y}\cos\phi + \underbrace{r\cos\theta}_{a_x}\sin\phi$$

$$b_x = a_x\cos\phi - a_y\sin\phi$$
$$b_y = a_x\sin\phi + a_y\cos\phi$$

$$\Rightarrow \quad \begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix}$$

$$\text{rotate}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

Why do we care about matricies?

- working in higher dim
- specialized hardware
- encode many operations

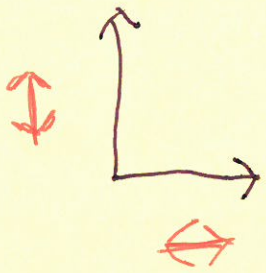
Let S be a scaling matrix
R be a rotation matrix
v be a vector

we want to scale and rotate

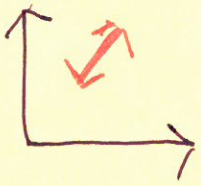
$$v_1 = Sv$$
$$v_2 = Rv_1$$

$$\Rightarrow v_2 = R(Sv) = \underbrace{(RS)}_{T=RS} v$$

Problem: I can scale in x or y



What if scale in ~~a~~ a different direction



rotate 45°

scale along Y

rotate -45°