# Camera transformations

Feb 20

constraints

- ~~geom in canonical view~~
- camera uses orthog projection
- ~~looking in -z~~

## Define

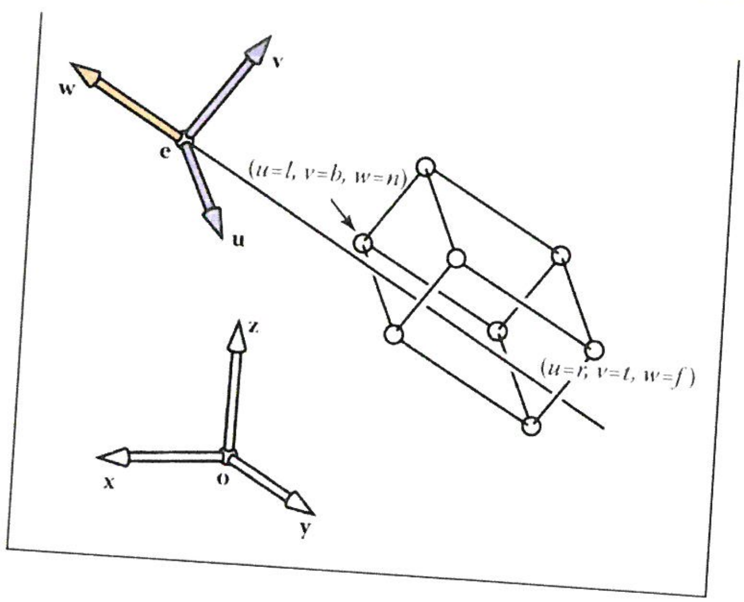
- e: eye position
- g: gaze direction
- t: viewer up direction

What we want:

$$w = \frac{-g}{\|g\|}$$

$$u = \frac{t \times w}{\|t \times w\|}$$

$$v = w \times u$$



I can write the coordinates of everything in the view box as

$$M_{cam} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

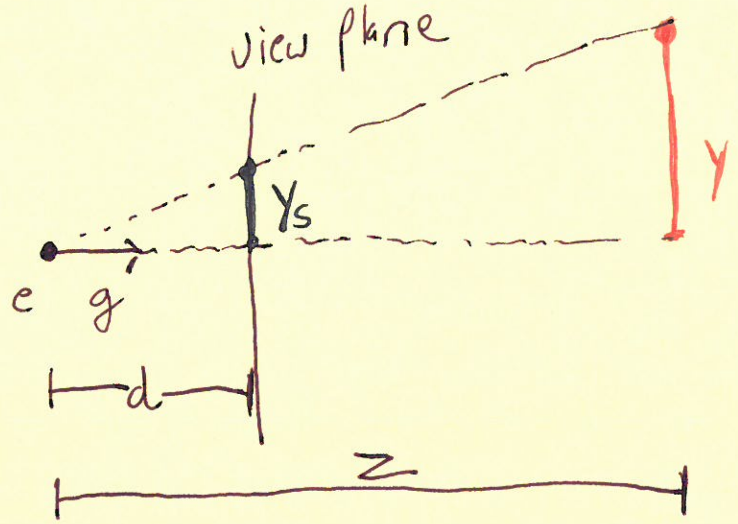
Remember

$M_{vp}$ is the viewport matrix

perform trans w/

$$\begin{bmatrix} x_{pixe} \\ y_{pixel} \\ z_{cannon} \\ 1 \end{bmatrix} = M_{vp} V_{orth} V_{cam} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Projective Transformations




$$y_s = \frac{d}{z} y$$ ← we can't generalize this w/ affine transformations

Define transformation

homogeneous coordinate → cartesean coordinate

$$(x, y, z, w) \longrightarrow (x/w,\ y/w,\ z/w)$$

linear trans

$$x' = a_1 x + b_1 y + c_1 z$$

affine transformations

$$x' = a_1 x + b_1 y + c_1 z + d_1$$

with $w$ as denom

$$x' = \frac{a_1 x + b_1 y + c_1 z + d_1}{ex + fy + gz + h} \quad ex + fy + gz + h = \tilde{w}$$

$$y' = \frac{a_2 x + b_2 y + c_2 z + d_2}{ex + fy + gz + h} \quad ex + fy + gz + h = \tilde{w}$$

$$z' = \text{---}$$

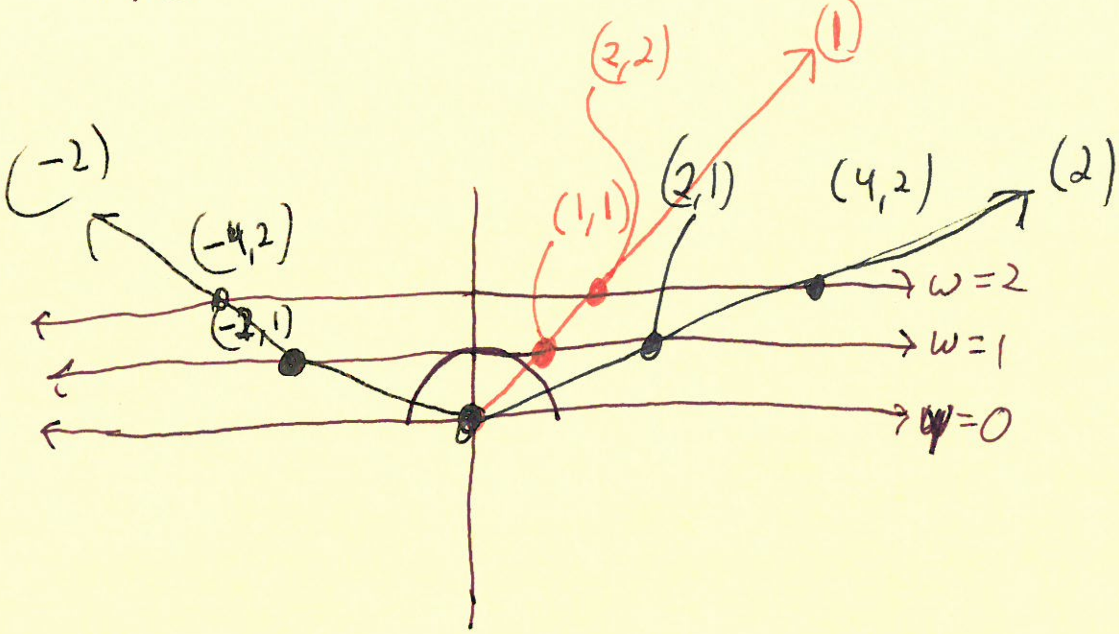
as a matrix

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ e & f & g & h \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \tilde{x}/\tilde{w} \\ \tilde{y}/\tilde{w} \\ \tilde{z}/\tilde{w} \end{bmatrix}$$

called a "projective transformation"

for cartesian coodinate $x$

$x \sim \alpha x \quad \forall x \neq 0$



perspective projection

my toy example"

$$\begin{bmatrix} y_s \\ 1 \end{bmatrix} = \begin{bmatrix} d & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix} \iff y_s = \frac{d}{z} y$$

"official matrix"

$z = n \equiv$ near plane

$z = f \equiv$ far plane

$n < 0$ (looking in $-z$ direction)

$$P = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

brings z coordinate along for the ride for suface ordering

perspective transform

$$\begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} nx \\ ny \\ (n+f)z - fn \\ z \end{bmatrix} \sim \begin{bmatrix} \frac{nx}{z} \\ \frac{ny}{z} \\ n+f-\frac{fn}{z} \\ 1 \end{bmatrix}$$

Lets put this together

$$M_{per} = M_{ort} P$$

Camera :

w/o perspective : $M_1 = M_{vp} M_{orth} M_{cam}$

w/ perspective : $M_2 = M_{vp} \underbrace{M_{orth} P}_{M_{per}} M_{cam}$

$$M_{per}^{(GL)} = \begin{bmatrix} \frac{2|n|}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2|n|}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{|n|+|f|}{|n|-|f|} & \frac{2|f||n|}{|n|-|f|} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$