Projections

Feb 15

Parallel projection

- move along a projection until
  hit a image plane

(a) orthographic projection
   image plane is $\perp$ to view direction

(b) oblique projection:
   image plane is $\not\perp$ to view direction

Why do we want this:

- parallel lines stay parallel
- size and shape of planar objects
  parallel to image plane are preserved

perspective projections

- all lines pass through a single point

Viewport transformation

Assume:

- geometry in in cannonical view
  (all objects are represented by
  verts w/ cartesian coordinates
  in $(x, y, z) \in [-1, 1]^3$

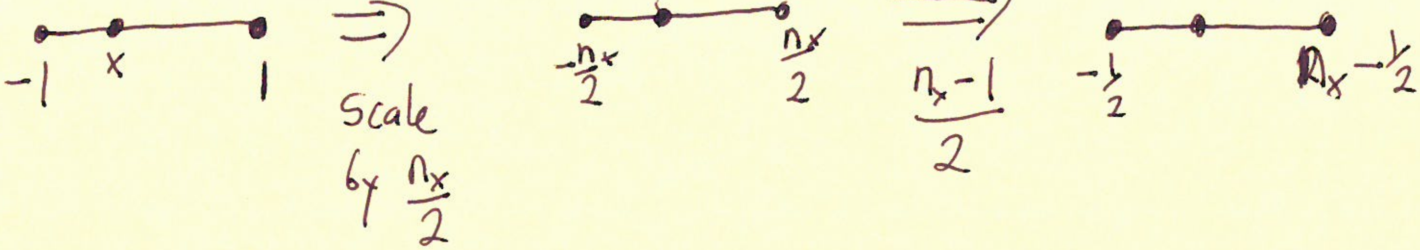
- camera using orthographic projection

- look in $-z$ directon,

Project

- $x = -1 \rightarrow$ ~~the~~ left of screen (px 0)
- $x = 1 \rightarrow$ right of screen (px n_x) $\leftarrow$ x width
- $y = 1 \rightarrow$ top of screen (p_x n_y) $\leftarrow$ y width
- $y = -1 \rightarrow$ bottom of screen (px 0)

in X



$$\begin{bmatrix} x_{screen} \\ y_{screen} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & \frac{n_y-1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{canon} \\ y_{canon} \\ 1 \end{bmatrix}$$

to add z

$$M_{vp} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic Projection

Assumption

- ~~geom in canonical view~~
- camera using orthographic projection
- lookin in $-z$

Define: view bounding box

$x = l \equiv$ left plane
$x = r \equiv$ right plane
$y = b \equiv$ bottom plane
$y = t \equiv$ top plane
$z = n \equiv$ near plane
$z = f \equiv$ far plane

Lets see for x-dim

we know $\ell \leq x \leq r$

Goal: $-1 \leq \alpha(x) \leq 1$

$$\ell \leq x \leq r$$

$$\Longleftrightarrow 0 \leq x - \ell \leq r - \ell$$

$$\Longleftrightarrow 0 \leq \frac{x-\ell}{r-\ell} \leq 1$$

$$\Longleftrightarrow 0 \leq 2\frac{x-\ell}{r-\ell} \leq 2$$

$$\Longleftrightarrow -1 \leq 2\frac{x-\ell}{r-\ell} - 1 \leq 1$$

$$\Longleftrightarrow -1 \leq 2\frac{x-\ell}{r-\ell} - \frac{r-\ell}{r-\ell} \leq 1$$

$$\Longleftrightarrow -1 \leq \frac{2x - 2\ell - r + \ell}{r-\ell} \leq 1$$

$$\Longleftrightarrow -1 \leq \frac{2x - \ell - r}{r-\ell} \leq 1$$

$$\Longleftrightarrow -1 \leq \frac{2x}{r-\ell} - \frac{r+\ell}{r-\ell} \leq 1$$

$$\frac{2x}{r-l} + 0y + 0z + \frac{-(r+l)}{(r-l)}1$$

$$M_{orth} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

result

$$\begin{bmatrix} x_{pixel} \\ y_{pixel} \\ z_{cannon} \\ 1 \end{bmatrix} = M_{vp} M_{orth} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

For open GL

we are looking at $-z$

so we start derivation w/

$$n \le -z \le f$$

$$
M_{orth}^{(GL)} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{-2}{-n+f} & -\frac{n+f}{-n+f} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$