Lectures in Astroparticle Phenomenology
I. Particle Cosmology

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Lecture Plan

Today: Particle Cosmology

- ΛCDM
- Power spectra of cosmological perturbations
- Reheating, Big Bang nucleosynthesis, cosmic strings

Tomorrow (SIfA Redfern): Dark Matter

- Theories
- Production
- Direct + indirect detection

Thursday (back here again): Global Fits

- Techniques, status and coming developments

Other cool stuff I won't be covering directly

(but that we can chat about at the end of one of lectures if you like)

- Neutrino mass models / GUTs and their observables
- Cosmic ray production + propagation
- Baryogenesis / leptogenesis
- Reionisation

Outline of Lecture 1

1. Cosmological Models
   - General
   - ΛCDM
2. Power spectra of cosmological perturbations
   - Background
   - Middle Universe observables
   - Rare objects
3. Specific particle/field processes **(optional)**
   - BBN
   - Cosmic strings
   - Phase transitions and reheating

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Setting up a framework for cosmology

Step 1.

Assume the Universe to be isotropic and homogeneous

$\Longrightarrow$ Friedmann-Robertson Walker (FRW) metric:

$$g_{\mu\nu}x^{\mu}x^{\nu} = \mathrm{d}t^2 + R(t)^2\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right). \tag{1}$$

- This is our distance measure in spacetime.
- Everything but $R(t)$ is determined by isotropy & homogeneity.

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Take the Einstein Field Equations from General Relativity:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}. \tag{2}$$

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$\rightarrow$ Tells us how geometry of space adjusts to mass/energy

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Solve $\mu = 0, \nu = 0$ of Einstein Eq. $\implies$ *Friedmann Equation*:

$$H(t)^2 \equiv \left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{k}{R(t)^2}. \tag{3}$$

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Critical density:

For a flat Universe $k = 0$. This defines

$$\textit{critical density:} \quad \rho_c = \frac{3H(t)}{8\pi G}$$

$$\textit{cosmological density:} \quad \Omega_x \equiv \frac{\rho_x}{\rho_c}$$

Equations of state

Equations of state:
1st law of thermodynamics ($\mu = 0$ in conservation of $T_{\mu\nu}$) is

$$\mathrm{d}(\rho R^3) = -p\mathrm{d}(R^3), \quad \text{i.e. } \Delta E = -p\Delta V \tag{4}$$

with a constant equation of state $\rho = wp$, we get energy density-scalefactor relations

$$\rho \propto R^{-3(1+w)} \tag{5}$$

For different types of energy:

Matter:	$w = 0$	$\Longrightarrow$	$\rho \propto R^{-3}$
Radiation:	$w = 1/3$	$\Longrightarrow$	$\rho \propto R^{-4}$
Vacuum (Λ):	$w = -1$	$\Longrightarrow$	$\rho \propto$ *constant*

This is basically enough to solve the Friedmann Equation.

Ingredients of ΛCDM

Ingredients required for a cosmological model	**Choices in ΛCDM**
A theory of gravity $+$ associated assumptions	GR $+$isotropy, homogeneity
Types of energy	radiation, matter, vacuum/dark energy
their equations of state	$w = 1/3,\ 0,\ -1$/other
their (self$-$)interactions	photons, baryonic (SM) matter $+$cold dark matter (CDM), ??
An initial spectrum of perturbations	approximately scale invariant on large scales

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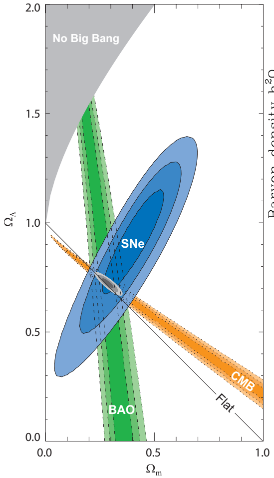
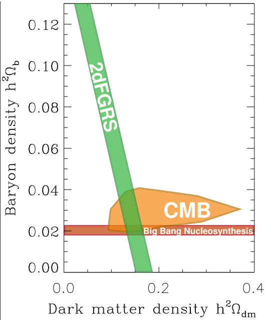
Cosmological probes & ‘concordance cosmology’

Joint fit to multiple cosmological observables gives a consistent set of parameter values:

$$\Omega_\Lambda \approx 0.73$$
$$\Omega_{\rm matter} \approx 0.27$$
$$=$$
$$\Omega_{\rm CDM} \approx 0.23 + \Omega_{\rm baryons} \approx 0.04$$
$$\rightarrow \Lambda\mathrm{CDM}$$



Kowalski et al *ApJ* 2008



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Due to inflation by definition? No.

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ΛCDM does not *demand* inflation, just as it does not *demand* any particular CDM

—inflation is just an idea for getting the required spectrum on CMB scales
—any particular DM model is just an idea for getting CDM

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Generation of perturbations

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$$\mathrm{pdf}(\delta) = \frac{1}{\sqrt{2\pi}\sigma^2_{\chi,H}(z_\mathrm{X}, R)} \exp\left(-\frac{\delta^2}{2\sigma^2_{\chi,H}(z_\mathrm{X}, R)^2}\right) \tag{8}$$

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Universe stops inflating, keeps expanding $\rightarrow$ catches up with the perturbations

$\Longrightarrow$ first out, last in (bigger-scale perturbations re-enter later)

Growth of perturbations

Post-inflation, Universe quickly becomes radiation-dominated

$\Longrightarrow$ baryons + photons coupled by electromagnetism

$\Longrightarrow$ growth of perturbations damped by radiation free-streaming

$\Longrightarrow$ growth of perturbations is logarithmic only

$$\delta \propto \log R \tag{9}$$

At $z \sim 3000$, baryons kinetically decouple as Universe becomes matter-dominated

$\Longrightarrow$ damping relieved

$\Longrightarrow$ perturbations grow linearly, structure growth begins

$$\delta \propto R \tag{10}$$

Observation of perturbations

Essentially all cosmological observables depend on 2 things:

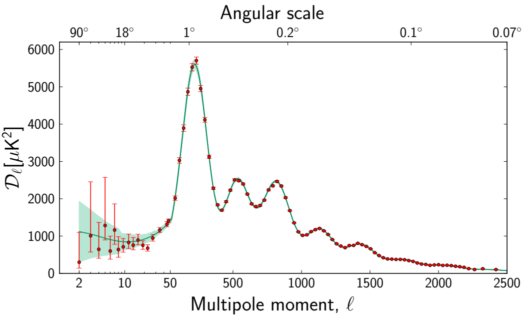
1. The initial spectrum of perturbations
   = distribution of amplitudes over scales: pdf(δ, k), $\mathcal{P}_\delta(k)$
   - $\rightarrow$ this (mostly) comes from your theory of inflation
2. How the perturbations + their consequences are processed
   - $\rightarrow$ the geometry of the Universe over time: $H(t)$
   - $\rightarrow$ the specific content of the Universe: $\mathcal{L}_{\mathrm{SM+BSM}}$
     - new particles
     - exotic objects (e.g. cosmic strings)
     - specific processing events associated with the content (BBN, phase transitions, etc)

$\Longrightarrow$ can use CMB, large scale structure, etc to test specific particle theories
(not just how much stuff with each w and its impact on $H(t)$)

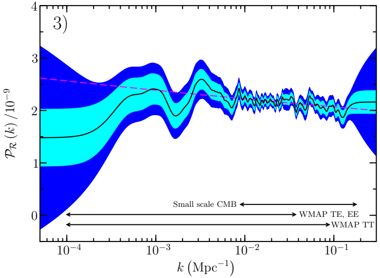
The cosmic microwave background (CMB)

Key is to look at amount of power on different scales for info on primordial spectrum and processing physics

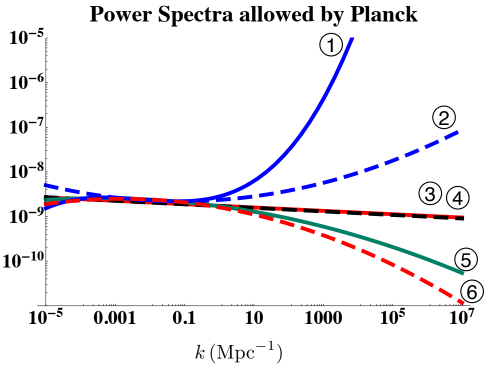
Planck Paper XV



Hunt & Sarkar (*JCAP* 2014)



The CMB – inflation-like examples

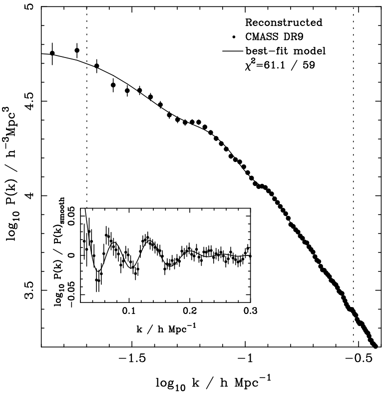


1. ΛCDM + $dn_s/d\ln k$ + $d^2(n_s)/d\ln k^2$, *Planck*+WMAP polarisation (WP)
2. ΛCDM + r + $dn_s/d\ln k$, *Planck*+WP
3. ΛCDM + r, *Planck*+WP
4. ΛCDM, *Planck*+WP
5. ΛCDM + $dn_s/d\ln k$, *Planck*+WP
6. ΛCDM + r + $dn_s/d\ln k$, *Planck*+WP+BAO

Shandera, Erickcek, PS & Yana Galarza *Phys. Rev. D* 2013

Large scale structure (LSS)

- Density perturbations $\equiv$ sound waves $\longrightarrow$ matter density oscillations
- Seen in CMB temperature, polarisation anisotropy at $z \sim 1100$
- Eventually grow to form galaxies, etc at $z \lesssim 20$



Some imprint of scales of density oscillations (=primordial spectrum) retained $\longrightarrow$ Baryon Acoustic Oscillations

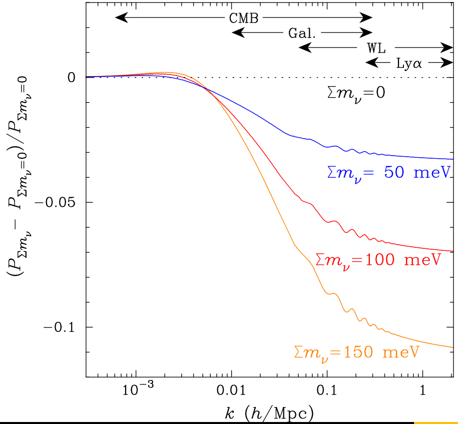
SDSS-III Data Release 9 *MNRAS* 2012

Large scale structure – neutrino mass example

Neutrinos are warm dark matter

$\Longrightarrow$ large free-streaming length, depends on m_ν
$\rightarrow$ escape from small collapsing perturbations

$\Longrightarrow$ Suppression in small-scale matter (processed) power spectrum



SNOWMASS 2013, arXiv:1309.5383

Rare objects: primordial black holes (PBHs)

Question

What is a *primordial* black hole?

- If density perturbations are big enough ($\delta > 0.3$), when they enter the horizon the whole thing collapses $\longrightarrow$ immediate black hole
- PBH mass is horizon mass at re-entry

$\Longrightarrow$ number of PBHs with M_{PBH} maps directly to amplitude of perturbations on some scale k

- For some distribution of perturbations $\mathrm{pdf}(\delta)$

$$\beta_{\mathrm{PBH}} \equiv \Omega_{\mathrm{PBH}}/\Omega_0 = \int_{\delta_{\mathrm{min}}}^{\infty} \mathrm{pdf}(\delta)\,\mathrm{d}\delta \qquad (11)$$

Can repeat at different k to get limits on primordial spectrum $\longrightarrow$ limits on inflationary theories

Rare objects: ultracompact minihalos (UCMHs)

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A DM halo that collapses shortly after matter-radiation equality
$\Longrightarrow$ from a large amplitude density perturbation

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'Shortly' means z_{collapse} is $O(100)$ or more

- $\Longrightarrow$ isolated collapse
- $\Longrightarrow$ formation by radial infall
- $\Longrightarrow$ very steep density profile $\rightarrow \rho \propto r^{-9/4}$
- $\Longrightarrow$ excellent indirect detection targets

PS & Sivertsson *Phys. Rev. Lett.* 2009
Lacki & Beacom *ApJL* 2010

Also good lensing prospects Ricotti & Gould *ApJ* 2009; Li et al *Phys. Rev. D* 2012

Rare objects: UCMH formation

Conditions for formation

- Seeded well before matter-radiation equality
- Requires $\delta \gtrsim \mathcal{O}(10^{-3})$
  (compare with normal inflationary perturbations $\delta \sim 10^{-5}$)
- $\longrightarrow$ much more likely than PBH formation ($\delta \gtrsim 0.3$)

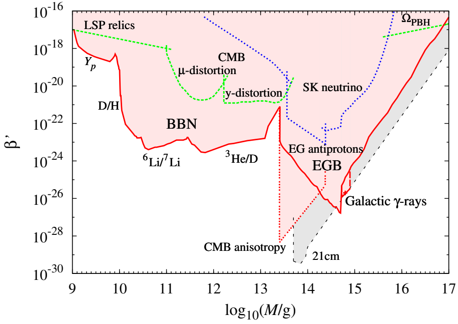
Usefulness

- Like PBHs, UCMH mass set by horizon scale at time of horizon entry
- $\implies$ specific UCMH mass $\equiv$ specific cosmological scale
- $\implies$ limit on abundance of specific mass halo $\equiv$ limit on power on specific scale k

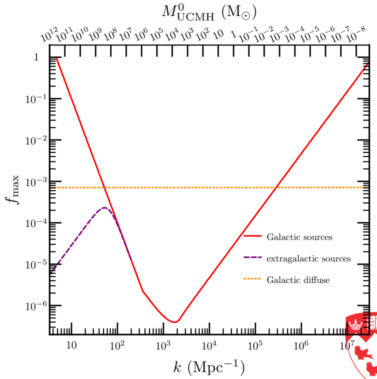
Rare objects: observational limits

PBHs: energetic particles from evaporation, lensing, binary disruption

UCMHs: energetic particles from DM annihilation, lensing



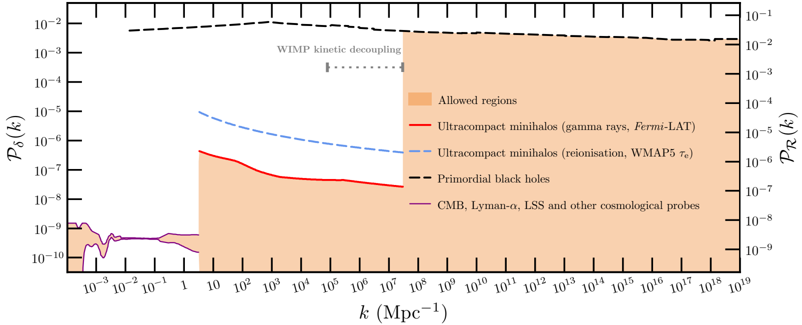
Josan et al, *Phys. Rev. D* 2009
Carr et al, *Phys. Rev. D* 2010



Bringmann, PS & Akrami, *Phys. Rev. D* 2012

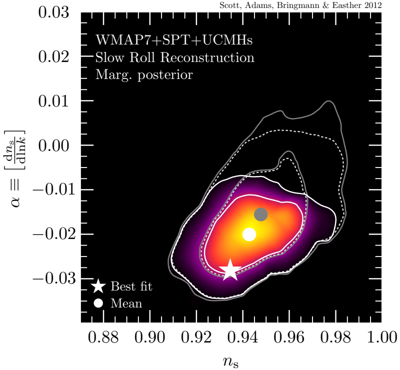
Rare objects: comparative limits on power spectrum

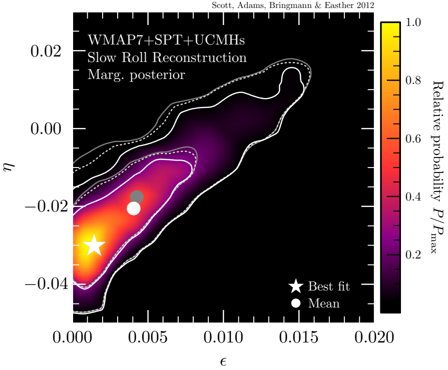
Limits on $\mathcal{P}_\delta$ from UCMHs $\sim$5 orders better than from PBHs
$\Longrightarrow$ strong limits on inflationary models



Bringmann, PS & Akrami, *Phys. Rev. D* 2012

Implications for inflation – slow-roll reconstruction





Impacts on slow-roll reconstruction
grey: original dashed: $z_c = 200$ colours: $z_c = 50$
(but beware extrapolation of α from WMAP scales)

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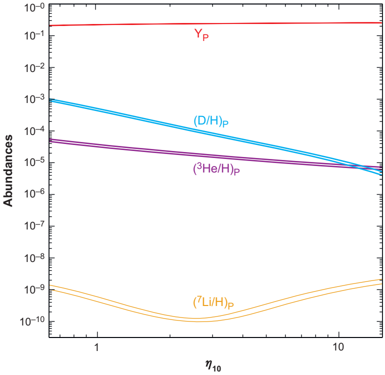
Big Bang Nucleosynthesis (BBN)



Iocco et al (*Phys. Repts.* 2009)

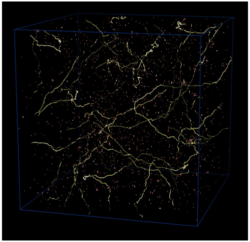
- Light elements (H, He, Li, B, Be) form when Universe cools to $T \sim$ a few MeV
- Relative amounts are sensitive to baryon-to-photon ratio η

  $\implies$ sensitive to Ω_b
- Can be messed up by additional energy injection from e.g.
  - Late-decaying or annihilating particles
  - Evaporation of PBHs

$\implies$ stringent bounds on new physics

Cosmic strings

- Nothing (necessarily) to do with string theory
- 1D topological defect caused by field transition to different vacua in causally disconnected regions
- Breaking of *any* $U(1)$ symmetry in the early Universe should produce cosmic strings
- Searches for presence of strings can constrain particle theories up the the GUT scale
- Crucial quantity is string tension $G\mu \propto$ symmetry-breaking scale^2
- Observational limits from
  - CMB position-space maps
  - 21 cm maps
  - pulsar timing
  - UCMH searches



Phase transitions and reheating

- As universe cools, vacuum goes through various phase transitions as symmetries break
  - Electroweak phase transition $\mathcal{O}(200\,\mathrm{GeV})$
  - QCD/chiral symmetry breaking $\mathcal{O}(200\,\mathrm{MeV})$
  - Breaking of symmetries associated with new physics
- Phase transitions may or may not produce:
  - defects like cosmic strings (depends on groups involved)
  - strong density perturbations at a particular k (depends on order of transition)
  - impacts on concurrent processes like kinetic decoupling of dark matter (more tomorrow)
- Drastic changes in field content have similar character
  - Reheating: mass in inflaton field at end of inflation converted into other particles, heating Universe
  - Particle genesis (baryo/lepto), creating matter asymmetry

Take-home points

- Cosmological observables are sensitive to
  - Initial distribution of density perturbations
  - Content of the Universe

$\Longrightarrow$ they can be used to test
  - Theories for inflation
  - Theories for new symmetries + particles beyond the Standard Model

- Inflation has few other observables to correlate this with
- …but many concrete Beyond Standard Model theories lead to correlated signals elsewhere $\rightarrow$ next 2 lectures