UNIVERSITY OF WESTMINSTER
SCHOOL OF COMPUTER SCIENCE & ENGINEERING

Module Code and title: 5BUIS002W/5BUIS006C Business Analytics

Exam period: May 2020

Main Exam - Solutions

Question 1 [25 Marks]

a. Formulate a linear programming model that can be used to determine how many Explorer and City Bags the company should produce per week in order to maximise profit. Precisely define each variable used and explain the purpose of each constraint. **[Total: 14 Marks]**

Decision variables:

x_1 = *number of explorer bags to produce per week*

x_2 = *number of city bags to produce per week*

The complete linear programming model for this problem can be summarised as follows:

Max $Z = 32x_1 + 24x_2$
Subject to

$2x_1 + 1x_2 \leq 1000$	(square-meter of the material used each week)
$x_1 \leq 500$	(max number of explorer bags sold per week)
$x_2 \leq 600$	(max number of city bags sold per week)
$45x_1 + 40x_2 \leq 24000$	(labour capacity per week)
$x_1 \geq 0,\ x_2 \geq 0$	(non-negativity constraints)

b. Write the LP problem in a standard form. **[Total: 6 Marks]**

Max $32x_1 + 24x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$
s.t.

$2x_1 + 1x_2 + s_1 = 1000$

$x_1 + s_2 = 500$

$x_2 + s_3 = 600$

$45x_1 + 40x_2 + s_4 = 24000$

$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$ where s_1, s_2, s_3, s_4 are slack variables

c. Suppose the managers specify that at least 10% of the total production must be Explorer Bags. How should the problem be formulated? **[Total: 5 Marks]**

Max $Z = 32\,x_1 + 24\,x_2$
Subject to

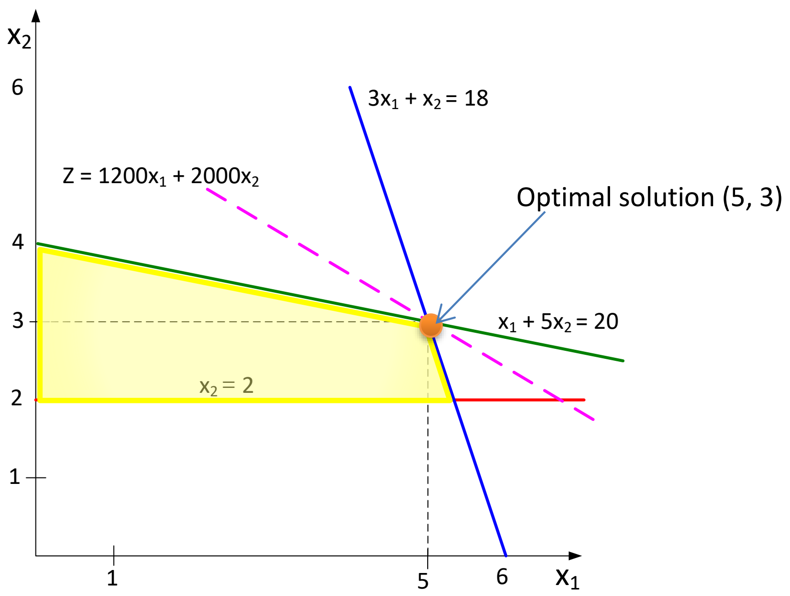
$2\,x_1 + 1\,x_2 \leq 1000$	(square-meter of the material used each week)
$x_1 \leq 500$	(max number of explorer bags sold per week)
$x_2 \leq 600$	(max number of city bags sold per week)
$45\,x_1 + 40\,x_2 \leq 24000$	(labour capacity per week)
$0.9\,x_1 - 0.1\,x_2 \geq 0$	Explorer bag production constraint
$x_1 \geq 0,\ x_2 \geq 0$	(non-negativity constraints)

Question 2 [25 Marks]

a. Using the graph provided, find the optimal solution: **[Total: 6 Marks]**

   i. Determine the optimal solution. How many city 1 and city 2 restaurants should the company open?

   ii. What is the projected total net weekly profit?



Optimal solution: $X_1=5$ (city 1), $X_2=3$ (city 2)

Z= 1200(5) + 2000(3) = 12000(£/week)

b. Identify the binding constraints. **[Total: 4 Marks]**

Two binding constraints : $X_1 + 5X_2 \leq 20$ and $3X_1 + X_2 \leq 18$

c. Is there any slack or surplus variable? If so, what is its value and how do you interpret it? If not, why? **[Total: 7 Marks]**

Constraint ($x_1 + 5x_2 \leq 20$) is binding → slack value = 0.

Constraints ($3x_1 + x_2 \leq 18$) is binding → slack value = 0.

Constraint ($x_2 \geq 2$) → surplus variable = 1 → the company should open 3 city 2 restaurants which is greater than the minimum requested by the company (which is 2) → the company should open 1 restaurant more than what they were wishing for.

d. If the objective function is changed to **Max $3000x_1 + 1000x_2$**, will the optimal solution change? **[Total: 8 Marks]**
   i. Determine the new optimal solution. **[6 Marks]**
   ii. Determine the value of the objective function. **[2 Marks]**

The optimal solution will change. The new optimal solution is an alternate optimal solution. The line segment between extreme point (5, 3) and extreme point (16/3, 2).

The new value of the objective function: $Z = 3000x_1 + 1000x_2 = 3000*5 + 1000*3 = 18000$

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Question 3 [25 Marks]

a. *What is the optimal solution?* **[Total: 4 Marks]**

X = 11.2 and Y = 2

Evidence: Table from Answer report or Sensitivity report:

From Answer report:

Variable Cells

Cell	Name	Original Value	Final Value	Integer
B11	X	0	11.2	Contin
C11	Y	0	2	Contin

From Sensitivity report:

Variable Cells

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
X	11.2	0	10	1E+30	5.714285714
Y	2	0	6	8	1E+30

b. *Identify the range of feasibility for the right-hand side values.* **[Total: 10 Marks]**

$39 \leq RHS_1 \leq 76.5$

$32.4 \leq RHS_2$

$RHS_3 \leq 11.2$

$0 \leq RHS_4 \leq 3.18$

Evidence: Sensitivity report:

Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Constraint #1	70	2	70	6.5	31
Constraint #2	32.4	0	35	1E+30	2.6
Constraint #3	11.2	0	5	6.2	1E+30
Constraint #4	2	-8	2	1.181818182	2

c. *Suppose that the right-hand side value of constraint 1 increases to 73 and the right-hand side value of constraint 4 decreases to 1. Would the dual price change? Use the 100 per cent rule and discuss.* **[Total: 11 Marks]**

Inputs and Results:

Constraint	RHS value	Change	Allowable increase/decrease	Percentage change
1	70	Increase 3	6.5	(3/6.5)*100=46.15%
4	2	Decrease 1	2	(1/2)*100=50%

Constraint 1: The 3 additional units are 46.15% of allowable increase in the RHS value of constraint 1
Constraint 4: The 1 less unit is 50% of allowable decrease in the RHS value of constraint 4

The sum of the percentage changes:
46.15% + 50% = 96.15% does not exceed 100%.

Therefore, the dual prices are applicable

Evidence: Sensitivity report:

Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Constraint #1	70	2	70	6.5	31
Constraint #2	32.4	0	35	1E+30	2.6
Constraint #3	11.2	0	5	6.2	1E+30
Constraint #4	2	-8	2	1.181818182	2

Question 4 [25 Marks]

a. If the inventor knows nothing about the probabilities of the three states of nature (three levels of sales), what is the recommended decision using the optimistic approach and the conservative approach? **[Total: 14 Marks]**

- Optimistic approach using maximax criterion (maximisation problem): select "Manufacture". The student should display how s/he worked it out following these steps:
    - Selecting criterion.
    - Use of selected criterion: determining maximum payoff + maximum of maximum payoff
    - Select decision alternative

Optimistic approach					
Decision	**State of Nature**			**Maximum**	**Recommended**
Alternative	**H**	**M**	**L**	**Payoff**	**Decision**
Manufacture	80	40	-20	80	Manufacturer
Royalties	50	30	10	50	
Sell	20	20	20	20	
	Best Payoff			80	

- Conservative approach using maximin criterion (maximisation problem): select "Sell". The student should display how s/he worked it out following these steps:
    - Selecting criterion
    - Use of selected criterion: determining minimum payoff + maximum of minimum payoff
    - Select decision alternative

Conservative approach					
Decision	**State of Nature**			**Minimum**	**Recommended**
Alternative	**H**	**M**	**L**	**Payoff**	**Decision**
Manufacture	80	40	-20	-20	
Royalties	50	30	10	10	
Sell	20	20	20	20	Sell
	Best Payoff			20	

b. Suppose the probability of high sales is 0.2, the probability of medium sales is 0.5, and the probability of low sales is 0.3. What decision should the inventor make using the expected value approach? **[Total: 11 Marks]**

- Calculating Expected values:
    - EV(Manufacture) = (0.2*80) + (0.5*40) + (0.3*(-20)) = 30
    - EV(Royalties) = (0.2*50) + (0.5*30) + (0.3*(10)) = 28
    - EV(Sell) = (0.2*20) + (0.5*20) + (0.3*(20)) = 20

- Optimal decision is “Manufacture” as it has the largest expected value.