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SkewHyperbolic: A Package for the Skew Hyperbolic Student-t Distribution

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Abstract

This paper presents the R package **SkewHyperbolic**, which includes functions relating to the skew hyperbolic Student-t distribution. The package has been available on the Comprehensive R Archive Network since November 2009. **SkewHyperbolic** includes functions to calculate the density, distribution, quantiles and random numbers from the distribution. There are functions relating to the moments and mode of the distribution, including a recursive method to calculate the moments of any order. Also included are functions that allow the user to fit the distribution to a data set and assess the goodness of fit.

Keywords: distribution, skew, hyperbolic, package, R.

1. Introduction

The R package **SkewHyperbolic** provides functions for the skew hyperbolic Student-t distribution (SHt). The SHt is a univariate, unimodal distribution that is analytically tractable and flexible. It gets its name by being both a limiting case of the generalized hyperbolic distribution, and a generalization of the Student-t distribution.

The SHt has four parameters: μ, a location parameter; δ, a scale parameter; and β and ν which are shape parameters. It is generally peaked compared to the normal, and can model extreme skewness in either the positive or negative direction. What distinguishes it from other skew t-distributions is its unique tail behaviour. When skew, the distribution has one heavy tail with polynomial behaviour, in the direction of skewness, and one semi-heavy tail with exponential behaviour. It is the only subclass of the generalized hyperbolic distribution to have this property.

The SHt was first introduced by Barndorff-Nielsen (1977). A full derivation of the distribution can be found in Paolella (2007) where it is called the hyperbolic asymmetric Student's t.

An extensive discussion appears in Aas and Hobæk Haff (2006) who call it the generalized hyperbolic skew Student's t-distribution, and explore its application to risk estimation and other financial problems. It is also mentioned in several other articles relating to GARCH models and risk estimation, including Prause (1999), Kim and McCulloch (2007) and Zhu and Galbraith (2009).

The SHt is of particular interest because of its potential applications, notably in finance, where it is common to encounter distributions of data that are peaked, asymmetrical, or have different behaviour in each tail. These are ideal situations in which the unique properties of the SHt can be exploited, and Aas and Hobæk Haff (2006) make a compelling argument for its use. There has already been a great deal of research done on both the generalized hyperbolic and other skew t-distributions such as the skew-t introduced by Azzalini and Capitanio (2003), and their application to many different situations, financial and otherwise. There is no doubt that the SHt is a promising distribution in many fields, and is worthy of further investigation and research.

The R package **SkewHyperbolic** was designed to facilitate the use of the SHt. Included are the density function, distribution function, quantile (inverse distribution) function and random number generation. In addition to this, there are methods to calculate the mode and first four moments of the distribution directly from formulae, as well as a recursive method for calculating the moments of any order and around any point. There are also new functions written to fit the SHt to data, and to assess its fit via Q-Q and P-P plots. We use the root `skewhyp` for the naming of the functions.

It is hoped that the **SkewHyperbolic** package will assist research into the distribution and its application by making the SHt distribution widely and available and easily used.

2. Package Description

2.1. Density

The density of the SHt is given by the formula

$$f_x(x) = \frac{2^{(1-\nu)/2}\delta^{\nu}|\beta|^{(\nu+1)/2}K_{(\nu+1)/2}\left(\sqrt{\beta^2\left(\delta^2+(x-\mu)^2\right)}\right)\exp\left[\beta(x-\mu)\right]}{\Gamma(\nu/2)\sqrt{\pi}\left[\sqrt{\delta^2+(x-\mu)^2}\right]^{(\nu+1)/2}}, \quad \beta \neq 0 \qquad (1)$$

and

$$f_x(x) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi}\delta\Gamma(\nu/2)}\left[1+\frac{(x-\mu)^2}{\delta^2}\right]^{-(\nu+1)/2}, \qquad \beta = 0 \qquad (2)$$

where $\beta \neq 0$ gives us the skew version, and $\beta = 0$ the symmetric. The density in Equation 2 when $\beta = 0$ can be recognized as a non-central Student-t distribution with ν degrees of freedom, expectation μ and variance $\delta^2/(\nu-2)$ (provided $\nu > 2$). Hence we can say that the SHt is a generalization of the Student-t distribution, which can be skewed.

The function that returns the value of the density is `dskewhyp`. As with all of the functions created, the parameters of the SHt can be given either individually or as a vector called `param`.

The supplied parameters are checked for their validity using `skewhypCheckPars`, which first checks that the `param` vector has four elements, and if so, it checks that the values given for δ and ν are greater than zero. If any of these conditions are violated the function will return a warning.

The function `dskewhyp` first calculates the log density, and then back transforms this by taking the exponential if necessary. The advantage of this is that numerical difficulties encountered in calculating the density when it is not logged are avoided. Looking at Equations 1 and 2 we can see that the powers and exponential parts of the density can easily become very large. There may also be numerical issues with the Bessel function, which can be largely avoided by calculating the exponentially-scaled version of the Bessel function and then adjusting for the scaling by adding another term to the density. The exponentially-scaled version of the Bessel function $K_\nu(x)$ returns the value $K_\nu(x)\exp(x)$, the exponential part of which can be canceled out by adding another term to the density.

2.2. Distribution and Quantiles

The cumulative distribution function and its inverse, the quantile function, are returned by `pskewhyp` and `qskewhyp` respectively. Numerical methods are used because neither function is available in closed form. Increased accuracy in determining the cumulative distribution function is obtained by breaking the real line into a number of regions and integrating over the regions separately.

In `pskewhyp` where we have task of integrating the density, the region to be integrated over is divided into eight smaller regions, which are integrated over in turn, as opposed to the more direct technique of simply calling `integrate` to evaluate the integral between some lower bound and the desired quantile.

The function `skewhypBreaks` is used to calculate suitable break points for the distribution. See Section 2.4 for an explanation of how this is done.

The function `qskewhyp` uses the same breakup of the real line as `pskewhyp`. Within each region the distribution function value is calculated at a number of points and a cubic spline interpolation is used to join them into a continuous approximation of the quantile function. The inverse is then found using `uniroot`.

2.3. Random Numbers

The function `rskewhyp` generates random variates from the SHt. We use of the following result from Aas and Hobæk Haff (2006). The generalized hyperbolic (GH) distribution can be represented as a normal variance-mean mixture with the generalized inverse Gaussian (GIG) distribution as a mixing distribution.

The GIG distribution has the density:

$$f(z;\lambda,\delta,\gamma) = \left(\frac{\gamma}{\delta}\right)^{\lambda} \frac{z^{\lambda-1}}{2K_\lambda(\gamma\delta)} \exp\left\{-\frac{1}{2}(\delta^2 z^{-1} + \gamma^2 z)\right\} \tag{3}$$

We note that this means $X \sim \mathrm{GH}(\lambda,\alpha,\beta,\delta,\mu)$ can be represented as

$$X = \mu + \beta Z + \sqrt{Z}Y \tag{4}$$

where

$$Y \sim \mathrm{N}(0,1) \quad \text{independently of} \quad Z \sim \mathrm{GIG}(\lambda,\delta,\gamma)$$

with $\gamma = \sqrt{\alpha^2 - \beta^2}$.

The SHt is the limiting case of the GH where $\alpha \to |\beta|$. Then the GIG distribution in the mixing formula (4) becomes an inverse gamma distribution with shape parameter $\nu/2$ and scale parameter $\delta^2/2$. In turn if X has a gamma distribution with shape parameter α and scale parameter β then $1/X$ has an inverse gamma distribution with shape parameter α again, and scale parameter $1/\beta$, by the usual method of transformations of random variables. Thus we can simulate observations from the SHt distribution by simulating Z as the inverse of a gamma distribution and Y as standard normal then using the mixing representation (4). Simulation of the SHt distribution is thus achieved using calls to the base R functions `rgamma` and `rnorm`.

2.4. Break Points

The function `skewhypBreaks` divides the support of the distribution, for given parameter values, into eight regions by defining seven break points, as illustrated in Figure 1

The very smallest (`xTiny`) and the very largest (`xHuge`) breakpoints are where the value of the density is `tiny` as defined by the user and set by default to 10^{-10}. In order to calculate these position of these points the function `skewhypCalcRange` is utilised, lowering the density curve by the value specified, in this case 10^{-10}, to find where the density curve now intercepts the x axis.

The next innermost break points, `xSmall` and `xLarge` are also calculated similarly by the function `skewhypCalcRange`, by using the value `small` with default value 10^{-6}.

The next two break points, `lowBreak` and `highBreak` are calculated to be the values to the left and right of the mode where the derivative at that point is `deriv` times the maximum derivative on that side of the mode. The derivative of the density is returned by the function `ddskewhyp`.

The final breakpoint is `modeDist`, the mode of the distribution, calculated by `skewhypMode`.

2.5. Moments and Mode

Functions are provided to calculate the mean, variance, skewness, kurtosis and mode of the distribution for a given set of parameters.

The function `skewhypMode` returns the mode of the distribution by numerically optimizing the density function `dskewhyp` using the optimizer `optim`.

The mean, variance, skewness (s) and kurtosis (k) on the other hand can be calculated directly from formulae, which are derived in Aas and Hobæk Haff (2006), and are as follows:

$$E[X] = \mu + \frac{\delta^2\beta}{\nu - 2} \tag{5}$$

$$\mathrm{Var}[X] = \frac{2\beta^2\delta^4}{(\nu-2)^2(\nu-4)} + \frac{\delta^2}{\nu-2} \tag{6}$$

$$s = \frac{2(\nu-4)^{1/2}\beta\delta}{[2\beta^2\delta^2 + (\nu-2)(\nu-4)]^{3/2}}\left[3(\nu-2) + \frac{8\beta^2\delta^2}{\nu-6}\right] \tag{7}$$

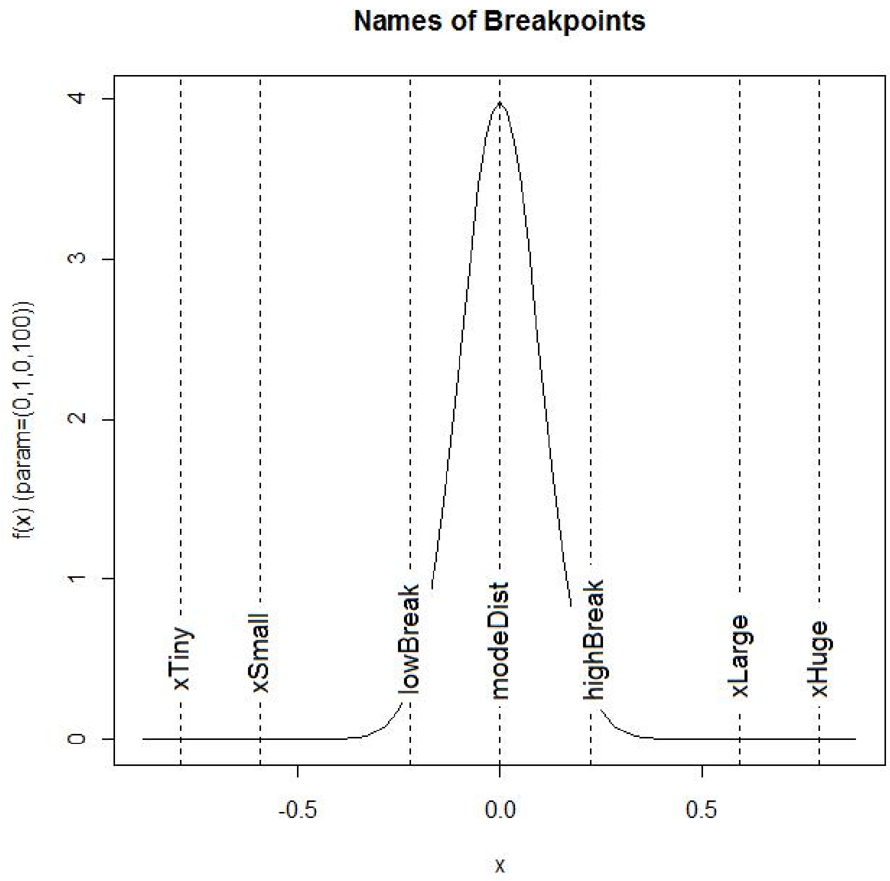

Figure 1: Names of the Breakpoints

$$k = \frac{6}{[2\beta^2\delta^2 + (\nu-2)(\nu-4)]^2}\left[(\nu-2)^2(\nu-4) + \frac{16\beta^4\delta^2(\nu-2)(\nu-4)}{\nu-6} + \frac{8\beta^4\delta^4(5\nu-22)}{(\nu-6)(\nu-8)}\right] \tag{8}$$

Note that while the mean always exists (as long as $\nu > 2$); the variance only exists when $\nu > 4$ when $\beta \neq 0$, and $\nu > 2$ when $\beta = 0$ (similarly to the Student-t); the skewness when $\nu > 6$; and the kurtosis when $\nu > 8$.

As an alternative to using the above formulae, the moments of any order (provided they exist) can be calculated using a recursive method implemented in `skewhypMom`. The theory can be found in Scott, Würtz, Dong, and Tran (2009), which outlines a method for calculating the moments of the generalized hyperbolic and several of its subclasses including the skew hyperbolic t-distribution. We find the following formula for the moments of the SHt around μ:

$$\overline{M}_k = \sum_{\ell=\lfloor (k+1)/2 \rfloor}^{k} a_{k,\ell}\beta^{2\ell-k}\left[\frac{\delta^{2\ell}\Gamma(\nu/2-\ell)}{\Gamma(\nu/2)2^{\ell}}\right] \tag{9}$$

In the above equation k is the order of the moment required, $a_{k,\ell}$ is the recursive coefficient, which can be calculated by the function `momRecursion`, and the other part of the equation marked [.] are the moments of the inverse gamma distribution which are provided by `gammaRawMom`. Both functions are implemented in the `HyperbolicDist` package.

To transform these moments to the central moments, moments about zero, or any other location we can use the moment transformation formulae from Kendall and Stuart (1969), which are found in the `HyperbolicDist` package as `momChangeAbout`.

2.6. Fitting

The fitting function, `skewhypFit`, first generates initial estimates of the parameter values for the SHt, if they are not already supplied by the user, then optimizes these to find the best fit.

Starting Values

To find starting values, the log-linear form of the tail density is utilised to calculate initial estimates of two of the parameters. The remaining two parameters are then calculated using a method of moments type approach. This is done in the new R function `skewhypFitStart` that is called by the fitting function `skewhypFit`.

The equations relating to the tail density of the SHt can be found in Paolella (2007). In the asymmetric case we find that after taking the log of the equation, the heavier tail has the form of a linear equation with one predictor:

$$\log f_x(x) \sim \text{const.} + (-\nu/2 - 1)\log(|x|) \quad \text{when} \begin{cases} \beta < 0 & \text{and} \quad x \to +\infty, \\ \beta > 0 & \text{and} \quad x \to -\infty. \end{cases} \tag{10}$$

and in the lighter tail it has the form of a linear equation with two predictors:

$$\log f_x(x) \sim \text{const.} + (-\nu/2 - 1)\log(|x|) - 2|\beta||x| \quad \text{when} \begin{cases} \beta < 0 & \text{and} \quad x \to +\infty, \\ \beta > 0 & \text{and} \quad x \to -\infty. \end{cases} \tag{11}$$

In the symmetric case we find that the density in both tails has the form of a linear equation with one predictor:

$$\log f_x(x) \sim \text{const.} + (-\nu/2 - 1)\log(|x|) \quad \text{as } x \to \pm\infty \tag{12}$$

Using a kernel density we can approximate the log of the density in the tails of our data. We use the R function `density`. We can then fit a linear model to this estimated log density using the R function `lm`, specifying the predictors that are appropriate in each case. We then equate the resulting coefficients with their theoretical form, and solve to find estimates for β and ν.

The final two parameters δ and μ are found by equating the sample mean and sample variance of the data to the first two moment equations, found in Aas and Hobæk Haff (2006). Note that for the mean and variance to exist we require that ν be greater than 4. This is advantageous compared to using solely the method of moments, as we would require the skewness and kurtosis also, and for these to exist we would require that ν be greater than 8. Avoiding using the skewness and kurtosis also improves the reliability of the resulting parameter estimates, as the sample skewness and kurtosis statistics are not robust.

To reduce the effect of individual observations on the linear models and hence on the parameter estimates, the function `hist` is called to divide the data into regions. The midpoints of these regions, along with the associated kernel density estimates at these points, are used to fit the linear models.

The heavy and light tails are identified by the sample skewness statistic. If it is greater than 0.1 in absolute value the distribution is considered skewed, and if the statistic is positive the heavy tail is the upper tail, if the statistic is negative the heavy tail is the lower tail.

It is also important to define the tails, as the density is only log linear as $x \to \infty$, however we do not want to lose too much data by defining the tails as too extreme. After some trial and error the boundaries of the tails were taken to be the 0.1 and 0.9 quantiles of the original data. This works well in most cases.

We can also rely on the fact that the initial estimates of the parameter values generated using this method are then passed to the optimizer for finer tuning.

Likelihood Maximization

The initial parameter estimates generated by `skewhypFitStart` are passed to `skewhypFit`, which optimizes these values to find the fit that maximises the likelihood function.

The likelihood is calculated by taking the sum of the log density evaluated at each of the original data points. This is continually re-evaluated as the parameters vary, starting from the initial conditions, until the log-likelihood is maximized.

The user has three different optimizers at their disposal. The first two are both options of the `optim` function: Nelder-Mead and BFGS. The third is the function `nlm`. Nelder-Mead is set as the default since it is considered quite robust. The user may find however that `nlm` works better in some cases. Additional arguments for each of the three methods can be passed into the function and this allows maximal control for the user of the optimization process.

The vector of parameters is optimized in the form $(\mu, \log(\delta), \beta, \log(\nu))$ as opposed to the usual $(\mu, \delta, \beta, \nu)$ in order to avoid having to specify boundary conditions and carrying out a constrained optimization, where we are more likely to have convergence problems.

2.7. Goodness of Fit

Once the fitting routine has been performed the user may inspect the goodness of fit from the output of `skewhypFit`. Four different diagnostic plots are currently implemented: a histogram of the data with the fitted density curve overlaid; a log-histogram, produced by the `logHist` function in the `HyperbolicDist` package with the log-density overlaid; a Q-Q plot; and a P-P plot. See Section 4.3 for examples of these.

The Q-Q plot is generated by the function `qqskewhyp`, which can also be used on its own. It produces the Q-Q plot by comparing the empirical quantiles of the observed data with the theoretical quantiles (calculated by using `qskewhyp`). Similarly the P-P plot is generated by the function `ppskewhyp`, and it too can be called independently of `skewhypFit`. It compares the empirical cdf with the theoretical cdf of the distribution.

3. Testing

In order to ensure that the functions are working as expected, and that they are reliable for a wide range of parameter values a variety of testing procedures were employed.

3.1. Testing Density and Random Numbers

To test the random number generation function and the density function, the approach used was to go through large numbers of different combinations of parameter values, and compare the histogram of random numbers with the curve of the density function.

The result of this being that numerical problems in calculating the density were identified and resolved. It was found that the distribution of the random numbers matches up well with the density, apart from some extreme cases.

3.2. Testing the Distribution and Inverse Distribution Functions

Testing was carried out on the distribution function `pskewhyp` and the inverse distribution function `qskewhyp` to assess their accuracy. This is of particular importance as the functions in question both use numerical integration techniques to approximate calculations, as discussed in Section 2.2.

To assess the accuracy of the functions, first `qskewhyp` was used to calculate the quantiles corresponding to selected points between zero and one. The function `pskewhyp` was then used to reverse the calculation. If both functions did a perfect job we would expect to get the same points back as those we started with, so the difference between the two can be used as a proxy for the cumulative error of the functions.

The resulting errors were compared to those generated from simple functions that did not use the broken integration techniques employed by `pskewhyp` and `qskewhyp`.

This procedure was repeated for several parameter sets.

It was found that the magnitude of the errors much less when using `pskewhyp` and `qskewhyp`. In addition, the errors in the simple functions were seen to follow a systematic pattern, which was not the case when the broken integration approach was used.

3.3. Testing the Fitting Functions

The fitting function `skewhypFit`, and related functions including the method for finding starting values, were also tested. These functions are detailed in Section 4.3.

A straightforward approach is to generate data from a known SHt distribution, and then to use `skewhypFit` and the linear approximation method to fit a SHt distribution to the generated data. We can then compare the fitted values to those used to generate the data.

It was found that the fits were very good, with the fitted densities being very close in shape to the generating densities. Any variation can be explained by sampling variability. This was corroborated by conducting Kolmogorov-Smirnov tests, which gave no evidence against the data having come from the fitted distribution.

Sometimes the fitted parameters do seem dissimilar to the parameters used for the generation of the data. This can be accounted for by the “flatness” of the GH likelihood function, discussed by Prause (1999). The functions in the package do a good job of fitting the SHt to data.

4. Usage and Examples

Details of the usage, and fully executable examples for each of the functions in the **SkewHyperbolic** package are available in the package documentation. The functions in the package use the root `skewhyp`, and have the general form

```
...skewhyp...(mu = 0, delta = 1, beta = 1, nu = 1,
              param = c(mu,delta,beta,nu), specific arguments , ...)
```

Note that the parameters of the distribution are set by default to (0,1,1,1) if not otherwise specified. If the user wishes to specify a different set of parameters they may submit them individually in the function call, or as a vector called `param`. When the user does specify their own parameters, they are checked by the function `skewhypCheckPars` to ensure that there are the correct number, and that the values are valid. If they are not then an error message is returned outlining the problem.

4.1. Usage of d, p, q and rskewhyp

The functions `dskewhyp`, `pskewhyp`, `qskewhyp` and `rskewhyp` return the value of the density function, the distribution function, the quantile function, and random variates respectively. These functions are discussed in Section 2. Related to these are the functions `ddskewhyp` which returns the value of the derivative of the density, and `skewhypBreaks` which returns the breakpoints that are used for integrations in `pskewhyp` and `qskewhyp`. This is explained in Section 2.4. These functions will in most cases not need to be called directly by the user.

The usage of `dskewhyp`, `pskewhyp`, and `qskewhyp` is similar. As the first argument a quantile (or percentile in the case of `qskewhyp`) or vector of quantiles must be supplied, followed by the parameters of the distribution, individually or as a vector. There are further optional arguments that allow the user to specify particulars of each function such as tolerances.

The usage of the functions is as follows, each argument is fully explained in the package documentation:

```
dskewhyp(x, mu = 0, delta = 1, beta = 1, nu = 1,
         param = c(mu, delta, beta, nu), log = FALSE,
         tolerance = .Machine$double.eps^0.5)
pskewhyp(q, mu = 0, delta = 1, beta = 1, nu = 1,
         param = c(mu,delta,beta,nu), log = FALSE, lower.tail = TRUE,
         small = 10^(-6), tiny = 10^(-10), subdivisions = 100,
         accuracy = FALSE, ...)
qskewhyp(p, mu = 0, delta = 1, beta = 1, nu = 1,
         param = c(mu,delta,beta,nu), small = 10^(-6), tiny = 10^(-10),
         deriv = 0.3, nInterpol = 100, subdivisions = 100, ...)
rskewhyp(n, mu = 0, delta = 1, beta = 1, nu = 1,
         param = c(mu,delta,beta,nu), log = FALSE)
```

For example, to compare the density and random numbers for a set of parameters one may use the code:

```
param <- c(0, 1, 40, 10)
range <- skewhypCalcRange(param = param, tol = 10^(-2))

data <- rskewhyp(1000, param = param)
curve(dskewhyp(x, param = param), range(data)[1], range(data)[2],
n = 1000, col = 2)
hist(data, freq = FALSE, add = TRUE)
```

Or to see the distribution function one may use the code:

```
curve(pskewhyp(x, param = param, small = 10^(-2), tiny = 10^(-4)),
range[1], range[2], n = 500)
```

Although it is sufficient just to use:

```
curve(pskewhyp(x, param = param), range[1], range[2])
```

4.2. Usage of functions relating to moments

There are several functions available that relate to the moments of the distribution, as discussed in Section 2.5.

To calculate the mean, variance, skewness and kurtosis directly from formulae, only the parameters need to be supplied. For example, to calculate the mean:

```
param <- c(10, 1, 5, 9)
skewhypMean(param = param)
```

And similarly for `skewhypVar` for the variance, `skewhypSkew` for the skewness, `skewhypKurt` for the kurtosis. The mode, `skewhypMode`, is calculated numerically so there is the option of also specifying `tolerance`.

There is also a recursive method to calculate the moments of any order and around any point. The usage of this function is as follows:

```
skewhypMom(order, mu = 0, delta = 1, beta = 1, nu = 1,
           param = c(mu, delta, beta, nu), momType = "raw", about = 0)
```

The types of moments accepted by `momType` are `"raw"`,`"mu"` for moments about μ, or `"central"`. If another location is required, a value for `about` may be specified, which overwrites `momType`.

For example:

```
param = c(1, 2, 3, 10)

### Raw moments of the skew hyperbolic t distribution
skewhypMom(3, param = param, momType = "raw")
```

```
skewhypSkew(param = param)   # same answers

### Moments about any location
skewhypMom(3, param = param, about = 5)
```

4.3. Usage of fitting functions

Fitting of the SHt to data uses the function `skewhypFit`. We apply it to two examples for which data is provided in the package **SkewHyperbolic**. The first example is the plasma ferritin concentration of athletes which is part of the Australian Institute of Sport dataset from Cook and Weisberg (1994). This data set was previously used by Azzalini as an example in the package **sn**, Azzalini (2006). For fitting the plasma ferritin data we have

```
> data(ais)
> Fe <- ais$Fe
> par(mfrow = c(2,2))
> Fefit <-skewhypFit(Fe, startValues = "US", paramStart = c(0,1,1,1))

Data:      Fe
Parameter estimates:
      mu     delta      beta        nu
-18.1905   29.4702    0.9624   10.7402
Likelihood:         -1029.897
Method:             Nelder-Mead
Convergence code:   0
Iterations:         303
```

The default is for the data to be plotted. With the parameter setting `par(mfrow = c(2,2))` we obtain the diagnostic graphs shown in Figure 2.

The data in this example is highly skewed to the right. Since the data values must be positive an alternative approach to modelling in this case might be to log the data first. The fitted model gives a very small probability of negative values occurring:

```
> pskewhyp(0, param = Fefit$param)
[1] 0.0002379412
```

Another data set provided with the package is the series of log returns of daily closing values from the Dow-Jones Index from January 4, 1999 to July 8, 2004. These log returns are close to symmetric. Fitting the SHt give the results:

```
> data(lrdji)
> par(mfrow = c(2,2))
> djfit <- skewhypFit(lrdji)

Data:      lrdji
Parameter estimates:
        mu       delta        beta          nu
```

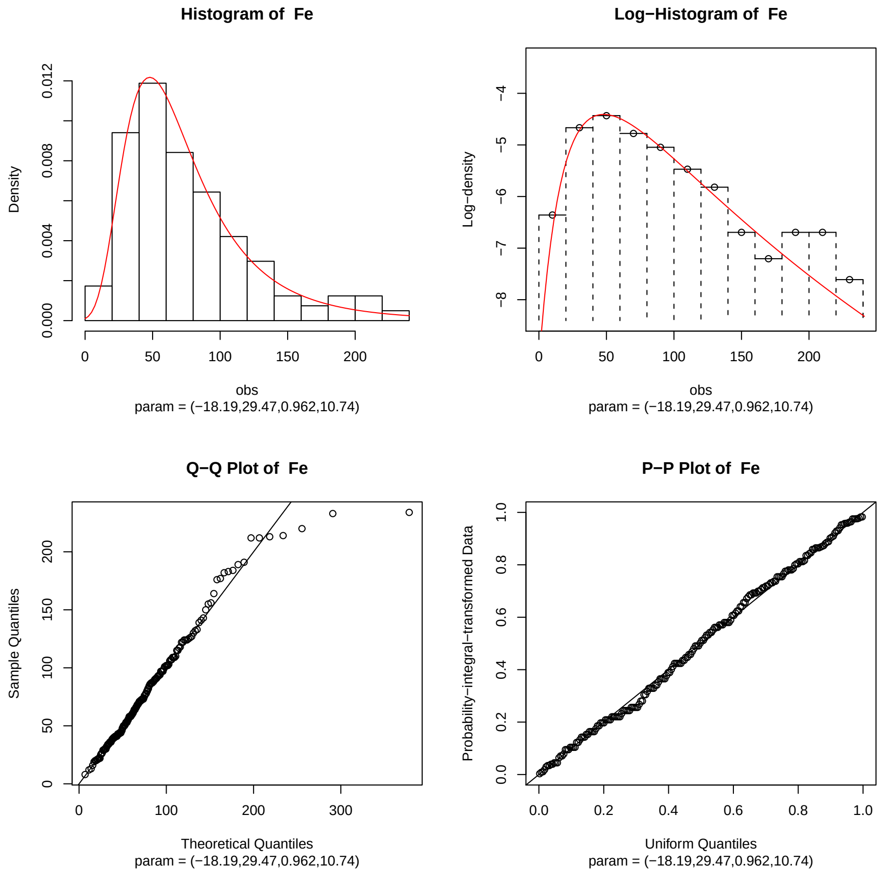

Figure 2: Diagnostic plots from fitting the SHt to the plasma ferritin concentration data from the Australian Institute of Sport data set.

```
-0.0004239   0.0282695   2.4133671   6.5334373
Likelihood:         3313.473
Method:             Nelder-Mead
Convergence code:   0
Iterations:         165
```

The diagnostic plots for the fit are shown in Figure 3.

4.4. Usage of plotting functions

The data can be examined using the log-histogram function `logHist`, and Q-Q or P-P plots using `qqskewhyp` or `ppskewhyp` respectively. Then for example, for the ferritin data of the previous section we obtain Figure 4.

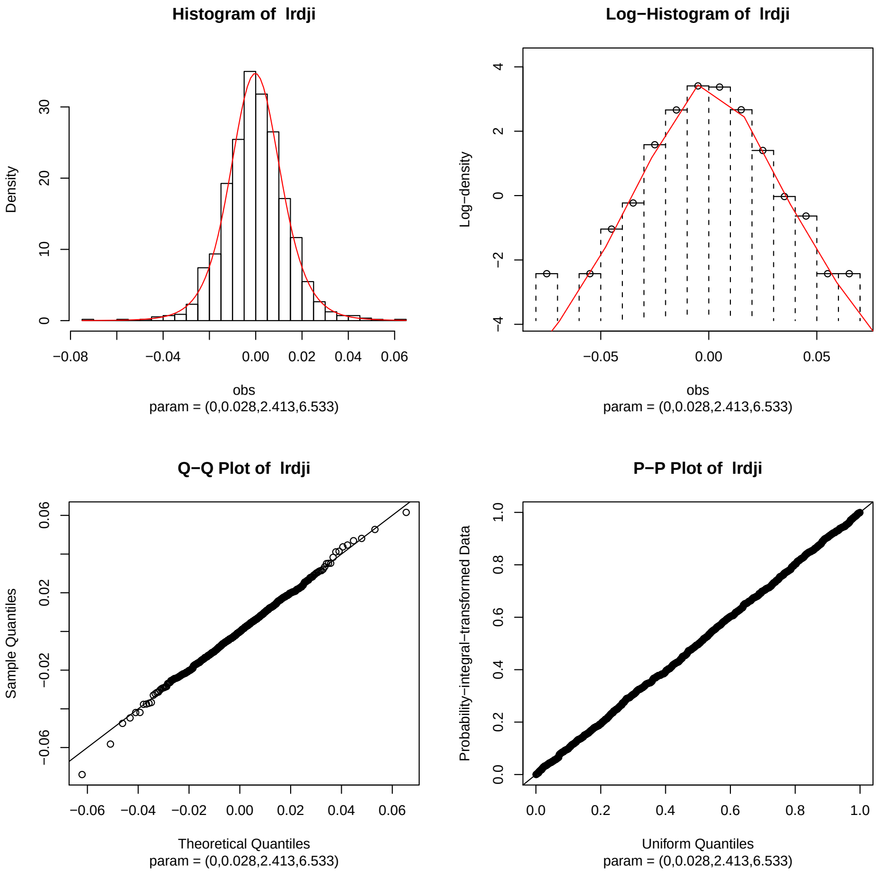

Figure 3: Diagnostic plots from fitting the SHt to log returns from the Dow-Jones Index.

5. Conclusion

This paper presented the functions included in the **SkewHyperbolic** R package. These included the density, distribution, quantiles and random numbers of the skew hyperbolic Student-t distribution, as well as functions that relate to the moments and mode of the distribution. There are also functions that allow the user to fit the distribution to a data set and assess the goodness of fit.

The package will be useful for assisting in the application of the distribution to financial problems. This is an area where the skew hyperbolic Student-t can be particularly useful due to its unique tail behaviour.

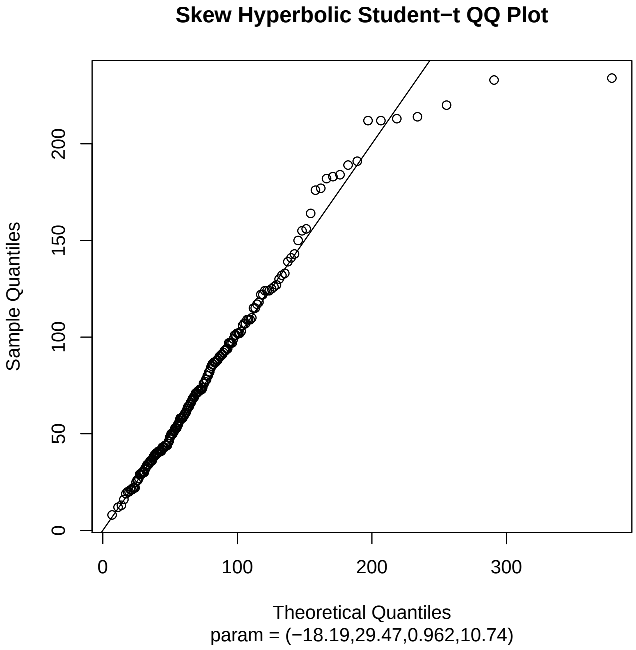

Figure 4: Q-Q plot of the plasma ferritin concentration data from the Australian Institute of Sport data set.

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