SkewedHyperbolicDistribution

Skewed Hyperbolic Student's t-Distribution

Description

Density function, distribution function, quantiles and random number generation for the Generalised Hyperbolic Skew Student's t-Distribution, with parameters β (skewness), δ (peakedness), μ (location) and ν (shape).

Usage

```
dskewhyp(x, mu = 0, delta = 1, beta = 1, nu = 1, param = c(mu, delta,
  beta, nu), log = FALSE, tolerance = .Machine$double.eps^0.5)
pskewhyp(q, mu = 0, delta = 1, beta = 1, nu = 1,param =
  c(mu,delta,beta,nu), log = FALSE, lower.tail = TRUE, small = 10^(-6),
  tiny = 10^(-10), subdivisions = 100, accuracy = FALSE, ...)
qskewhyp(p, mu = 0, delta = 1, beta = 1, nu = 1, param =
  c(mu,delta,beta,nu), small = 10^(-6), tiny = 10^(-10), deriv = 0.3,
  nInterpol = 100, subdivisions = 100, ...)
rskewhyp(n, mu = 0, delta = 1, beta = 1, nu = 1, param =
  c(mu,delta,beta,nu), log = FALSE)
```

Arguments

x,q	Vector of quantiles.
p	Vector of probabilities.
n	Number of random variates to be generated.
mu	Location parameter μ, default is 0.
delta	Peakedness parameter δ, default is 1.
beta	Skewness parameter β, default is 1.
nu	Shape parameter ν, default is 1.
param	Specifying the parameters as a vector of the form `c(mu, delta, beta, nu)`.
log	Logical; if `log = TRUE`, probabilities are given as log(p).
lower.tail	Logical; if `lower.tail = TRUE`, the cumulative density is taken from the lower tail.
tolerance	Specified level of tolerance when checking if parameter beta is equal to 0.
small	Size of a small difference between the distribution function and 0 or 1.
tiny	Size of a tiny difference between the distribution function and 0 or 1.
subdivisions	The maximum number of subdivisions used to integrate the density and determine the accuracy of the distribution function calculation.

accuracy	Logical; if accuracy = TRUE, accuracy calculated by integrate to try and determine the accuracy of the distribution function calculation.
deriv	Value between 0 and 1 which determines the point at which the value of the derivative becomes substantial compared to its maximal value, see **Details**.
nInterpol	Number of points used in qskewhyp for cubic spline interpolation of the distribution function.
...	Passes additional arguments to integrate.

Details

Users may either specify the values of the parameters individually or as a vector. If both forms are specified, then the values specified by the vector param will overwrite the other ones.

The density function is

$$f(x) = \frac{2^{(\frac{1-\nu}{2})}\delta^{\nu}|\beta|^{(\frac{\nu+1}{2})}K_{(\frac{\nu+1}{2})}\sqrt{(\beta^2(\delta^2+(x-\mu)^2))}exp(\beta(x-\mu))}{\Gamma(\frac{\nu}{2})\sqrt{(\pi)}\sqrt{(\delta^2+(x-\mu)^2)^{(\frac{\nu+1}{2})}}}$$

when $\beta \neq 0$, and

$$f(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{(\pi)}\delta\Gamma(\frac{\nu}{2})}\left(1+\frac{(x-\mu)^2}{\delta^2}\right)^{\frac{-(\nu+1)}{2}}$$

when $\beta = 0$, where $K_{nu}(.)$ is the modified Bessel function of the third kind with order nu, and Γ is the gamma function.

pskewhyp breaks the real line in to 8 regions in order to determine the integral of dhyperb. The breakpoints determining the regions are calculated by skewhypBreaks, based on the values of small, tiny, and deriv.

The inner area is divided into two regions above and two below the mode. The breakpoint that divides these is calculated to be where the derivative of the density function is deriv times the value of the maximum derivative on that side of the mode. In the extreme tails of the distribution where the probability is tiny the probability is taken to be zero. In the remaining regions the integral of the density is calculated using the numerical routine safeIntegrate (a wrapper for integrate).

qhyperb Used the same breakup of the real line as pskewhyp. For quantiles that fall in the two extreme regions the quantile is returned as Inf or -Inf as appropriate. In the remaining regions splinefun is used to fit values of the distribution function calulated by pskewhyp. The quantiles are then found by the uniroot function.

Value

dskewhyp gives the density function, pskewhyp gives the distribution function, qskewhyp gives the quantile function and rskewhyp generates random variates.

An estimate of the accuracy of the approximation to the distribution function can be found by setting accuracy=TRUE in the call to pskewyhp which returns a list with components value and error.

Author(s)

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References

Aas, K. and Haff, I. H. (2006). The generalised hyperbolic skew Student's *t*-distribution, *Journal of Financial Econometrics*, **4**, 275–309.

See Also

`safeIntegrate`, `integrate` for its shortfalls, `skewhypBreaks`

`qqskewhyp` *Skewed Hyperbolic Student's t-Distribution Quantile-Quantile and Percent-Percent Plots*

Description

`qqskewhyp` produces a skewed hyperbolic t-distribution Q-Q plot of the values in `y`, `ppskewhyp` produces a skewed hyperbolic t-distribution P-P (percent-percent) plot or probability plot of the values in `y`. Graphical parameters may be given as arguments to `qqskewhyp` and `ppskewhyp`.

Usage

```
qqskewhyp(y, mu = 0, delta = 1, beta = 1, nu = 1, param = c(mu, delta,
beta, nu), main = "Skewed Hyperbolic Student's-t QQ Plot", xlab =
"Theoretical Quantiles", ylab = "Sample Quantiles", plot.it = TRUE, line
= TRUE, ...)
ppskewhyp(y, beta = NULL, delta = NULL, mu = NULL, nu = NULL, param =
c(mu, delta, beta, nu), main = "Skewed Hyperbolic Student's-t P-P Plot",
xlab = "Uniform Quantiles", ylab =
"Probability-integral-transformed Data", plot.it = TRUE, line = TRUE, ...)
```

Arguments

`y`	The sample data.
`mu`	Location parameter μ, default is 0.
`delta`	Peakedness parameter δ, default is 1.
`beta`	Skewness parameter β, default is 1.
`nu`	Shape parameter ν, default is 1.
`param`	Specifying the parameters as a vector of the form `c(mu, delta, beta, nu)`.
`main,xlab,ylab`	Plot labels.

plot.it	Logical; if plot.it = TRUE the results will be plotted.
line	Logical; if line = TRUE a line is added through the origin with unit slope.
...	Further graphical parameters.

Details

Users may either specify the values of the parameters individually or as a vector. If both forms are specified, then the values specified by the vector param will overwrite the other ones.

Value

For qqskewhyp and ppskewhyp, a list with components:

x	The x coordinates of the points to be plotted.
y	The y coordinates of the points to be plotted.

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See Also

ppoints, qqplot, dskewhyp

SkewedHyperbolic-package

The Package 'SkewedHyperbolic': Summary Information

Description

This package provides a collection of functions for working with the skewed hyperbolic Student's t-distribution.

Functions are provided for the density function (dskewhyp), distribution function (pskewhyp), quantiles (qskewhyp) and random number generation (rskewhyp). There are also functions that fit the distribution to data (skewhypFit). The mean, variance, skewness, kurtosis and mode can be found usig the functions skewhypMean, skewhypVar, skewhypSkew, skewhypKurt and skewhypMode respectively. To assess goodness of fit, there are also functions to generate a Q-Q plot (qqskewhyp) and a P-P plot (ppskewhyp). S3 methods print, plot and summary are provided for the output of skwewhypFit.

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Type:	Package
Version:	0.1-1
Date:	2009-08-28
License:	GPL(>=2)
LazyLoad:	yes

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References

Aas, K. and Haff, I. H. (2006). The generalised hyperbolic skew Student's *t*-distribution, *Journal of Financial Econometrics*, **4**, 275–309.

See Also

`dskewhyp`, `skewhypMean`, `skewhypFit`, `skewhypFitStart`, `skewhypBreaks`, `qqskewhyp`, `HyperbolicDist`.

`skewhypBreaks` *Break points for the Skewed Hyperbolic Student's t-Distribuiton*

Description

Utility routines that calculate suitable breakpoints for use in determining the distribution function, and the derivative of the density function.

Usage

```
skewhypBreaks(mu = 0, delta = 1, beta = 1, nu = 1, param =
  c(mu, delta, beta, nu), small = 10^(-6), tiny = 10^(-10),
  deriv = 0.3, ...)
skewhypCalcRange(mu = 0, delta = 1, beta = 1, nu = 1,
  param = c(mu,delta,beta,nu), tol= 10^(-5), ...)
ddskewhyp(x, mu = 0, delta = 1, beta = 1, nu = 1, param =
  c(mu,delta,beta,nu),log = FALSE, tolerance =
  .Machine$double.eps ^ 0.5)
```

Arguments

`x`	Vector of quantiles
`mu`	Location parameter μ, default is 0.
`delta`	Peakedness parameter δ, default is 1.
`beta`	Skewness parameter β, default is 1.
`nu`	Shape parameter ν, default is 1.
`param`	Specifying the parameters as a vector of the form `c(mu, delta, beta, nu)`.
`small`	Size of a small difference between the distribution function and 0 or 1.
`tiny`	Size of a tiny difference between the distribution function and 0 or 1.
`deriv`	Value between 0 and 1 which determines the point at which the value of the derivative becomes substantial compared to its maximal value, see **Details**.

log	Logical; if log = TRUE, probabilities are given as log(p).
tolerance	Specified level of tolerance when checking if parameter beta is equal to 0 in ddskewhyp.
tol	Tolerance used in skewhypCalcRange, see **Details**.
...	Passes additional arguments to uniroot.

Value

ddskewhyp gives the derivative of dskewhyp.

skewhypBreaks returns a list with components:

xTiny	Value such that the probabilities to the left are less than tiny.
xSmall	Value such that probabilities to the left are less than small.
lowBreak	Point to the left of the mode such that the derivative of the density at that point is deriv times its maximum value on that side of the mode.
highBreak	Point to the right of the mode such that the derivative of the density at that point is deriv times the value of the maximum value on that side of the mode.
xLarge	Value such that the probabities to the right are less than xSmall.
xHuge	Value such that probability to the right is less than tiny.
modeDist	The mode of the given skewed hyperbolic distribution, calculated by skewhypMode.

skewhypCalcRange returns the quantile values at the endpoints of the region of the density function where the probablity is greater than tol.

Author(s)

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See Also

uniroot, dskewhyp, skewhypMode

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skewhypFit *Fit the Skewed Hyperbolic Student's t-Distribution to Data*

Description

Fits a skewed hyperbolic t-distribution to given data. Displays the histogram, log-histogram (both with fitted densities), Q-Q plot and P-P plot for the fit which has maximum likelihood.

Usage

```
skewhypFit(x, freq = NULL, breaks = NULL, startValues = "LA",
  paramStart = NULL, method = "Nelder-Mead", hessian = TRUE,
  plots = TRUE, printOut = TRUE, controlBFGS = list(maxit = 200),
  controlNM = list(maxit = 1000), maxitNLM = 1500, ...)
## S3 method for class 'skewhypFit':
plot(x, which = 1:4, plotTitles = paste(c(
"Histogram of ", "Log-Histogram of ", "Q-Q Plot of ", "P-P Plot of "),
x$obsName, sep = ""), ask = prod(par("mfcol")) < length(which) &&
dev.interactive(), ...)
## S3 method for class 'skewhypFit':
print(x,digits = max(3, getOption("digits") - 3),...)
## S3 method for class 'skewhypFit':
summary(object,...)
```

Arguments

x	Data Vector for `skewhypFit`. Object of class `"skewhypFit"` for `plot.skewhypFit`, `print.skewhypFit` and `summary.skewhypFit`.
freq	Vector of weights with length equal to length of x.
breaks	Breaks for histogram, defaults to those generated by `hist(x, plot=FALSE, right=FALSE)`. If `startValues = "LA"` then 30 breaks are used by default.
startValues	Code giving the method of determining starting values for finding the maximum likelihood estimates of the parameters.
paramStart	If `startValues = "US"` the user must specify a vector of starting parameter values in the form `c(mu, delta, beta, nu)`.
method	Different optimisation methods to consider, see **Details**.
hessian	Logical; if `hessian = TRUE` the value of the hessian is returned.
plots	Logical; if `plots = TRUE` the histogram, log-histogram, Q-Q and P-P plots are printed.
printOut	Logical; if `printOut = TRUE` results of the fitting are printed.
controlBFGS	A list of control parameters for `optim` when using the `"BFGS"` optimisation.
controlNM	A list of control parameters for `optim` when using the `"Nelder-Mead"` optimisation.
maxitNLM	A positive integer specifying the maximum number of iterations when using the `"nlm"` optimisation.
which	If a subset of plots is required, specify a subset of the numbers `1:4`.
plotTitles	Titles to appear above the plots.
ask	Logical; if `TRUE` the user is asked before plot change, see `par(ask = .)`.
digits	Desired number of digits when the object is printed.
object	Object must be of class "skewhypFit"
...	Passes arguments to `optim`, `nlm`, `hist`, `logHist`, `qqskewhyp` and `ppskewhyp`.

Details

startValues can be either "US"(User-supplied) or "LA" (Linear approximation) If startValues = "US" then a value for paramStart must be supplied. For the details concerning the use of startValues and paramStart see skewhypFitStart.

The three optimisation methods currently available are:

"BFGS" Uses the quasi-Newton method "BFGS" as documented in optim.

"Nelder-Mead" Uses an implementation of the Nelder and Mead method as documented in optim.

"nlm" Uses the nlm function in R.

For the details of how to pass control information using optim and nlm, see optim and nlm.

Value

skewhypFit returns a list with components:

param	A vector giving the maximum likelihood estimates of the parameters in the form c(mu, delta, beta, nu).
maxLik	The value of the maximised log-likelihood.
hessian	If hessian was set to TRUE, the value of the hessian, not present otherwise.
method	Optimisation method used.
conv	Convergence code. See optim or nlm for details.
iter	Number of iterations of optimisation routine.
x	The data used to fit the distribution.
xName	Character stirng with the actual x argument name.
paramStart	Starting values of the parameters returned by skewhypFitStart.
svName	Name of the method used to find starting values.
startValues	Acronym of method used to find starting values.
breaks	Cell boundaries found by a call to hist.
midpoints	The cell midpoints found by a call to hist.
empDens	The estimated density found by a call to hist if startValues = "US" or density if startValues = "LA".

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See Also

optim, nlm, par, hist, logHist, qqskewhyp, ppskewhyp and skewhypFitStart.

`skewhypFitStart` *Find Starting Values for Fittting a Skewed Hyperbolic Student's t-Distribution*

Description

Finds starting values for input to a maximum likelihood routine for fitting a skewed hyperbolic t-distribution to data.

Usage

```
skewhypFitStart(x, breaks = NULL, startValues = "LA", paramStart = NULL)
skewhypFitStartLA(x, breaks = NULL)
```

Arguments

`x`	Data vector.
`breaks`	Breaks for histogram. If missing defaults to those generated by `hist(x, right = FALSE, plot =FALSE)`. If `startValues = "LA"` then 30 breaks are used by default.
`startValues`	Code giving the method of determining starting values for finding the maximum likelihood estimates of the parameters.
`paramStart`	If `startValues = "US"` the user must specify a vector of starting parameter values in the form `c(mu, delta, beta, nu)`.

Details

`startValues` can be either `"US"`(User-supplied) or `"LA"` (Linear approximation).

If `startValues = "US"` then a value for `paramStart` must be supplied.

If `startValues = "LA"` a linear approximation is made to the log-density in each of the tails, from which the estimates for ν and β are found. The remaining two parameters, δ and μ are found by solving the moment equations for mean and variance. Since the variance does not exist for values of $\nu \leq 4$, the estimate of ν will be at least 4.1.

Value

`skewhypFitStart` returns a list with components:

`paramStart`	A vector of the form `c(mu, delta, beta, nu)` giving the generated starting values of the parameters.
`breaks`	The cell boundaries found by a call to `hist`.
`midpoints`	The cell midpoints found by a call to `hist`.
`empDens`	The estimated density at the midpoints found by a call to `hist` if `startValues = "US"` or `density` if `startValues = "LA"`.
`svName`	Name of the method used to find the starting values.

Author(s)

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References

Aas, K. and Haff, I. H. (2006). The Generalised Hyperbolic Skew Student's t-Distribution. In *Jorunal of Financial Econometrics*, 26-Jan-2006

See Also

`hist`, `density`, `dskewhyp`, `skewhypFit`

`skewhypMean` *Moments and Mode of the Skewed Hyperbolic Student's t-Distribution.*

Description

Functions to calculate the mean, variance, skewness and kurtosis of a specified skewed hyperbolic t-distribution.

Usage

```
skewhypMean(mu = 0, delta = 1, beta = 1, nu = 1, param = c(mu, delta,
  beta, nu))
skewhypVar(mu = 0, delta = 1, beta = 1, nu = 1, param = c(mu, delta,
  beta, nu))
skewhypSkew(mu = 0, delta = 1, beta = 1, nu = 1, param = c(mu, delta,
  beta, nu))
skewhypKurt(mu = 0, delta = 1, beta = 1, nu = 1, param = c(mu, delta,
  beta, nu))
skewhypMode(mu = 0, delta = 1, beta = 1, nu = 1, param = c(mu, delta,
  beta, nu), tolerance = .Machine$double.eps ^ 0.5)
```

Arguments

`mu`	Location parameter μ, default is 0.
`delta`	Peakedness parameter σ, default is 1.
`beta`	Skewness parameter β, default is 1. Negative values give a negative skew, positive values give a positive skew.
`nu`	Shape parameter ν, default is 1.
`param`	Specifying the parameters as a vector of the form `c(mu, delta, beta, nu)`.
`tolerance`	A difference smaller than this value is taken to be zero.

Details

Users may either specify the values of the parameters individually or as a vector. If both forms are specified, then the values specified by the vector `param` will overwrite the other ones.

The moments are calculated as per formulae in Aas&Haff(2006) and the mode is calculated by numerical optimisation of the density function using `optim`.

Note that the mean does not exist when $\nu = 2$, the variance does not exist for $\nu \leq 4$, the skewness does not exist for $\nu \leq 6$, and the kurtosis does not exist for $\nu \leq 8$.

Value

`skewhypMean` gives the mean of the skewed hyperbolic t-distribution, `skewhypVar` the variance, `skewhypSkew` the skewness, `skewhypKurt` the kurtosis and `skewhypMode` the mode.

Author(s)

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References

Aas, K. and Haff, I. H. (2006). The Generalised Hyperbolic Skew Student's t-Distribution. In *Jorunal of Financial Econometrics*, 26-Jan-2006

See Also

`dskewhyp`, `optim`