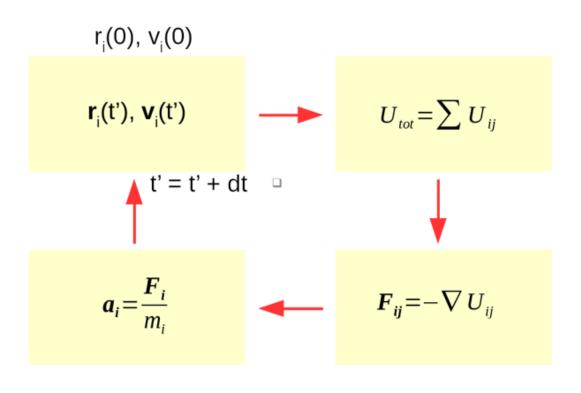
Objective

- Molecular dynamics (MD)
- Short-range and long-range forces
- Neighbor lists Verlet and cell lists
- Ingredients for running MD simulations
- Properties from MD simulations
- MD Fortran code for Argon
- Generate initial configurations
- Visualize the trajectory in VMD

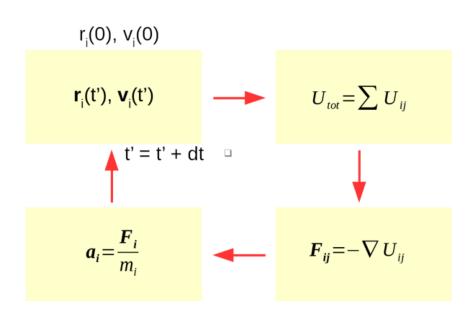
Molecular dynamics (MD)



Molecular dynamics (MD)

Mainly involves two parts

- Compute forces on all atoms/particles computationally expensive
- Integration to update the positions and velocities



Simple MD program

```
program MD
call initialization
t=0
do while (t<tmax)
                                      computationally
   call force
                                          expensive
   call integrate
   t = t + delt
enddo
stop
end program MD
```

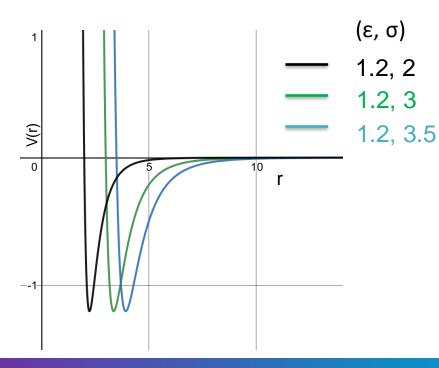
Calculating forces

- Bonded forces (bond, angle, etc.) are local computations not expensive
- Non-bonded forces
 - Van der Waals forces (short-range) moderately expensive
 - Electrostatic forces (long-range) very expensive
- Parallelization of MD is mainly associated with
 - parallelization of force calculations and
 - Improve serial performance reducing the number of iterations $(< O(N^2))$ for force calculation

vdW forces

Van der Waals forces (short-range) – moderately expensive

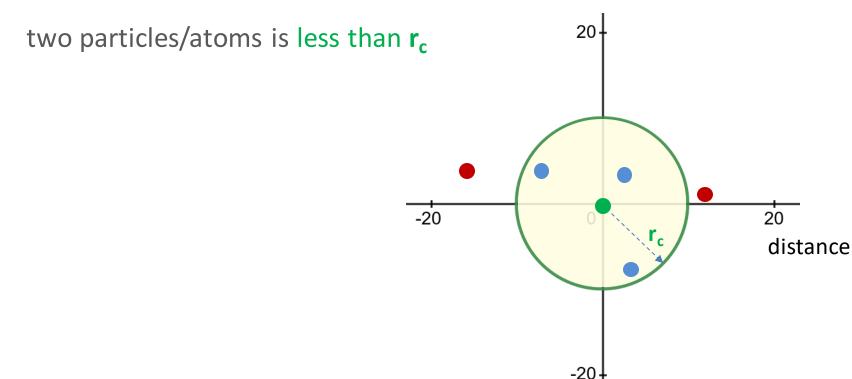
$$V(r) = 4\varepsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^{6} \right]$$



- Interaction goes to zero beyond a certain distance depends on the value of $\boldsymbol{\sigma}$
- Recommended cutoff 2.5σ
- Computational complexity O(N²)

vdW forces – speed up the calculations

Energy and forces are computed only when the distance between



vdW forces – sample Fortran code

```
Force=0.d0
do atom1=1, TotAtom
  do atom2=
           1,TotAtom
    dr=r(atom1,:)-r(atom2,:)
    dr=dr-Box*anint(dr/Box)
    r2=dot product(dr,dr)
    if(r2<=R2cut) then
                                ! r2cut is square of rcut
      r2=1/r2
      fac2=r2*Sig*Sig
      fac6=fac2*fac2*fac2
      df=48.d0*Eps*r2*fac6*(fac6-0.5d0)
      fc(1)=df*dr(1)
      fc(2)=df*dr(2)
      fc(3)=df*dr(3)
      Force(atom1,:)=Force(atom1,:)+fc(:)
      Force(atom2,:)=Force(atom2,:)-fc(:)
      PE=PE+4.d0*Eps*fac6*(fac6-1)-Ecut
                                               !shifted to zero at cutoff
    endif
  enddo
enddo
```

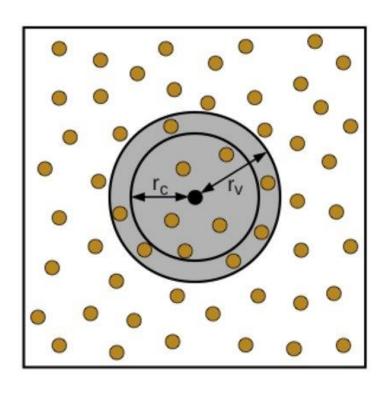
vdW forces

```
Force=0.d0
do atom1=1,TotAtom-1
do atom2=atom1+1,TotAtom
```

- Even after truncating the potential, we still have to compute N*(N-1)/2 pair distances
- Without any tricks, the time taken for force evaluation is proportional to $\ensuremath{\mathsf{N}}^2$
- Speedup can be achieved with
 - Verlet list
 - Cell (or linked) list
 - Verlet + Cell list

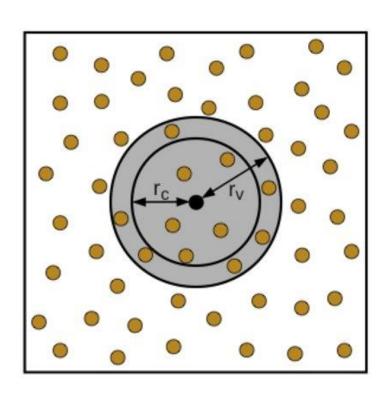
Improves serial performance

Neighbour lists – Verlet neighbor lists



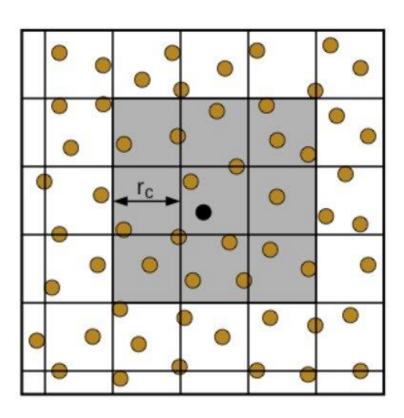
- Verlet list is a bookkeeping technique that contains list of all particles within the radius r_{i,v} around each atom 'i'.
- List is prepared for each atom before calculating the forces whenever the maximum displacement of the atoms is greater than r_c r_v
- Scales as O(N x Nv) where 'N' is the total number of particles, 'Nv' is the avg. number of particles within cutoff

Neighbour lists – Verlet neighbor lists



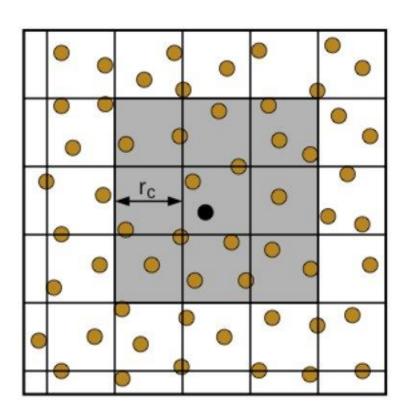
- $r_v r_c = s = skin distance$
- Larger skin distance (s), less often we build neighbor lists, but more pair of atoms will be checked for force calculation at every timestep
- Optimal value of skin distance must be chosen for achieving maximum computational efficiency

Neighbour lists – cell lists method



Scales as O(N x Nc) where 'N' is the total number of particles, 'Nc' is the avg. number of particles in 9 cells

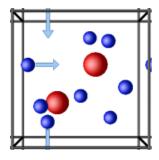
Neighbour lists – verlet + cell lists method



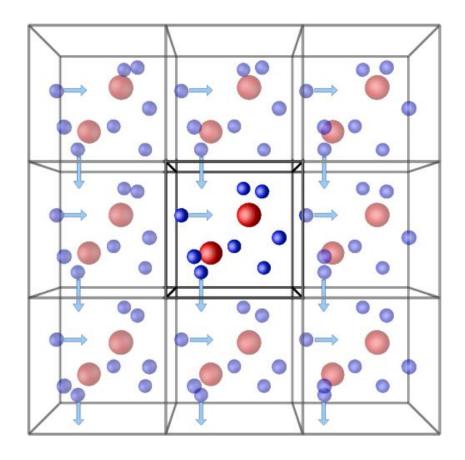
Used when neighbour list needs to be regenerated

Surface atoms

- Atoms near the faces of the box have different environment
- Surface atoms are present



Periodic boundary conditions



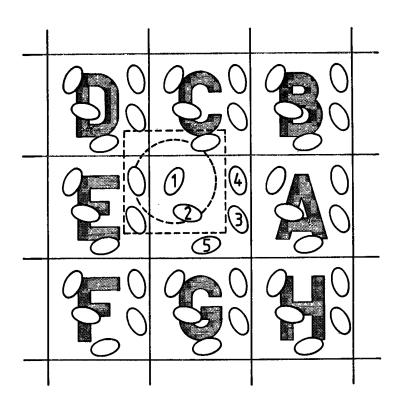
No surface atoms!

Periodic boundary conditions

- Advantages: simulations can be performed in a relatively small number of atoms/particles
- If an atom/particle leaves simulation box, it is replaced by its image atom/particle
- 2D box − 8 neighbors; 3D box − 26 neighbors
- Disadvantage: small box sizes can lead to inconsistency in force/energy calculation
- Coordinates of the Image atoms/particles can be obtained from

$$x' = x + n1L$$
 $y' = y + n2L$ $z' = z + n3L$ where $n1/n2/n3=-1,0,1$

Minimum image convention

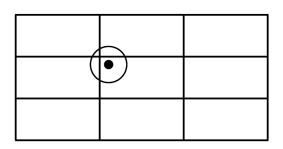


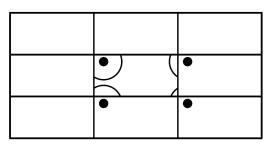
- Each ith particle interacts with other particles as well as their images i_A,
 i_B...in neighboring boxes.
- Infinite no. of calculations
- Easy to calculate short-range forces
- Molecule 1 interacts with all those which lie within a certain radius,
- Each atom interacts with just one image of the other atoms

Computer simulations of liquids; Allen and Tildesley; Chapter 1

Verlet list when PBC is used

In two dimensions,





 Avoids nearest image calculation in the inner loop of MD

 each periodic image of a particle has its own Verlet list that contains particles only those in the central

In three dimensions,

$$\mathbf{F}_{i} = \sum_{i=1}^{N} \sum_{k=-13}^{13} {}' \mathbf{F}_{i(j,k)} \iff \mathbf{F}_{i} = \sum_{j=1}^{N} \sum_{k=-13}^{13} {}' \mathbf{F}_{(i,k)j}$$
box.

$$F_i = \sum_{j=1}^{N} \sum_{k=-13}^{13} {}^{\prime}$$

Box index $k = 9n_x + 3n_y + n_z$ Translation vector $t_k = n_x L_x + n_y L_y + n_z L_z$

where $n_x/n_y/n_z=-1,0,1$

Understanding Molecular Simulation, Daan Frenkel and Berend Smit, see Appendix F; An Efficient, Box Shape Independent Non-Bonded Force and Virial Algorithm for Molecular Dynamics, Bekker, et al.

Fortran code – Verlet method

```
makes a new Verlet list
SUBROUTINE new_vlist
                                      initialize list
do i=1, npart
  nlist(i) = 0
                                      store position of particles
  xv(i)=x(i)
enddo
do i=1, npart-1
  do j=i+1, npart
    xr=x(i)-x(j)
    if (xr.gt.hbox) then
                                      nearest image
      xr=xr-box
    else if (xr.lt.-hbox) then
      xr = xr + box
    endif
                                     add to the lists
    if (abs(xr).lt.rv) then
      nlist(i)=nlist(i)+1
      nlist(j) = nlist(j) + 1
      list(i,nlist(i))=j
      list(j,nlist(j))=i
    endif
  enddo
enddo
return
end
```

Understanding Molecular Simulation, Daan Frenkel and Berend Smit, see Appendix F

Fortran code – cell list method

```
rn=box/int(box/rc)
do icel=0,ncel-1
  hoc(icel)=0
enddo
do i=1,npart
  icel=int(x(i)/rn)
  ll(i)=hoc(icel)
  hoc(icel)=i
enddo
return
end
```

makes a new cell list with cell size r_c using a linked-list algorithm determine size of cells $r_n \geq r_c$

set head of chain to 0 for each cell

loop over the particles determine cell number link list the head of chain of cell icel make particle i the head of chain

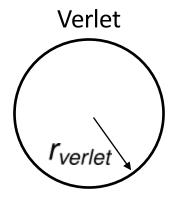
Fortran code – cell list method

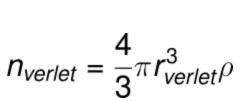
```
SUBROUTINE new_nlist(rc)
                                                                           <u>i=5</u>
                                   <u>i=1</u>
                                                      <u>i=3</u>
                                        icel = 0
                                                                                icel = 1
                                                           icel = 0
rn=box/int(box/rc)
                                        II(1) = 0
                                                                                II(5) = 4
                                                           II(3) = 1
do icel=0, ncel-1
                                        hoc(0)=1
                                                                                hoc(1)=5
  hoc(icel) = 0
                                                           hoc(0)=3
enddo
                                   <u>i=2</u>
                                                       <u>i=4</u>
do i=1, npart
  icel=int(x(i)/rn)
                                        icel = 1
                                                            icel = 1
  11(i) = hoc(icel)
                                        II(2) = 0
                                                            II(4) = 2
  hoc(icel)=i
                                        hoc(1)=2
                                                            hoc(1)=4
enddo
return
end
                                   icel =0
                                                                     icel = 1
                                   hoc(0)=3,II(3)=1,II(1)=0
                                                                     hoc(1)=5, II(5)=4, II(4)=2,
                                                                     II(2)=0
```

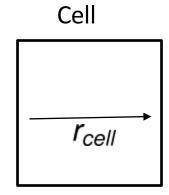
Understanding Molecular Simulation, Daan Frenkel and Berend Smit, see Appendix F

Neighbor summary

- Typical update frequency 10 to 20 MD steps
 - Too high --> no gain in the performance of the code
 - Too low --> forces calculated incorrectly







$$n_{cell} = 27 r_{cell}^3 \rho$$

$$\frac{n_{cell}}{n_{verlet}} = 5$$

Parallel performance



Journal of Computational Physics
Volume 117, Issue 1, 1 March 1995, Pages 1-19



Regular Article

Fast Parallel Algorithms for Short-Range Molecular Dynamics

Steve Plimpton

https://www.sciencedirect.com/science/article/pii/S002199918571039X

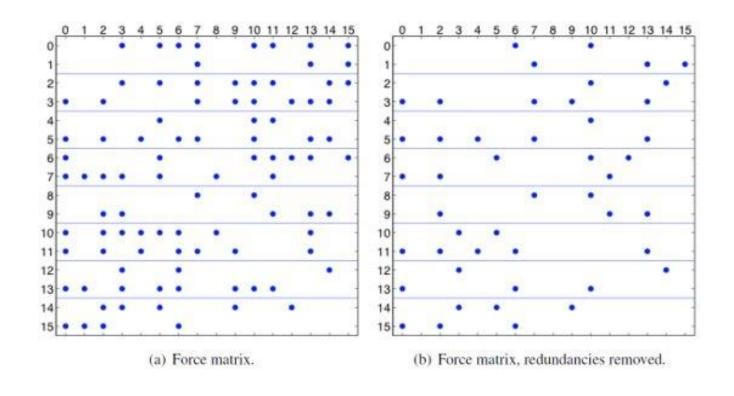
Parallel MD algorithms

- Force calculations and positions/velocity updates
- Good algorithm ensures
 - Load balancing
 - Less communication among processes
- Assign a set of force computations to each processor and it remains fixed throughout the simulations
 - Atom decomposition
 - Force decomposition
- Assign a portion of the physical simulation domain and processes exchange atoms throughout the simulations
 - Spatial decomposition (geometric methods)

Particle decomposition

- Identical copies of atom information are stored on all processors
- Each processor (P) is assigned N/P atoms where N is total atoms
- At every MD step, the communication is needed among all processes to update the forces, etc
- Cost for communication is mainly dictated by number of messages and total volume of data (sent and received)
- Communications scales as N, independent of P ---> limits the performance for large systems
- Load balancing is maintained

Particle (or atomic) Decomposition

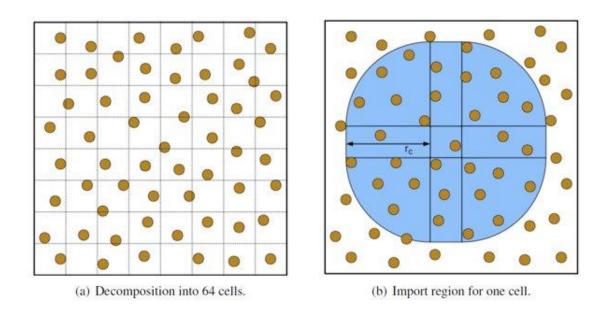


Atom decomposition: showing a force matrix of 16 particles distributed among 8 processors.

Spatial decomposition

- Space is decomposed into cells
- Each cell is assigned to a processor, and it is responsible for computing the forces on particles that lie inside this cell
- Assignment of particles to cells changes during simulation
- Communication is required between processors corresponding to nearby particles
- For high P, the import region is large compared to the number of particles contained inside each cell

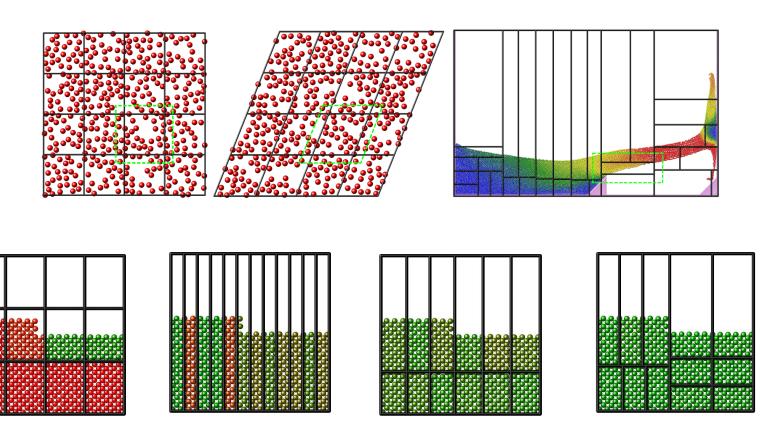
Spatial decomposition



Spatial decomposition: showing particles in a 2-D computational box, partitioned into 64 cells (left), import region for one cell (right)

Introduction to High Performance Scientific Computing, Victor Eijkhout

Spatial decomposition

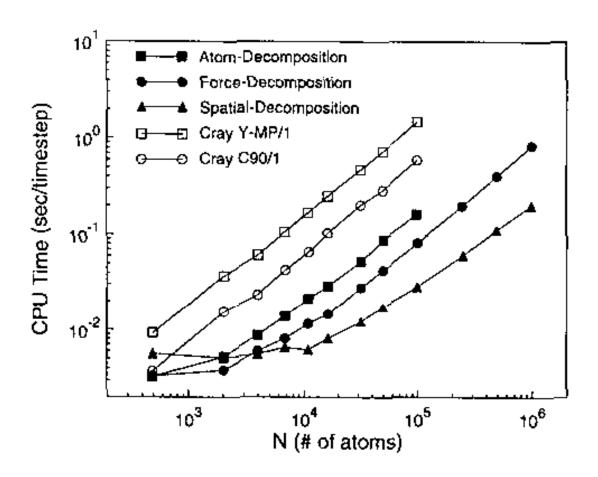


Different decompositions for a 2d system with 12 MPI ranks

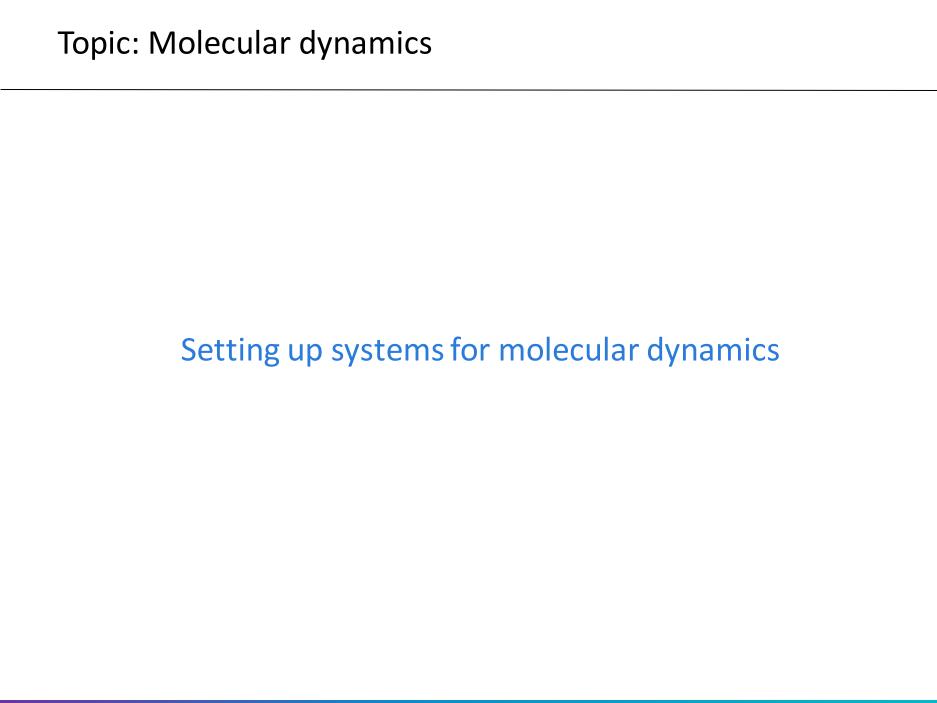
Load imbalance is reduced

https://docs.lammps.org/Developer_par_part.html

Parallel performance

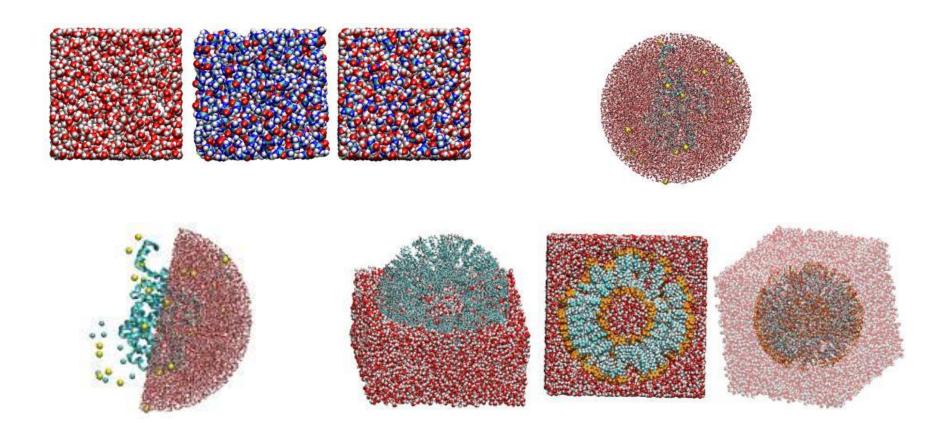


https://www.sciencedirect.com/science/article/pii/S002199918571039X



Initial configuration

• Use packmol http://m3g.iqm.unicamp.br/packmol/



MD softwares

- LAMMPS
- DL POLY
- NAMD
- Gromacs
- CHARMM
- AMBER

and many more

Time scales associated various phenomena

S. No.	Internal motion	Timescale [s]	
1	Protein folding	10 ⁻⁵ -10 ⁻¹	
2	Interior-sidechain rotation (proteins)	10 ⁻⁴ -10 ⁰	
3	Site-juxtaposition (super helical DNA)	10 ⁻⁶ -10 ⁰	
4	Collective subgroup motion for e.g. For hinge bending, for allosteric transitions Global DNA bending	10 ⁻¹¹ -10 ⁻⁷ 10 ⁻¹⁰ -10 ⁻⁷	
5	Surface-side chain rotation (Protein)	10 ⁻¹¹ -10 ⁻¹⁰	
6	Sugar puckering (Nucleic acids)	10 ⁻¹² -10 ⁻⁹	
7	Global DNA twisting	10 ⁻¹²	
8	Heavy atom angle bend	5x10 ⁻¹⁴	
9	Heavy atom bond stretch	3x10 ⁻¹⁴	
10	Light atom angle bend	2x10 ⁻¹⁴	
11	Double bond stretch	2x10 ⁻¹⁴	
12	Light atom bond stretch	10 ⁻¹⁴	

Properties from MD trajectories

- Potential energy
- density
- Temperature

Pressure

$$P = \frac{Nk_BT}{V} + \frac{\sum_{i}^{N'} r_i \bullet f_i}{dV}$$

Enthalpy of vaporization

$$\Delta H_{vap} = H_{gas} - H_{liquid} = U_{gas} - U_{liquid} + P(V_{gas} - V_{liquid})$$

• Isothermal compressibility

$$\kappa_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

• Thermal expansion coefficient

$$\alpha_P = -\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_P$$

Properties from MD trajectories

Self-diffusion coefficient

$$D = \frac{1}{3} \int_0^\infty \langle v_i(t)v_i(0)\rangle dt$$

Static dielectric constant

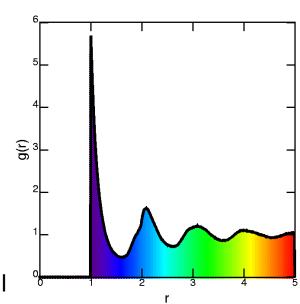
Surface tension

$$\epsilon = 1 + \frac{4\pi}{3Vk_B \langle T \rangle} \left(\langle \mathbf{M}^2 \rangle - \langle \mathbf{M} \rangle^2 \right)$$

$$\gamma = \frac{1}{2}L_z \left\langle P_{zz} - \frac{1}{2} \left(P_{xx} + P_{yy} \right) \right\rangle$$

• Pair correlation function or

radial distribution function



http://www.physics.emory.edu/faculty/weeks//idl/gofr.html

LAMMPS Benchmark

Potential	System	Atoms	Timestep	CPU	LJ Ratio
Granular	chute flow	32000	$0.0001 \mathrm{tau}$	5.08e-7	0.34x
FENE bead/spring	polymer melt	32000	0.012 tau	5.32e-7	0.36x
Lennard-Jones	LJ liquid	32000	$0.005 \mathrm{tau}$	1.48e-6	1.0x
DPD	pure solvent	32000	$0.04 \mathrm{\ tau}$	2.16e-6	1.46x
EAM	bulk Cu	32000	5 fmsec	3.59e-6	2.4x
Tersoff	bulk Si	32000	1 fmsec	6.01e-6	4.1x
Stillinger-Weber	bulk Si	32000	1 fmsec	6.10e-6	4.1x
EIM	crystalline NaCl	32000	$0.5 \mathrm{fmsec}$	9.69e-6	6.5x
SPC/E	liquid water	36000	2 fmsec	1.43e-5	9.7x
CHARMM + PPPM	solvated protein	32000	2 fmsec	2.01e-5	13.6x
MEAM	bulk Ni	32000	5 fmsec	2.31e-5	15.6x
Peridynamics	glass fracture	32000	$22.2 \mathrm{nsec}$	2.42e-5	16.4x
Gay-Berne	ellipsoid mixture	32768	$0.002 \mathrm{tau}$	4.09e-5	28.3x
AIREBO	polyethylene	32640	$0.5 \mathrm{fmsec}$	8.09e-5	54.7x
COMB	crystalline SiO2	32400	$0.2 \mathrm{fmsec}$	4.19e-4	284x
m eFF	H plasma	32000	$0.001~\mathrm{fmsec}$	4.52e-4	306x
ReaxFF	PETN crystal	16240	$0.1 \; \mathrm{fmsec}$	4.99e-4	337x
ReaxFF/C	PETN crystal	32480	$0.1 \; \mathrm{fmsec}$	2.73e-4	185x
VASP/small	water	192/512	$0.3 \mathrm{fmsec}$	26.2	17.7e6
VASP/medium	CO2	192/1024	$0.8 \mathrm{fmsec}$	252	170e6
VASP/large	Xe	432/3456	2.0 fmsec	1344	908e6

https://www.lammps.org/bench.html

Projects

- Submit one page report along with modified code in two weeks.
- **Project 2**: Implement cell list algorithm in the given MD code and check for its performance by plotting the CPU/wall time for different system size. Also include a note on changes in wall time before and after implementing the neighbour lists.
- **Project 3**: Implement Verlet+cell list (neighbour list) algorithm in the given MD code and check for its performance by plotting the CPU/wall time for different system size. Also include a note on changes in wall time before and after implementing the neighbour lists.
- **Project 4**: Implement MPI model in the given MD code and check for its performance by plotting the parallel speedup and parallel efficiency, for different system size.

Summary

- Simple algorithm
- Force/energy calculation is computationally expensive part
- Serial performance is improved using neighbor lists
- PBC and minimum image convention

Topic:

Reading material

- Understanding Molecular Simulation. From Algorithms to Applications, Daan Frenkel and Berend Smit
 - For cell and Verlet lists, read Appendix F "Saving CPU Time"
- Computer simulations of liquids; Allen and Tildesley
- Fast Parallel Algorithms for Short-Range Molecular Dynamics, Steve Plimpton
- For packmol, See http://m3g.iqm.unicamp.br/packmol/home.shtml