Topic 6.2: AI and Digital Finance

Machine Learning Transforms Financial Services

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Learning Objectives

By the end of this topic, you will be able to:

1. **Identify** the major AI applications in finance (robo-advisory, fraud detection, credit scoring, algorithmic trading)
2. **Understand** how robo-advisors make investment decisions (the logic behind automated portfolio balancing)
3. **Analyze** AI risks including model opacity, adversarial attacks, and systemic risks
4. **Evaluate** AI claims in finance using a critical framework
5. **Apply** these concepts through hands-on robo-advisor simulation

Key Question: How is artificial intelligence transforming financial services, and what are the associated risks and limitations?

Prerequisites and Background

What We've Learned So Far:

- Money and financial systems (Day 1)
- Digital payments and APIs (Day 2)
- Blockchain and consensus (Day 3)
- Smart contracts and DeFi (Day 4)
- Risks and regulation (Day 5)

Course Context:

- Day 1: Foundations – Money, FinTech vs. DeFi
- Day 2: Payments, APIs, Data-Driven Finance
- Day 3: Cryptography, Blockchain, Wallets
- Day 4: Smart Contracts, DeFi, Stablecoins
- Day 5: Risks, Regulation, DAOs, Privacy
- **Today: AI as force multiplier**

No Prior Knowledge Needed

We will introduce every concept as we go – no prior knowledge of AI, programming, mathematics, or finance is required.

Connection to Previous Topics

AI intersects with all previous material: AI-powered DeFi, algorithmic stablecoin management, automated compliance, and AI-driven trading on blockchain platforms.

The AI Revolution in Finance

Why AI Matters Now:

- Massive data availability
- Computational power explosion
- Algorithm sophistication
- Cost reduction pressure
- Customer experience demands

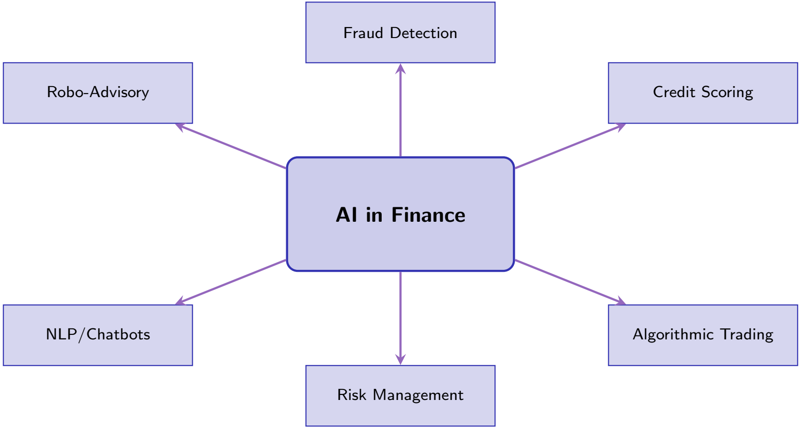
Scale of Transformation:

- $1.4T+ in robo-advisor AUM globally (2024, Statista)
- 75%+ of US equity trading volume is algorithmic (SEC estimates)
- Billions saved in fraud prevention
- Millions gaining credit access

Critical Perspective

AI in finance is both **genuinely transformative** and **frequently overhyped**. This topic gives you tools to distinguish between the two.

AI in Finance: The Landscape



AI Application 1: Robo-Advisory

What Is It?

Automated investment management using algorithms to construct, monitor, and rebalance portfolios based on client goals and risk tolerance.

How it works:

1. Client inputs: goals, timeline, risk tolerance
2. Algorithm maps to asset allocation
3. Automated portfolio construction
4. Continuous monitoring and rebalancing
5. Tax-loss harvesting (if applicable)

Key Players:

- Betterment
- Wealthfront
- Schwab Intelligent Portfolios
- Vanguard Digital Advisor

Scale:

- \$1.4T+ AUM globally (2024, Statista)
- Fees: 0.25–0.50% vs. 1%+ traditional
- Growing 20%+ annually

See Notebook NB13 for hands-on robo-advisor simulation

Robo-Advisory: The Intuition Behind It

Modern Portfolio Theory in Plain English

Core idea: Don't put all your eggs in one basket. By mixing investments that don't move together, you can get better returns for the same level of risk.

What the robo-advisor's algorithm does:

1. **Estimates** how much each investment might return and how risky it is
2. **Measures** how investments move relative to each other (when one goes up, does the other go down?)
3. **Finds** the best mix given how much risk you are comfortable with
4. **Adjusts** the mix over time as markets change

How Your Risk Tolerance Maps to Investments:

Risk Profile	Typical Stock Allocation
Conservative (cautious)	20–40%
Moderate (balanced)	40–60%
Aggressive (growth-seeking)	60–90%

For the mathematical formulas behind this, see the Appendix. Modern robo-advisors also add constraints: ESG filters, tax efficiency, position limits.

Portfolio Concepts: Return, Risk, and the Sharpe Ratio

These concepts power robo-advisors behind the scenes:

Portfolio Return:
Your portfolio's expected gain is simply the weighted average of each investment's expected gain. If you put 60% in stocks (expected 8%) and 40% in bonds (expected 3%), your expected return is about 6%.

Portfolio Risk:
The overall risk of your portfolio depends not just on how risky each investment is, but on how they move relative to each other. If stocks zig when bonds zag, combining them reduces your overall risk.

Key Insight:
Diversification works because investments rarely move in perfect lockstep.

The Sharpe Ratio*:
A simple scorecard for investments:

How to read it

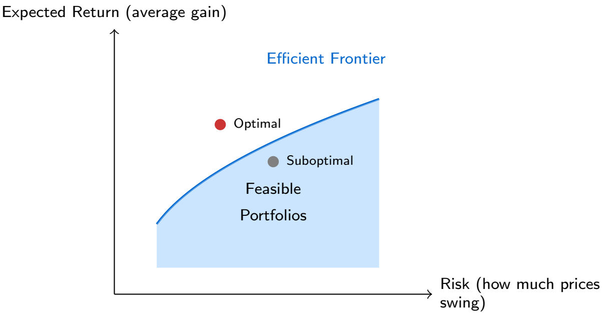
$$\textit{Sharpe Ratio} = \frac{\text{Portfolio return} - \text{Risk-free return}}{\text{Portfolio risk (volatility)}}$$

- **Numerator:** How much extra return you earn above a "safe" investment (like a government bond)
- **Denominator:** How bumpy the ride is
- **Higher is better:** More reward per unit of risk

*Sharpe ratio = a measure of return per unit of risk. Higher is better—it tells you how much extra reward you earn for each unit of "bumpiness" in your portfolio.

For the full mathematical formulas, see the Appendix.

The Efficient Frontier



Definition:
The set of portfolios that maximize return for a given risk level, or minimize risk for a given return.

Key Properties:

- Portfolios below are suboptimal
- Portfolios above are impossible
- Rational investors choose only frontier portfolios

Risk Profiling: From Questionnaire to Portfolio

Risk Assessment Factors:

1. Investment time horizon
2. Reaction to hypothetical losses
3. Financial goals (growth vs. income)
4. Investment experience
5. Income stability
6. Liquidity needs

Scoring Methodology:

Score Range	Profile
0–20	Conservative
21–40	Moderately Conservative
41–60	Moderate
61–80	Moderately Aggressive
81–100	Aggressive

From Score to Portfolio

Each risk profile maps to a target asset allocation (e.g., 60/40 stocks/bonds), which the robo-advisor's algorithm then fine-tunes to find the best mix of investments within those targets.

Portfolio Rebalancing Strategies

Calendar-Based:

- Rebalance on fixed schedule
- Monthly, quarterly, or annually
- Simple to implement
- May miss significant drifts

Threshold-Based:

- Rebalance when drift exceeds threshold
- Typically 5% deviation trigger
- More responsive to market moves
- Minimizes unnecessary trading

Hybrid Approach:

- Check thresholds daily
- Mandatory annual review
- Tax-efficient timing
- Cash flow integration

Trade-off

More frequent rebalancing = better risk control but higher costs. Robo-advisors optimize this balance automatically.

Tax-Loss Harvesting

Definition

Selling investments at a loss to offset capital gains* and reduce taxes, then replacing with similar assets to maintain market exposure.

How It Works:

1. Identify positions with unrealized losses
2. Sell to realize the loss
3. Immediately buy similar (not identical) asset
4. Use loss to offset gains

The Wash Sale Rule:**

- Cannot buy "substantially identical" security within 30 days before/after sale
- Loss disallowed if violated
- Solution: Use similar but not identical ETFs

*Capital gains tax = tax on the profit you make when you sell an investment for more than you paid.

**Wash sale rule = you cannot sell an investment at a loss and immediately rebuy the same one just to claim the tax benefit. You must wait 30 days.

Robo-Advisor Advantage: Automated systems can monitor all positions daily and harvest losses systematically—impractical for human advisors at scale.

International Note

Tax-loss harvesting is primarily a US concept. International equivalents exist in some countries (e.g., capital gains offsetting in the UK, *Verlustverrechnung* in Germany) but rules vary significantly. The Wash Sale Rule described above is US-specific (IRS).

Fee Comparison: Robo vs. Traditional

Service Type	Annual Fee
Pure Robo-Advisor	0.15–0.50%
Hybrid Robo + Human	0.30–0.50%
Independent RIA	0.50–1.50%
Traditional Broker	1.00–2.00%
Private Wealth Mgmt	1.00–2.50%

Long-Term Impact:
On $100,000 over 30 years at 7% return:

- 0.25% fee: $700,000 final
- 1.00% fee: $574,000 final
- **Difference: $126,000**

The Compounding Effect:
Small fee differences compound into large wealth differences over time.

AI Application 2: Fraud Detection

AI helps catch fraud that humans would miss—by learning patterns from millions of past transactions and spotting subtle anomalies in real time.

Traditional Rules-Based:

- If transaction ¿ \$10,000 → flag
- If international + new merchant → flag
- Static thresholds
- High false positive rate
- Easily gamed once known

ML-Based Detection:

- Learns from historical fraud patterns
- Dynamic thresholds per user
- Behavioral biometrics
- Network analysis
- Adapts to new attack vectors

Common ML Approaches

- **Supervised**: Random forests, gradient boosting (an AI technique that combines many simple predictions into one strong one), neural networks on labeled fraud data
- **Unsupervised**: Anomaly detection using isolation forests (an algorithm that detects unusual patterns by isolating outliers) and autoencoders (AI models that learn to compress and reconstruct data, useful for spotting anomalies)
- **Graph-based**: Network analysis to detect fraud rings

Fraud Detection: Performance Metrics

Key Metrics:

- **Precision**: Of flagged transactions, how many are actually fraud?
- **Recall**: Of all fraud, how much do we catch?
- **False Positive Rate**: Legitimate transactions incorrectly flagged

The Trade-off:
Higher recall (catching more fraud) typically increases false positives (blocking legitimate users).

Industry Benchmarks:

System	Recall	FPR
Rules-based	60%	5%
Basic ML	85%	2%
Advanced ML	95%	0.5%

Business Impact:
Reducing FPR from 5% to 0.5% = 90% fewer frustrated customers.

AI Application 3: Credit Scoring

Traditional FICO (a widely used US credit score):

- Payment history (35%)
- Amounts owed (30%)
- Length of history (15%)
- New credit (10%)
- Credit mix (10%)

Limitations:

- "Credit invisible" populations
- Backward-looking only
- Limited data sources

AI/Alternative Data Scoring:

- Bank transaction patterns
- Utility/rent payment history
- Employment data
- Education records
- Behavioral signals
- Social connections

Players:

- Upstart, ZestFinance, Lenddo
- Claims: 75% fewer defaults at same approval rate

Credit Scoring: Ethical and Legal Considerations

Potential Benefits:

- Financial inclusion for "thin file" consumers
- More accurate risk assessment
- Lower costs passed to consumers
- Faster decisions

Potential Risks:

- Proxy discrimination
- Lack of transparency
- Data privacy concerns
- Perpetuating historical biases

Regulatory Requirements

- **ECOA** (Equal Credit Opportunity Act, US): Cannot discriminate on race, religion, national origin, sex
- **FCRA** (Fair Credit Reporting Act, US): Right to know reasons for adverse actions
- **GDPR** (EU): Right to explanation for automated decisions

Note: ECOA and FCRA are US-specific; other jurisdictions have equivalent frameworks.

AI Application 4: Algorithmic Trading

Evolution:

- **1980s–90s**: Statistical arbitrage (using math to find temporary price differences)
- **2000s**: High-frequency trading emerges
- **2010s**: Factor investing at scale
- **2020s**: Deep learning, NLP, alternative data

Modern AI Trading Signals:

- Satellite imagery (parking lots, shipping)
- Social media sentiment
- Patent filings, job postings
- Earnings call NLP analysis
- Web traffic data

Reality Check

- Most AI trading strategies don't beat benchmarks net of fees
- Overfitting is rampant
- Alpha (returns above the market average) decays quickly once discovered
- Data quality issues pervasive

Institutional players:
Two Sigma, Citadel, Renaissance, DE Shaw

Algorithmic Trading: Strategy Types

Market Making:

- Provide liquidity
- Earn bid-ask spread*
- High frequency
- Low margin, high volume

Statistical Arbitrage:

- Exploit price relationships
- Mean reversion**
- Pairs trading***
- Market neutral****

Momentum/Trend:

- Follow price trends
- Technical signals
- Factor-based
- Longer holding periods

*Bid-ask spread = the difference between the highest price a buyer will pay and the lowest a seller will accept.
**Mean reversion = the idea that prices tend to return to their historical average over time.
***Pairs trading = betting that two historically related assets will converge after temporarily diverging.

****Market neutral = a strategy designed to profit regardless of whether the overall market goes up or down.

The Alpha Challenge

- Markets are highly efficient
- Any profitable pattern gets arbitraged away
- Need continuous innovation to maintain edge
- Most retail AI trading systems fail

AI in DeFi: Emerging Applications

Current Applications:

- MEV (Maximal Extractable Value) bots – profit that blockchain validators can extract by reordering transactions, like a postal worker who reads your letters and acts on the information before delivering them
- Liquidation bots
- Yield farming optimizers
- Portfolio rebalancing agents
- Smart contract auditing

Emerging/Experimental:

- AI-managed DAOs
- Autonomous trading agents
- On-chain ML inference
- LLM-powered governance
- AI agents holding wallets

Example: Yearn Finance

Yearn uses algorithms to automatically move deposits between yield strategies, optimizing for the highest risk-adjusted returns. Not “AI” in the deep learning sense, but algorithmic automation of DeFi.

AI Risks in Finance: Model Opacity

The Black Box Problem

Complex ML models (neural networks, gradient boosting) often cannot explain *why* they make specific predictions. This creates regulatory and ethical challenges in finance.

Why It Matters:

- Regulatory requirements (ECOA, GDPR)
- Right to explanation for credit denials
- Model risk management
- Debugging and improvement
- Trust and accountability

Mitigation Approaches:

- Interpretable models (logistic regression)
- SHAP/LIME (tools that explain why an AI made a specific decision)
- Surrogate models
- Feature importance analysis
- Regulatory sandboxes

Tension: More complex models → better predictions → less interpretability

AI Risks in Finance: Adversarial Attacks

Definition

Deliberately crafted inputs designed to fool ML models while appearing normal to humans.

Finance-Specific Threats:

- **Credit fraud**: Manipulate application to get approval
- **Trading**: Poison training data with fake signals
- **KYC bypass**: Adversarial images for identity verification
- **Spam**: Evade NLP-based filters

Example: Credit Application

- Attacker knows model features
- Slightly modifies spending patterns
- Opens strategic accounts
- Model sees "good" applicant
- Fraud not detected until default

Deepfake Fraud

AI-generated voice/video for social engineering attacks. A CFO "calls" to authorize a wire transfer—but it's a deepfake. Losses in the tens of millions already documented.

AI Risks: Systemic and Concentration

AI in finance creates new types of systemic risk—when many firms rely on similar AI models, their correlated behavior can amplify market shocks.

Herding Risk:

- Many firms use similar models
- Trained on same data
- Similar predictions → same trades
- Amplifies market moves
- Flash crashes

Historical Example:
August 2007 quant meltdown—many hedge funds used similar AI-driven factor models. When one large fund began selling to cover losses, it triggered a chain reaction: others held the same positions and also had to sell, causing billions in losses within days. This demonstrated how model "herding" can amplify market shocks.

Concentration Risk:

- Few dominant AI providers
- Cloud infrastructure concentration
- Model monoculture
- Single points of failure

Regulatory Response:

- EU AI Act (risk classification)
- Fed/OCC model risk guidance
- Explainability requirements

AI Claims: A Critical Evaluation Framework

When you hear "AI will disrupt X in finance," ask:

1. **What's the benchmark?** Is AI actually better than existing methods, or just newer?
2. **Is there enough data?** ML needs large, clean datasets. Rare events (like financial crises) are hard to learn from.
3. **Is the environment stationary?** Financial markets change; models trained on past data may fail.
4. **What are the feedback loops?** If everyone uses the same AI, does it still work?
5. **What's the adversarial dynamic?** Are there incentives to game the model?
6. **What's the regulatory status?** Can this legally be deployed?

Healthy Skepticism

Most AI finance claims are overhyped. Demand evidence: backtests (with proper methodology), out-of-sample performance, real-world deployments.

Hands-On: Robo-Advisor Simulation (NB13)

What you'll do in the notebook:

1. Load historical return data for major asset classes
2. Implement mean-variance optimization
3. Map risk tolerance scores to portfolio allocations
4. Visualize efficient frontier
5. Simulate portfolio performance under different risk profiles
6. Add constraints (position limits, ESG filters)

Key Learning Objectives

- Understand how robo-advisors translate preferences to portfolios
- See the math behind automated investing
- Recognize the assumptions and limitations

Notebook: NB13_Robo_Advisor.ipynb

NB13: Implementation Details

Data Used:

- US Stocks (SPY proxy)
- International Stocks (VEU)
- US Bonds (AGG)
- International Bonds (BWX)
- Real Estate (VNQ)
- Commodities (GSG)

Time Period:

10+ years of daily returns for robust estimation

Key Code Steps:

1. Calculate expected returns for each asset
2. Estimate how assets move together
3. Find the best portfolio mix
4. Generate the efficient frontier chart
5. Map risk scores to allocations
6. Backtest (test on historical data)

Extensions:

- Add position limits
- Implement rebalancing
- Add tax-loss harvesting

Discussion: AI vs. Human Advisors

Robo-Advisor Advantages

- Lower costs (0.25% vs. 1%+)
- No minimum investment barriers
- Consistent, disciplined execution
- 24/7 availability
- No emotional decision-making
- Tax optimization at scale

Human Advisor Advantages

- Complex financial planning
- Behavioral coaching
- Life event guidance
- Estate and tax planning
- Relationship and trust
- Flexibility for unique situations

The Hybrid Model

Leading platforms now offer "hybrid" services combining algorithmic management with human advice access (e.g., Vanguard Personal Advisor Services at 0.30%).

Discussion: Market-Wide Implications of AI

Efficiency Arguments:

- Better price discovery
- Increased liquidity
- Lower transaction costs
- Faster information incorporation
- Reduced human error

Stability Concerns:

- Flash crashes
- Correlated strategies
- Feedback loops
- Reduced diversity of views
- Unknown failure modes

Discussion Questions

1. Does AI make markets more or less stable?
2. Should there be "speed limits" on algorithmic trading?
3. Who is liable when AI systems cause market disruptions?

Application: Career Implications

Jobs at Risk of Automation:

- Basic financial analysis
- Routine trading operations
- Standard compliance checking
- Simple customer service
- Manual data entry

Growing Demand:

- FinTech product managers
- Compliance analysts who understand AI
- Business strategists evaluating AI claims
- Risk officers assessing algorithmic systems
- AI ethics specialists

Key Skills for the Future

- Combination of finance domain knowledge + technical skills
- Understanding of AI capabilities AND limitations
- Ability to explain complex models to stakeholders
- Critical thinking about AI claims

Application: Regulatory Landscape for AI in Finance

Current Frameworks:

- **US**: SEC guidance on robo-advisors; Fed SR 11-7 on model risk
- **EU**: MiFID II suitability; GDPR data rights; EU AI Act
- **UK**: FCA guidance on AI/ML
- **Global**: FSB reports on AI/ML risks

Key Requirements:

- Suitability assessments
- Disclosure of algorithmic use
- Explainability for adverse actions
- Model risk management
- Fair lending compliance
- Data protection

EU AI Act Classification

Financial AI applications (credit scoring, insurance pricing) are classified as “high-risk” requiring conformity assessments, transparency, and human oversight.

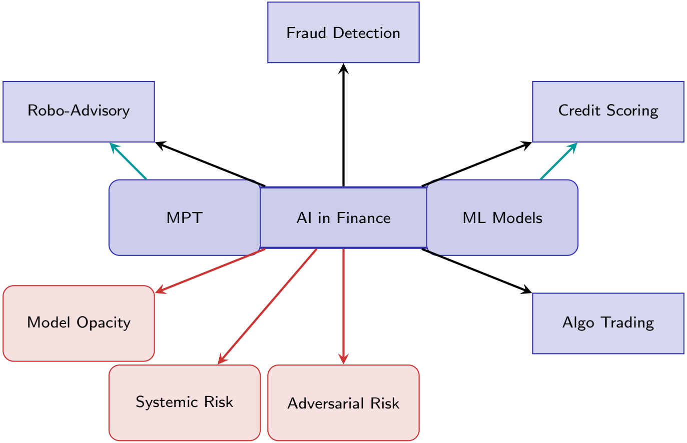
Executive Summary

Key Takeaways from Topic 6.2

1. **AI applications** span robo-advisory, fraud detection, credit scoring, and algorithmic trading—each with distinct benefits and risks
2. **Robo-advisors** democratize investment management through Modern Portfolio Theory, reducing fees from 1%+ to 0.25%
3. **ML in credit scoring** can expand access but raises fairness and explainability concerns
4. **Algorithmic trading** dominates markets but most strategies fail to beat benchmarks
5. **Key risks** include model opacity, adversarial attacks, systemic herding, and concentration
6. **Critical evaluation** is essential—most AI claims are overhyped

Bottom Line: AI is transforming finance, but with clear limitations and risks that require careful management.

Concept Map: AI and Digital Finance



Fraud Detection

Robo-Advisory

Credit Scoring

MPT

AI in Finance

ML Models

Model Opacity

Algo Trading

Systemic Risk

Adversarial Risk

Key Terms (1/2)

Robo-Advisor Automated platform providing digital investment management using algorithms to construct and rebalance portfolios

Modern Portfolio Theory (MPT) Framework for constructing portfolios to maximize return for a given risk level through diversification

Mean-Variance Optimization Mathematical approach to find optimal portfolio weights by balancing expected return against variance

Efficient Frontier Set of optimal portfolios offering highest return for each level of risk

Sharpe Ratio A scorecard measuring how much extra return you earn per unit of risk – higher is better

Tax-Loss Harvesting Selling losing investments to realize losses for tax benefits while maintaining market exposure

Key Terms (2/2)

Wash Sale Rule US IRS rule prohibiting repurchase of substantially identical securities within 30 days of selling at a loss. Other countries have similar concepts (e.g., “bed and breakfasting” rules in the UK, *Verlustverrechnung* rules in Germany)

Alternative Data Non-traditional data sources (satellite imagery, social media, web traffic) used for financial predictions

Model Opacity Inability to explain why complex ML models make specific predictions (“black box” problem)

Adversarial Attack Deliberately crafted inputs designed to fool ML models

Herding Risk Systemic risk from many institutions using similar AI models and making correlated decisions

SHAP/LIME Tools that explain *why* an AI made a specific decision – like showing which factors most influenced a loan approval or denial

Common Misconceptions

Misconception 1

“AI trading systems consistently beat the market”

Reality: Most fail after accounting for transaction costs, fees, and survivorship bias. Markets are highly efficient.

Misconception 2

“More data always means better predictions”

Reality: Data quality matters more than quantity. Noisy, biased, or non-stationary data can harm models.

Misconception 3

“AI removes human bias from finance”

Reality: AI can amplify historical biases present in training data, potentially creating systematic discrimination.

Misconception 4

“Robo-advisors are only for small investors”

Reality: Sophisticated investors increasingly use algorithmic tools; the technology scales to any portfolio size.

Self-Assessment Questions (1/2)

Question 1 (From Quiz 6.2, Q2)

What is the typical annual fee range for robo-advisors compared to traditional advisors?

A. Robo: 2-3%, Traditional: 4-5%

B. Robo: 0.15-0.50%, Traditional: 1.00-2.00%

C. Robo: 5-10%, Traditional: 1-2%

D. Both charge the same: 1-2%

Question 2 (From Quiz 6.2, Q8)

In the Sharpe ratio formula $(r_p - r_f)/\sigma_p$, what does r_f typically represent?

A. The inflation rate

B. The expected market return

C. The risk-free rate, such as the Treasury bill rate

D. The firm's cost of capital

Self-Assessment Questions (2/2)

Question 3 (From Quiz 6.2, Q17)

Why is the “efficient frontier” important for robo-advisors?

A. It guarantees maximum returns with zero risk

B. It identifies portfolios that offer the best return for each level of risk

C. It eliminates the need for diversification

D. It predicts future stock prices accurately

Self-Assessment Questions (2/2)

Question 3 (From Quiz 6.2, Q17)

Why is the “efficient frontier” important for robo-advisors?

A. It guarantees maximum returns with zero risk

B. It identifies portfolios that offer the best return for each level of risk

C. It eliminates the need for diversification

D. It predicts future stock prices accurately

Answers

- Question 1: **B** – Robo-advisors charge 0.15-0.50%, significantly less than traditional 1-2%
- Question 2: **C** – The risk-free rate (typically T-bills) is the baseline return with no risk
- Question 3: **B** – The efficient frontier shows the optimal risk-return tradeoff; portfolios below it are suboptimal

What's Next: Topic 6.3 – Synthesis Framework

Preview of T6.3:

- Building your digital finance worldview
- The Innovation Scorecard framework
- Six questions for any innovation
- Applying multiple analytical lenses
- Capstone exercise (NB14)

Connection to Today:

- AI as one technology to evaluate
- Framework applies to AI claims
- Integration with regulatory lens
- Critical evaluation skills

Preparation

Think about how you would evaluate a new AI-powered financial service:

- What problem does it solve?
- What are the risks and tradeoffs?
- Who benefits and who bears costs?

Resources for Further Learning

Academic/Technical:

- Markowitz, H. (1952) "Portfolio Selection"
- Black & Litterman (1992) "Global Portfolio Optimization"
- Advances in Financial Machine Learning (Lopez de Prado)
- Journal of Financial Economics

Industry Reports:

- FSB: "Artificial Intelligence and Machine Learning in Financial Services"
- BIS: "Big Tech in Finance"
- Federal Reserve: SR 11-7 (Model Risk Management)

Practical/News:

- Risk.net (AI in finance coverage)
- The Block, CoinDesk (AI+crypto)
- a16z crypto blog
- MIT Technology Review

Tools and Platforms:

- Python: PyPortfolioOpt, cvxpy
- Betterment, Wealthfront (try them)
- QuantConnect (algo trading platform, free tier available)

Questions?

Topic 6.2: AI and Digital Finance

Machine Learning Transforms Financial Services

Next: Topic 6.3 – Building Your Digital Finance Worldview

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Optional Deep Dive: Mathematical Details (1/2)

This appendix contains the formal mathematics behind the robo-advisory concepts discussed earlier. This material is optional and not required for the course.

Mean-Variance Optimization (Markowitz, 1952):

$$\min_{w} \frac{1}{2} w^T \Sigma w - \lambda \mu^T w \tag{1}$$

- w = portfolio weights (how much of each investment to hold)
- Σ = covariance matrix of returns (measures how investments move relative to each other)
- μ = expected returns vector (estimated future gains for each investment)
- λ = risk aversion parameter (higher = more cautious investor)

In words: Find the portfolio weights w that minimize risk (first term) while maximizing expected return (second term), with λ controlling the trade-off.

Optional Deep Dive: Mathematical Details (2/2)

Portfolio Expected Return:

$$r_p = w^T \mu = \sum_{i=1}^{n} w_i \mu_i \tag{2}$$

The portfolio return is the weighted average of individual asset returns.

Portfolio Variance:

$$\sigma_p^2 = w^T \Sigma w \tag{3}$$

Portfolio risk depends on both individual asset risks and how assets move together (correlations). When correlations are below 1, diversification reduces overall risk.

The Sharpe Ratio:

$$\mathrm{SR} = \frac{r_p - r_f}{\sigma_p} \tag{4}$$

Measures excess return (above the risk-free rate r_f, e.g. government bonds) per unit of risk. Higher Sharpe ratios indicate better risk-adjusted performance.