Topic 3.1: Cryptographic Building Blocks

Hashing, Keys, and Digital Signatures

Joerg Osterrieder

Digital Finance

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Learning Objectives

By the end of this topic, you will be able to:

1. **Explain** what cryptographic hash functions are and their essential properties
2. **Describe** how public-key cryptography enables secure identity without a central authority
3. **Understand** how digital signatures provide authentication, integrity, and non-repudiation
4. **Connect** these three primitives to how blockchain systems establish trust
5. **Apply** these concepts in hands-on exercises using Python

Why This Matters

Cryptography is the foundation that allows blockchain to replace institutional trust with mathematical proof.

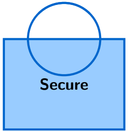
What is Cryptography?

Definition: The science of secure communication in the presence of adversaries.

Everyday Examples:

- **HTTPS** – Secure websites (the padlock icon)
- **WhatsApp** – End-to-end encrypted messages
- **ATM PINs** – Encrypted card data
- **Passwords** – Stored as hashes, not plaintext

Key Insight: You already use cryptography daily!



Cryptography protects your data

In This Topic:
We focus on *what* cryptographic tools guarantee, not the complex math behind them.

The Trust Problem: Why Cryptography Matters for Finance

Traditional Trust Model

- Banks verify your identity
- Courts enforce contracts
- Governments back currency
- Intermediaries everywhere

Problem: Single points of failure

Cryptographic Trust Model

- Mathematics verifies identity
- Code enforces agreements
- Network backs value
- Trust is distributed

Solution: Trust through verification

Key Insight: Cryptography lets us replace “trust me” with “verify this”

Three Cryptographic Primitives

Hash Functions
Digital fingerprints

Public-Key Crypto
Identity without authority

Digital Signat
Unforgeable pro

Integrity
Data hasn't changed

Identity
You are who you claim

Non-repudiatio
You can't deny sig

Focus: What these tools *guarantee*, not how the math works

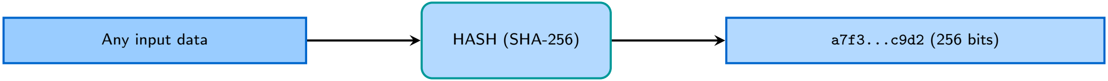
These three primitives are the atoms of decentralized trust

Think of it like: A fingerprint machine for data

- Any person → unique fingerprint (fixed size)
- Any data → unique hash (fixed size: 256 bits)

Definition: A hash function takes *any* input and produces a fixed-size output called a **hash** (or digest).

Hash Functions: Digital Fingerprints



Deterministic: Same input → same output, always

One-way: Cannot reverse to find input

Collision-resistant: Practically impossible to find two inputs with same hash

Avalanche effect: Tiny change → completely different output

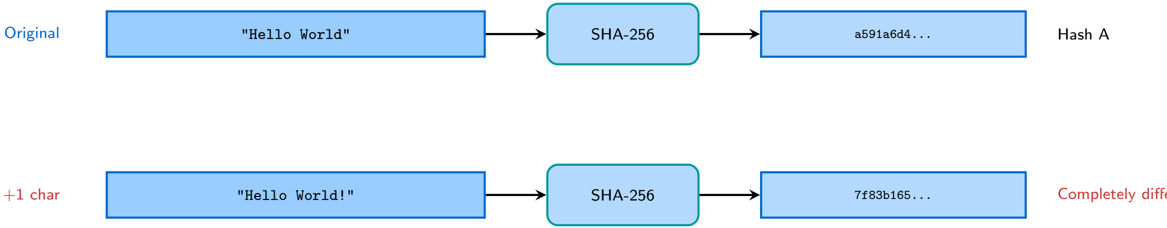
Five Essential Properties of Hash Functions

1. **Deterministic**
   Hash("Bitcoin") today = Hash("Bitcoin") tomorrow = Hash("Bitcoin") forever
2. **Fixed Output Size**
   SHA-256 always produces 256 bits (64 hex characters), regardless of input size
3. **One-Way (Preimage Resistant)**
   Given a hash, you cannot compute the original input
4. **Collision Resistant**
   Practically impossible to find two different inputs with the same hash
5. **Avalanche Effect**
   Change 1 bit of input → approximately 50% of output bits change

Why These Matter

Together, these properties make hashes reliable "digital fingerprints" for data integrity.

The Avalanche Effect Visualized



Why this matters for blockchain:

- Change one transaction → entire block hash changes
- This change cascades through all subsequent blocks
- Tampering becomes immediately detectable

How Secure is SHA-256?

SHA-256 produces 2^{256} possible outputs.

How big is that number?

$$2^{256} \approx 10^{77}$$

This is close to the estimated number of
atoms in the observable universe ($\approx 10^{80}$)

To find a collision by brute force:

- Even trying 1 billion hashes per second
- Using every computer on Earth
- Would take longer than the age of the universe

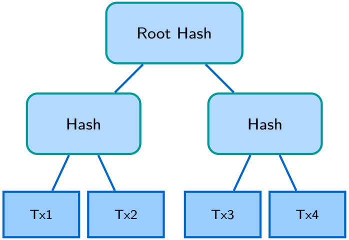
Practical Security

SHA-256 is considered cryptographically secure for all practical purposes.

What Hash Functions Guarantee

Use Cases in Blockchain

- **Data integrity:** Verify nothing changed
- **Block linking:** Each block contains hash of previous
- **Transaction IDs:** Unique identifier for every transaction
- **Mining puzzles:** Finding hashes with specific properties
- **Merkle trees:** Efficiently verify large data sets



Merkle Tree

Like a family tree where you can verify any member by checking their parents

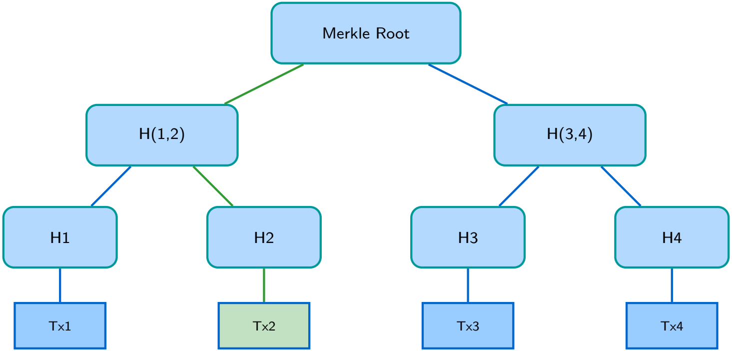
Verify any transaction with $O(\log n)$ hashes

The Guarantee

If two hashes match, the data is identical (with overwhelming probability).

Problem: How do you verify one transaction out of thousands without downloading everything?

Like verifying one page of a book by checking chapter summaries – you don't need to read every page



To verify Tx2: Only need H1, H(3,4), and Merkle Root (3 items, not 4 transactions)

With 1 million transactions, you only need about 20 hashes to verify any single one

The Problem: How can two strangers communicate securely without meeting first to exchange a secret password?

Traditional (Symmetric) Encryption:

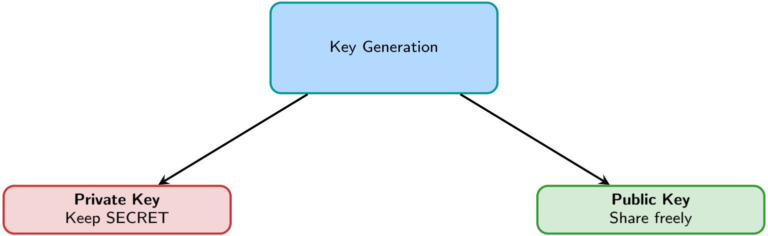
- Same key encrypts and decrypts
- Problem: How do you safely share the key?

Public-Key (Asymmetric) Solution:

- Two mathematically related keys
- One key encrypts $\rightarrow$ only the other decrypts
- Share one key publicly, keep the other secret

Breakthrough (1976): Diffie, Hellman, and later RSA showed this was mathematically possible

Public-Key Cryptography: Identity Without Authority



Mathematical relationship: Public key is *derived* from private key

Easy: Private $\rightarrow$ Public   Impossible: Public $\rightarrow$ Private

Public-Key Cryptography: How It Works

To send encrypted message to Bob:

1. Alice encrypts with Bob's public key
2. Only Bob can decrypt with his private key

Key insight: Only the person with the private key can decrypt

The Mathematical Intuition (No Math Required!)

Analogy: The Padlock and Key

Traditional Lock

- Same key locks and unlocks
- Must physically give key to someone
- If copied, security is broken

Public-Key "Lock"

- Public key = open padlock (share freely)
- Private key = the only key that opens it
- Anyone can lock (encrypt), only you can open (decrypt)

Message $+$ Public Key $\rightarrow$ Encrypted $+$ Private Key $\rightarrow$

From Public Key to Blockchain Address

What You Control

- Private key = your identity
- Whoever has it controls the funds
- Lose it = lose everything
- Share it = share everything

What You Share

- Address = your "account number"
- Safe to share publicly
- Used to receive payments
- Cannot derive private key from it

"Not your keys, not your coins" – a fundamental principle of crypto

Why don't blockchains use RSA?

Two different mathematical approaches to creating unforgeable digital signatures:

- **RSA:** Based on factoring large prime numbers
- **ECDSA (Elliptic Curve Digital Signature Algorithm):** Based on elliptic curve mathematics

Property	**RSA**	**ECDSA (secp256k1)**
Key Size (equiv. security)	3072 bits	256 bits
Signature Size	Large	Small
Speed	Slower	Faster
Used by Bitcoin/Ethereum	No	Yes

secp256k1: The specific mathematical curve Bitcoin and Ethereum use for signatures

- Provides 256-bit security (more combinations than atoms in the observable universe) with smaller keys
- Smaller signatures = lower transaction fees
- Well-studied and standardized

In NB05, we use RSA for simplicity, but real blockchains use ECDSA with secp256k1

What is a Digital Signature?

Definition: A mathematical scheme that proves:

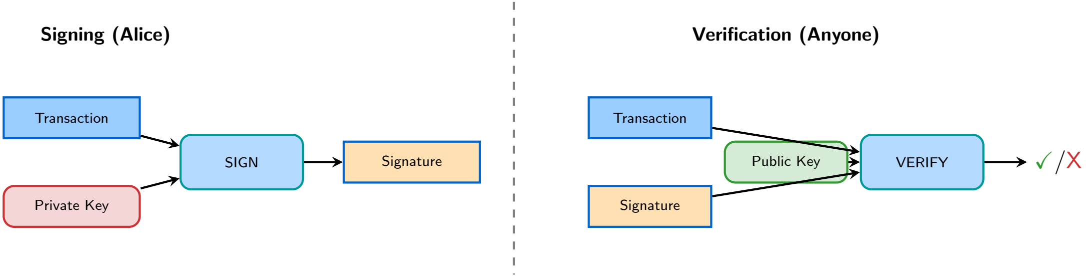
1. **Who** created or approved a message (authentication)
2. That the message **hasn't been changed** (integrity)
3. The signer **can't deny** signing (non-repudiation)

Physical vs Digital Signatures:

Property	Handwritten	Digital
Can be forged?	Yes	Practically no
Tied to document?	No	Yes (any change invalidates)
Remotely verifiable?	No	Yes
Provably unique?	No	Yes

Key Point: Digital signatures are *stronger* than handwritten ones!

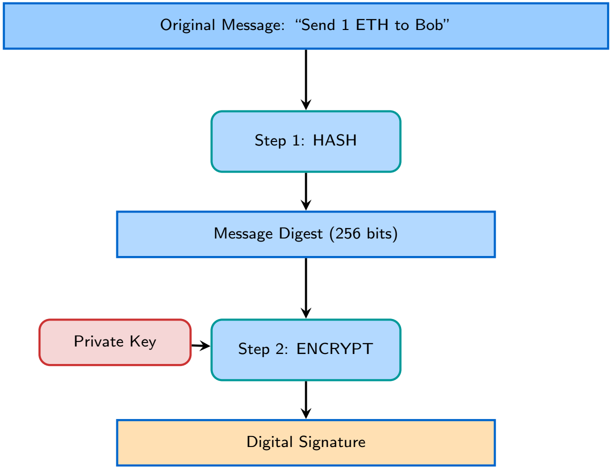
Digital Signatures: Unforgeable Proof



What Digital Signatures Guarantee:

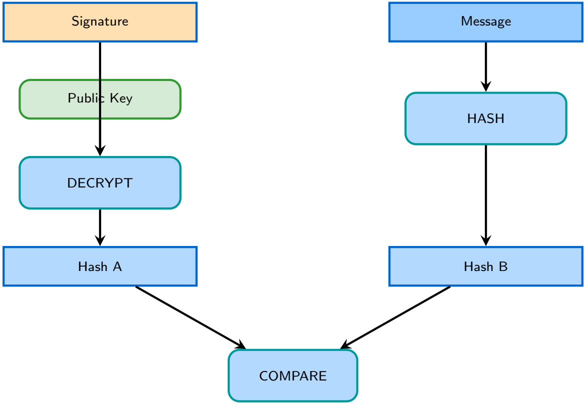
- **Authentication:** Only private key holder could create this signature
- **Integrity:** The message hasn't been altered
- **Non-repudiation:** Signer cannot deny having signed

The Signing Process: Step by Step



Why hash first?

- Messages can be any size; hashes are always fixed (256 bits)
- Encrypting a small hash is much faster than encrypting a large message



Hash A = Hash B?

✓Valid X Invalid

If hashes match: Signature is valid – message is authentic and unaltered

Digital Signatures in Blockchain Transactions

Every blockchain transaction includes a digital signature proving authorization:

Field	**Value**
From	0x7a2b...9f3c
To	0x9c4d...e8f2
Value	1.5 ETH
Nonce	42
Gas Limit	21,000
Gas Price	50 gwei
Signature	*v, r, s (proves authorization)*

How it works in practice:

1. Alice creates transaction: "Send 1.5 ETH to Bob"
2. Alice signs with her private key
3. Network verifies signature using Alice's public key
4. If valid: transaction is processed. If invalid: rejected

Summary: The Cryptographic Trust Stack

Blockchain = All three combined at scale

Digital Signatures → Authorization & Non-repudiation

Public-Key Crypto → Identity & Ownership

Hash Functions → Data Integrity

Key Takeaway

Cryptography transforms trust from "believe the institution" to "verify the math." No bank, government, or third party needed – just mathematics.

Hands-On Exercise: NB05 Cryptographic Operations

What you'll do in the Colab notebook:

1. **Hash Functions**
   - Compute SHA-256 hashes of different inputs
   - Observe the avalanche effect firsthand
   - Verify that same input = same output
2. **Key Generation**
   - Generate a public-private key pair
   - Derive a wallet address from the public key
   - Understand the one-way relationship
3. **Digital Signatures**
   - Sign a message with your private key
   - Verify the signature with the public key
   - See what happens when verification fails

Access: NB05 – Cryptographic Operations
No installation required (runs in browser)

Hashing in Python:

```python
import hashlib
message = "Hello, Blockchain!"
hash_result = hashlib.sha256(message.encode()).hexdigest()
print(hash_result)  # 64 hex characters
```

Creating a signature:

```python
# Sign with private key
signature = private_key.sign(
    message_hash,
    algorithm=hashes.SHA256()
)
```

Verifying a signature:

```python
# Verify with public key
public_key.verify(signature, message_hash)
# Raises exception if invalid!
```

Complete code and explanations in NB05 notebook

Discussion: Where Do You See These Concepts?

Think-Pair-Share: Where else are these cryptographic primitives used?

Hash Functions

- Password storage
- File integrity (checksums)
- Git version control
- Digital forensics
- Deduplication systems

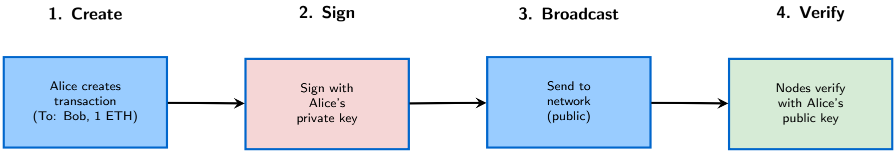
Digital Signatures

- Software updates
- Email (S/MIME, PGP)
- PDF documents
- Code signing
- SSL/TLS certificates

Discussion Questions

1. Why might a company hash passwords instead of encrypting them?
2. What happens to digital signatures if quantum computers become powerful?

Application: How a Blockchain Transaction Stays Secure



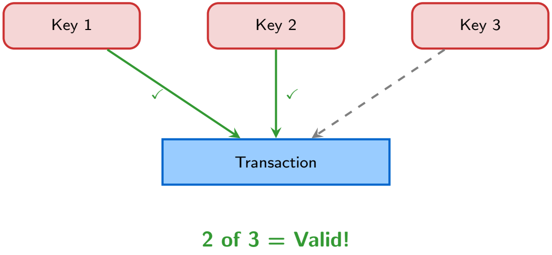
Security guarantees at each step:

- **Step 2:** Only Alice can sign (private key)
- **Step 3:** Transaction is public but tamper-evident (hash)
- **Step 4:** Anyone can verify Alice authorized it (public key)

Result: No one can forge, alter, or deny the transaction

Problem: What if one private key is stolen or lost?

Solution: Require multiple signatures (e.g., 2-of-3 multisig)



Use Cases:

- Corporate treasury (CFO + CEO approval)
- Family inheritance (multiple heirs)
- Exchange cold storage (security team)

Multi-sig combines multiple digital signatures for enhanced security

Application: Trust Implications for Finance

How cryptography changes financial trust:

Aspect	Traditional	Cryptographic
Identity verification	Bank/Government	Public key
Authorization	Signature card	Digital signature
Transaction integrity	Bank records	Hash chains
Dispute resolution	Courts	Mathematical proof
Account recovery	ID documents	Seed phrase

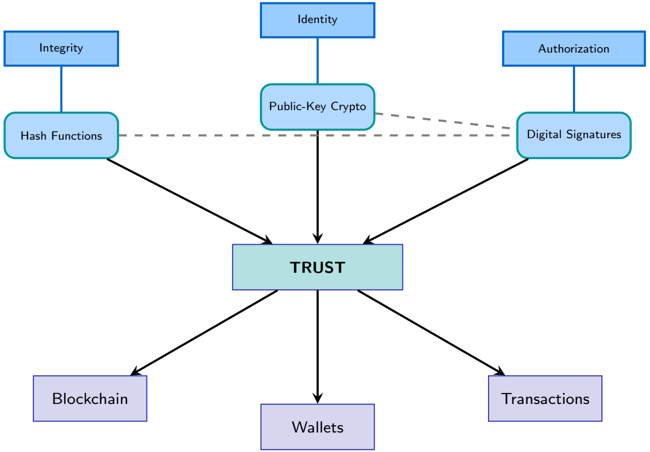
Trade-off

Benefit: No intermediaries, censorship-resistant, 24/7 operation
Risk: "With great power comes great responsibility" – lose your keys, lose your funds

Executive Summary: 5 Key Takeaways

1. **Hash functions** create unique digital fingerprints – any change to data produces a completely different hash, enabling tamper detection
2. **Public-key cryptography** allows secure identity without central authorities – you control your identity through your private key
3. **Digital signatures** provide three guarantees: authentication (who signed), integrity (not altered), and non-repudiation (can't deny signing)
4. **These primitives combine** in blockchain to enable trustless transactions – math replaces institutional trust
5. **Security responsibility shifts** from institutions to individuals – "not your keys, not your coins"

Concept Map: How It All Connects



Key relationship: Digital signatures combine hash functions (for efficiency) with public-key crypto (for authentication)

Key Terms & Definitions (Part 1)

Hash Function A mathematical function that converts any input into a fixed-size output (digest). One-way and deterministic.

SHA-256 Secure Hash Algorithm producing 256-bit output. Used in Bitcoin and many blockchains.

Avalanche Effect Property where a tiny input change causes a dramatically different output hash.

Collision Resistance Property that makes it computationally infeasible to find two different inputs with the same hash.

Merkle Tree A tree structure where each leaf is a hash of data, and each non-leaf is a hash of its children. Enables efficient verification.

Key Terms & Definitions (Part 2)

Public Key The shareable part of a key pair, used to verify signatures or encrypt messages.

Private Key The secret part of a key pair, used to create signatures or decrypt messages. Must never be shared.

Digital Signature Cryptographic proof that a message was approved by the holder of a specific private key.

ECDSA Elliptic Curve Digital Signature Algorithm. Used by Bitcoin and Ethereum for smaller, faster signatures.

Non-repudiation The property that a signer cannot deny having signed a message, as only their private key could have created the signature.

Common Misconceptions: Myth vs Reality

Myth 1:
"Hashing encrypts data"
Reality:
Hashing is one-way – you cannot "decrypt" a hash. Encryption is reversible; hashing is not.

Myth 2:
"If my public key is exposed, I'm hacked"
Reality:
Public keys are *meant* to be public! Only exposure of your private key is dangerous.

Myth 3:
"Longer passwords = stronger hashes"
Reality:
Hash output size is fixed regardless of input. A 256-bit hash is 256 bits whether input is "a" or a novel.

Myth 4:
"Quantum computers will break all crypto"
Reality:
Some algorithms are vulnerable, but post-quantum cryptography already exists. Hash functions remain largely secure.

Self-Assessment: Test Your Understanding

Question 1: What is the primary purpose of a cryptographic hash function?

A. To encrypt data so it can be decrypted later
B. To create a fixed-size unique fingerprint of any input data
C. To generate random numbers for cryptographic operations
D. To compress large files into smaller ones

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Answer: B
A cryptographic hash function takes any input and produces a fixed-size output called a hash or digest. This serves as a unique "fingerprint" of the data. Unlike encryption, hashing is one-way and cannot be reversed.

Self-Assessment: Test Your Understanding

Question 2: What three properties does a digital signature provide?

A. Encryption, compression, and speed
B. Authentication, integrity, and non-repudiation
C. Confidentiality, availability, and scalability
D. Hashing, signing, and verification

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A digital signature provides: (1) **Authentication** – proves who signed, (2) **Integrity** – proves the message wasn't altered, and (3) **Non-repudiation** – the signer cannot deny having signed.

Question 3: What is the "birthday attack" in hash functions?

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Question 3: What is the "birthday attack" in hash functions?
Finding a collision requires only 2^{128} attempts for SHA-256 (not 2^{256}) due to probability theory – still astronomically large but worth knowing!

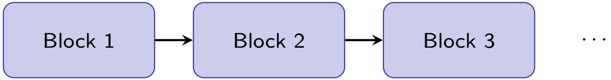
Now that you understand the building blocks, we'll see how they assemble:

Topics Covered:

- What is a blockchain?
- How blocks link together
- The block structure
- Consensus mechanisms
- The blockchain trilemma

You'll Learn:

- Why tampering is detectable
- How networks agree on truth
- Trade-offs in blockchain design
- Proof of Work vs Proof of Stake



Preview: The hash of each block is included in the next – creating an unbreakable chain

Resources for Further Learning

Hands-On:

- **NB05:** Cryptographic Operations (Colab notebook)
- Online SHA-256 calculator: `https://emn178.github.io/online-tools/sha256.html`

Reading:

- Antonopoulos, A. (2017). *Mastering Bitcoin*, Chapter 4: Keys & Addresses
- Narayanan et al. (2016). *Bitcoin and Cryptocurrency Technologies*, Chapter 1

Video:

- 3Blue1Brown: "How secure is 256-bit security?" (YouTube)
- Computerphile: "Hashing Algorithms and Security" (YouTube)

Interactive:

- Anders Brownworth's Blockchain Demo: `https://andersbrownworth.com/blockchain/`

Questions?

Topic 3.1: Cryptographic Building Blocks

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Next: Topic 3.2 – Blockchain Mechanics

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