

# Week 0: Introduction to Machine Learning & AI

## Foundations, Algorithms, and Modern Applications

Machine Learning for Smarter Innovation

BSc-Level Course Series

September 28, 2025



# Part 1: Machine Learning Foundations

## Theory, Definitions, and Core Concepts

## Tom Mitchell's Definition (1997)

A computer program learns from:

- **Experience**  $E$  with respect to
- **Task**  $T$  and
- **Performance measure**  $P$

if its performance at task  $T$ , as measured by  $P$ , improves with experience  $E$ .

### Example:

E: Email database

T: Classify spam vs non-spam

P: Classification accuracy

## Mathematical View

Learn function  $f : \mathcal{X} \rightarrow \mathcal{Y}$

Given training set:

$$\mathcal{D} = \{(x_i, y_i)\}_{i=1}^n$$

Find:

$$\hat{f} = \underset{f \in \mathcal{F}}{\arg\min} \sum_{i=1}^n L(y_i, f(x_i)) + \lambda R(f)$$

where:

- $L$ : Loss function
- $R$ : Regularization term
- $\lambda$ : Regularization strength

# Three Paradigms of Machine Learning

## Supervised



$$\{(x_i, y_i)\}_{i=1}^n \rightarrow \hat{f}$$

### Applications:

- Email spam detection
- Medical diagnosis
- Stock price prediction
- Image recognition

### Key Algorithms:

- Linear Regression
- Random Forest
- Neural Networks

## Unsupervised



$$\{x_i\}_{i=1}^n \rightarrow \text{Structure}$$

### Applications:

- Customer segmentation
- Anomaly detection
- Data compression
- Market basket analysis

### Key Algorithms:

- K-means clustering
- PCA
- Autoencoders

## Reinforcement



$$(s_t, a_t, r_t, s_{t+1}) \rightarrow \pi^*$$

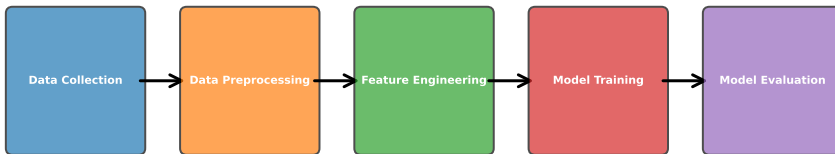
### Applications:

- Game playing (Chess, Go)
- Autonomous vehicles
- Robotics control
- Resource allocation

### Key Algorithms:

- Q-Learning
- Policy Gradients
- Actor-Critic

## Machine Learning Pipeline



*Iterative Process with Feedback Loops*

Data Collection → Preprocessing → Feature Engineering → Model Training → Validation → Deployment

## Mathematical Framework

For any learning algorithm, the expected error can be decomposed as:

$$\text{Error} = \text{Bias}^2 + \text{Variance} + \text{Noise}$$

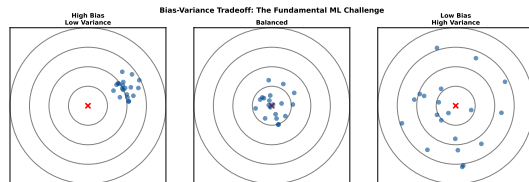
**Bias:** Error from oversimplifying assumptions

$$\text{Bias}[\hat{f}(x)] = E[\hat{f}(x)] - f(x)$$

**Variance:** Error from sensitivity to training data

$$\text{Var}[\hat{f}(x)] = E[(\hat{f}(x) - E[\hat{f}(x)])^2]$$

**Key Insight:** There's a fundamental tradeoff between bias and variance



### Model Complexity Examples:

- **High Bias:** Linear models on nonlinear data
- **Balanced:** Regularized models
- **High Variance:** Deep trees, k-NN with small k

## Traditional Programming

### Process:

- Write explicit rules
- Code logic step by step
- Handle edge cases manually
- Deterministic outputs

### Example: Email Classification

- IF contains "FREE" AND "LIMITED TIME"
- THEN classify as spam
- Requires manual rule updates

### Limitations:

- Rules become complex
- Hard to handle exceptions
- Doesn't adapt to new patterns

## Machine Learning

### Process:

- Provide example data
- Algorithm learns patterns
- Generalizes to new cases
- Probabilistic outputs

### Example: Email Classification

- Train on 10,000 labeled emails
- Learn complex word patterns
- Automatically adapts to new spam

### Advantages:

- Handles complex patterns
- Adapts to new data
- Discovers hidden relationships



## Why Split Data?

### Training Set (60%):

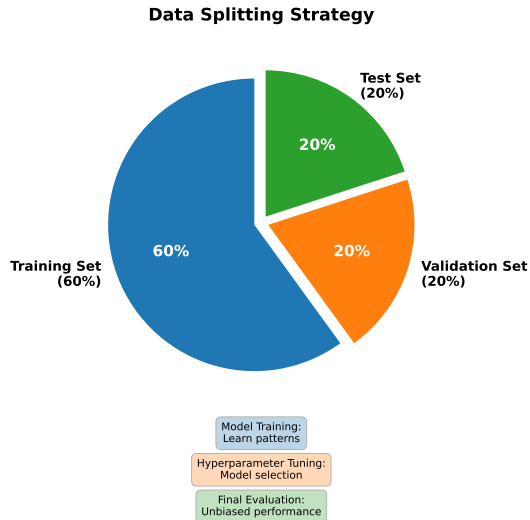
- Used to fit model parameters
- Algorithm learns from this data
- Larger is generally better

### Validation Set (20%):

- Used for hyperparameter tuning
- Model selection and comparison
- Prevents overfitting to training data

### Test Set (20%):

- Final unbiased evaluation
- Never seen during development
- Estimates real-world performance



## Cross-Validation:

## Classification Metrics

**Accuracy:**

$$\text{Accuracy} = \frac{\text{Correct Predictions}}{\text{Total Predictions}}$$

**Precision:**

$$\text{Precision} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Positives}}$$

**Recall (Sensitivity):**

$$\text{Recall} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Negatives}}$$

**F1-Score:**

$$F1 = 2 \cdot \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$

## Regression Metrics

**Mean Squared Error:**

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

**Root Mean Squared Error:**

$$RMSE = \sqrt{MSE}$$

**Mean Absolute Error:**

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$$

**R-squared:**

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

## Part 2: Supervised Learning Methods

### Prediction and Classification Algorithms

## Linear Regression Family

### Ordinary Least Squares:

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

$$\min_{\beta} \|y - X\beta\|_2^2$$

### Ridge Regression (L2):

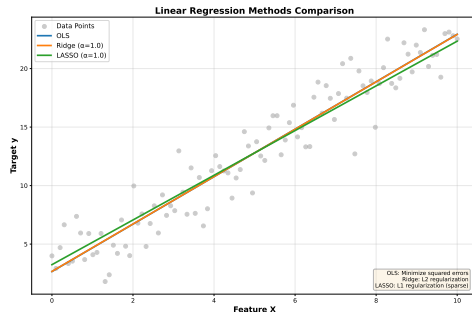
$$\hat{\beta}_{ridge} = (X^T X + \lambda I)^{-1} X^T y$$

$$\min_{\beta} \|y - X\beta\|_2^2 + \lambda \|\beta\|_2^2$$

### LASSO (L1):

$$\min_{\beta} \|y - X\beta\|_2^2 + \lambda \|\beta\|_1$$

No closed form - use coordinate descent



### Applications:

- House price prediction
- Sales forecasting
- Medical diagnosis
- Scientific modeling

### Elastic Net (Best of Both):

## Mathematical Framework

Logistic Function:

$$p(y = 1|x) = \frac{1}{1 + e^{-(\beta_0 + \beta^T x)}}$$

Odds Ratio:

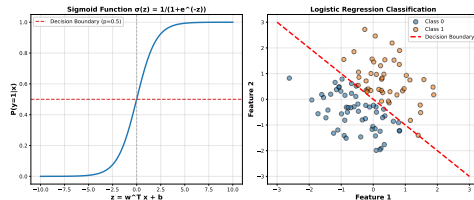
$$\frac{p}{1 - p} = e^{\beta_0 + \beta^T x}$$

Log-Likelihood:

$$\ell(\beta) = \sum_{i=1}^n [y_i \log p_i + (1 - y_i) \log(1 - p_i)]$$

No closed form solution

- Use gradient descent
- Newton-Raphson method
- Iteratively reweighted least squares



Decision Boundary:

$$\beta_0 + \beta^T x = 0$$

Applications:

- Email spam detection
- Medical diagnosis
- Marketing response
- Credit approval

## Optimization Problem

Primal Form:

$$\min_{w,b} \frac{1}{2} \|w\|^2$$

$$\text{s.t. } y_i(w^T x_i + b) \geq 1, \forall i$$

Dual Form:

$$\max_{\alpha} \sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j x_i^T x_j$$

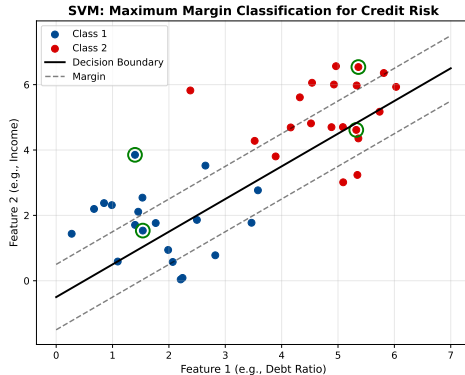
$$\text{s.t. } \alpha_i \geq 0, \sum_i \alpha_i y_i = 0$$

Kernel Trick:

$$K(x_i, x_j) = \phi(x_i)^T \phi(x_j)$$

Common kernels:

- RBF:  $K(x, z) = e^{-\gamma \|x - z\|^2}$



**Key Concepts:**

- **Support Vectors:** Data points on margin
- **Maximum Margin:** Optimal separating hyperplane
- **Kernel Trick:** Nonlinear classification

**Advantages:**

## Tree Construction

### Splitting Criterion:

*Gini Impurity:*

$$G = \sum_{k=1}^K p_k(1 - p_k)$$

*Information Gain:*

$$IG = H(\text{parent}) - \sum_j \frac{n_j}{n} H(\text{child}_j)$$

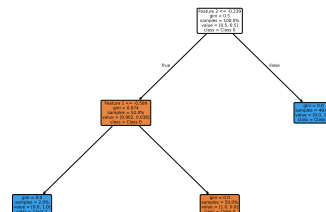
*Entropy:*

$$H = - \sum_{k=1}^K p_k \log_2 p_k$$

### CART Algorithm:

- 1 Find best split across all features
- 2 Partition data based on split

Decision Tree Structure (Max Depth = 3)



### Advantages:

- Highly interpretable
- Handles mixed data types
- No assumptions about distribution
- Captures interactions automatically

### Disadvantages:

- Prone to overfitting
- Unstable (small data changes)

## Ensemble Method

### Bootstrap Aggregating (Bagging):

- 1 Draw  $B$  bootstrap samples
- 2 Train tree on each sample
- 3 Average predictions (regression)
- 4 Vote on class (classification)

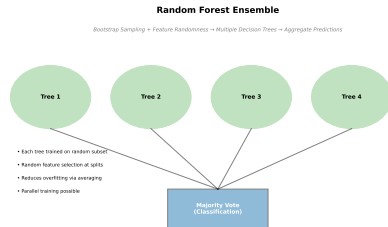
### Random Feature Selection:

- At each split, randomly select  $m$  features
- Typically  $m = \sqrt{p}$  for classification
- Typically  $m = p/3$  for regression

### Final Prediction:

$$\hat{y} = \frac{1}{B} \sum_{b=1}^B T_b(x)$$

**Key insight:** Averaging reduces variance while maintaining low bias



### Advantages:

- Reduces overfitting
- Handles missing values
- Provides feature importance
- Works well out-of-the-box

### Feature Importance:

- Mean decrease in impurity
- Permutation importance
- Out-of-bag importance



## Boosting Algorithm

### Sequential Model Building:

$$F_m(x) = F_{m-1}(x) + \gamma_m h_m(x)$$

where  $h_m$  is trained on residuals:

$$r_{im} = -\frac{\partial L(y_i, F_{m-1}(x_i))}{\partial F_{m-1}(x_i)}$$

### XGBoost Objective:

$$\mathcal{L} = \sum_i l(y_i, \hat{y}_i) + \sum_k \Omega(f_k)$$

$$\Omega(f) = \gamma T + \frac{1}{2} \lambda \|w\|^2$$

### Key Features:

- Regularization prevents overfitting
- Second-order derivatives
- Handles missing values

### Gradient Boosting Process

$$F_M(x) = f_0(x) + \sum_{n=1}^M \alpha_n h_n(x)$$



1. Start with initial prediction  $f_0$
2. Compute residuals (errors)
3. Train weak learner on residuals
4. Update model with weighted learner
5. Iterate M times

### Popular Implementations:

- **XGBoost:** Extreme Gradient Boosting
- **LightGBM:** Fast gradient boosting
- **CatBoost:** Categorical features

### Applications:

- Kaggle competitions
- Click-through rate prediction
- Risk modeling
- Ranking problems

## Non-parametric Method

### Algorithm:

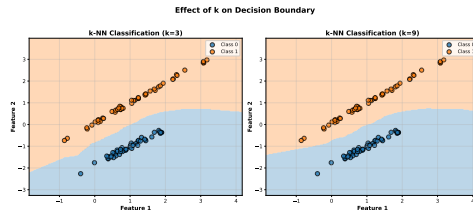
- 1 Store all training data
- 2 For new point, find k nearest neighbors
- 3 Classification: majority vote
- 4 Regression: average target values

### Distance Metrics:

- Euclidean:  $d(x, z) = \sqrt{\sum_i (x_i - z_i)^2}$
- Manhattan:  $d(x, z) = \sum_i |x_i - z_i|$
- Minkowski:  $d(x, z) = (\sum_i |x_i - z_i|^p)^{1/p}$

### Choosing k:

- Small k: Low bias, high variance
- Large k: High bias, low variance
- Use cross-validation to select



### Advantages:

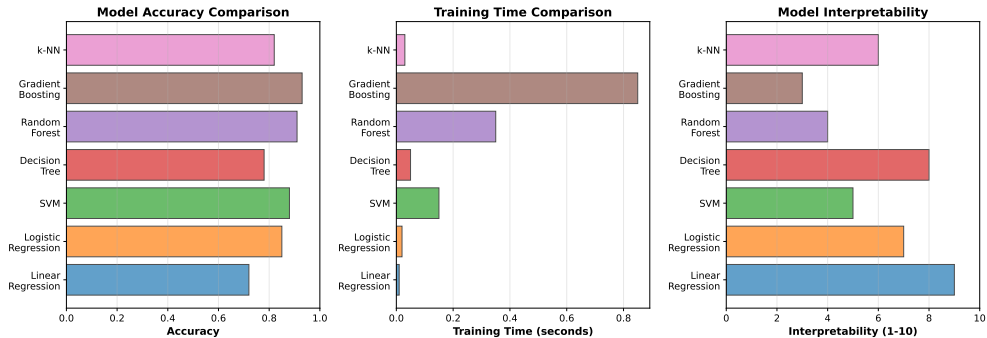
- Simple to understand and implement
- No assumptions about data distribution
- Adapts to local patterns
- Works well with small datasets

### Disadvantages:

- Computationally expensive for large datasets
- Sensitive to irrelevant features
- Curse of dimensionality
- Sensitive to data scaling

# Supervised Learning: Algorithm Comparison

Supervised Learning Algorithm Comparison



Algorithm	Interpretability	Training Speed	Prediction Speed	Accuracy
Linear Regression	High	Fast	Fast	Low-Medium
Logistic Regression	High	Fast	Fast	Medium
Decision Tree	High	Medium	Fast	Medium
Random Forest	Medium	Slow	Medium	High
XGBoost	Low	Slow	Medium	Very High
SVM	Low	Slow	Fast	High
k-NN	Medium	Fast	Slow	Medium-High

## Part 3: Unsupervised Learning Methods

### Discovering Hidden Structure in Data

## Algorithm

Objective Function:

$$J = \sum_{i=1}^n \sum_{k=1}^K w_{ik} \|x_i - \mu_k\|^2$$

K-means Algorithm:

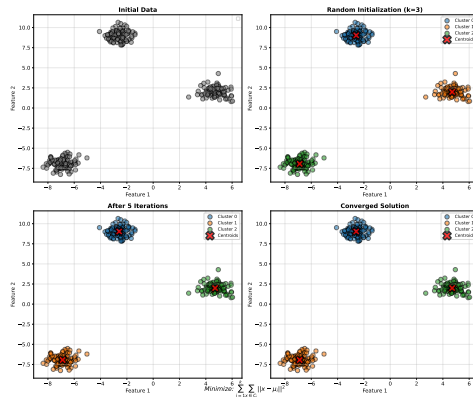
- 1 Initialize K cluster centers randomly
- 2 Assign each point to nearest center
- 3 Update centers to mean of assigned points
- 4 Repeat until convergence

Update Rules:

$$w_{ik} = \begin{cases} 1 & \text{if } k = j \text{ } \|x_i - \mu_j\|^2 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_k = \frac{\sum_{i=1}^n w_{ik} x_i}{\sum_{i=1}^n w_{ik}}$$

K-Means Clustering Process



Choosing K:

- **Elbow Method:** Plot within-cluster sum of squares
- **Silhouette Analysis:** Measure cluster separation
- **Gap Statistic:** Compare to random data

## Agglomerative Approach

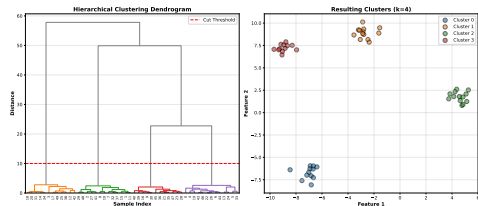
### Algorithm:

- 1 Start with each point as its own cluster
- 2 Merge closest pair of clusters
- 3 Repeat until single cluster remains
- 4 Cut dendrogram at desired level

### Linkage Criteria:

- **Single:**  $\min(d(a, b))$  where  $a \in A, b \in B$
- **Complete:**  $\max(d(a, b))$  where  $a \in A, b \in B$
- **Average:**  $\frac{1}{|A||B|} \sum_{a \in A} \sum_{b \in B} d(a, b)$
- **Ward:** Minimize within-cluster variance

**Time Complexity:**  $O(n^3)$  for naive implementation



### Advantages:

- No need to specify number of clusters
- Produces hierarchy of clusters
- Deterministic results
- Works with any distance metric

### Disadvantages:

- Computationally expensive
- Sensitive to outliers
- Difficult to handle large datasets

## Density-Based Approach

### Key Concepts:

- **Core Point:**  $\geq \text{minPts}$  neighbors within  $\epsilon$
- **Border Point:** In neighborhood of core point
- **Noise Point:** Neither core nor border

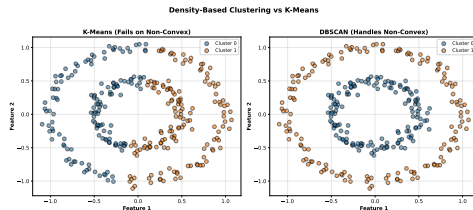
### Algorithm:

- 1 For each unvisited point
- 2 If core point, start new cluster
- 3 Add all density-reachable points
- 4 Mark non-core points as noise

### Parameters:

- $\epsilon$ : Neighborhood radius
- minPts: Minimum points for core

$$\text{Density} = \frac{\text{Points in } \epsilon\text{-neighborhood}}{|\epsilon\text{-neighborhood}|}$$



### Advantages:

- Finds arbitrary-shaped clusters
- Automatically determines cluster count
- Robust to outliers
- Identifies noise points

### Applications:

- Anomaly detection
- Image segmentation
- Fraud detection
- Social network analysis

## Mathematical Framework

**Objective:** Find directions of maximum variance

**Covariance Matrix:**

$$C = \frac{1}{n-1} X^T X$$

**Eigendecomposition:**

$$C = V \Lambda V^T$$

where  $V$  contains eigenvectors (principal components) and  $\Lambda$  contains eigenvalues.

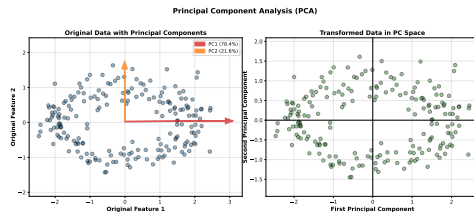
**Dimensionality Reduction:**

$$Z = XW$$

where  $W$  contains the first  $k$  principal components.

**Variance Explained:**

$$\text{Explained Variance} = \frac{\sum_{i=1}^k \lambda_i}{\sum_{i=1}^p \lambda_i}$$



**Steps:**

- 1 Standardize the data
- 2 Compute covariance matrix
- 3 Find eigenvalues and eigenvectors
- 4 Sort by eigenvalue magnitude
- 5 Select top  $k$  components
- 6 Transform data

**Applications:**

- Data visualization
- Noise reduction
- Feature extraction



## Architecture

Encoder:

$$z = f(Wx + b)$$

Decoder:

$$\hat{x} = g(W'z + b')$$

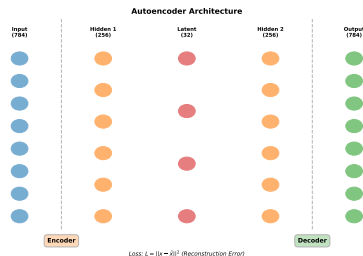
Objective:

$$\min_{W, W'} ||x - \hat{x}||^2$$

Types of Autoencoders:

- **Vanilla:** Basic encoder-decoder
- **Denoising:** Add noise to input
- **Sparse:** Encourage sparse representations
- **Variational:** Probabilistic latent space

**Bottleneck Layer:** Forces compression and learning of important features



Advantages over PCA:

- Nonlinear transformations
- Better reconstruction for complex data
- Can learn hierarchical features
- Flexible architecture

Applications:

- Image denoising
- Anomaly detection
- Data compression

## t-SNE

**t-Distributed Stochastic Neighbor Embedding**  
High-dimensional similarities:

$$p_{j|i} = \frac{\exp(-||x_i - x_j||^2/2\sigma_i^2)}{\sum_{k \neq i} \exp(-||x_i - x_k||^2/2\sigma_i^2)}$$

Low-dimensional similarities:

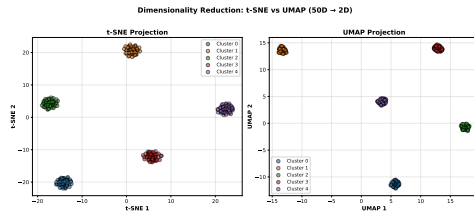
$$q_{ij} = \frac{(1 + ||y_i - y_j||^2)^{-1}}{\sum_{k \neq i} (1 + ||y_i - y_k||^2)^{-1}}$$

**Objective:** Minimize KL divergence

$$C = \sum_i KL(P_i || Q_i)$$

**Key Features:**

- Preserves local structure
- Heavy-tailed distribution in low-dim
- Stochastic optimization



## UMAP (Uniform Manifold Approximation)

- Faster than t-SNE
- Preserves global structure better
- Consistent results across runs
- Can embed to higher dimensions

**When to Use:**

- **t-SNE:** Detailed local clustering
- **UMAP:** Balance of local and global
- **PCA:** Linear structure, fast

## Internal Metrics

**Silhouette Score:**

$$s(i) = \frac{b(i) - a(i)}{\max(a(i), b(i))}$$

where  $a(i)$  = avg distance within cluster,  $b(i)$  = avg distance to nearest cluster

**Calinski-Harabasz Index:**

$$CH = \frac{SS_B / (k - 1)}{SS_W / (n - k)}$$

**Davies-Bouldin Index:**

$$DB = \frac{1}{k} \sum_{i=1}^k \max_{j \neq i} \frac{\sigma_i + \sigma_j}{d(c_i, c_j)}$$

**Inertia (Within-cluster sum of squares):**

$$WCSS = \sum_{i=1}^k \sum_{x \in C_i} \|x - \mu_i\|^2$$

## External Metrics

**Adjusted Rand Index:**

$$ARI = \frac{RI - E[RI]}{\max(RI) - E[RI]}$$

**Normalized Mutual Information:**

$$NMI = \frac{MI(U, V)}{\sqrt{H(U)H(V)}}$$

**Homogeneity and Completeness:**

- Homogeneity: Each cluster contains only one class
- Completeness: All members of class in same cluster

**V-measure:** Harmonic mean of homogeneity and completeness

**Note:** External metrics require ground truth labels

## Clustering

### Customer Segmentation:

- Group customers by behavior
- Targeted marketing campaigns
- Product recommendations

### Market Basket Analysis:

- Find product associations
- Store layout optimization
- Cross-selling opportunities

### Image Segmentation:

- Medical image analysis
- Computer vision
- Object recognition

## Dimensionality Reduction

### Data Visualization:

- Explore high-dimensional data
- Identify patterns and outliers
- Present insights to stakeholders

### Feature Engineering:

- Reduce computational cost
- Remove noise and redundancy
- Improve model performance

### Compression:

- Image and audio compression
- Efficient data storage
- Fast data transmission

## Anomaly Detection

### Fraud Detection:

- Credit card transactions
- Insurance claims
- Online account activity

### Network Security:

- Intrusion detection
- Malware identification
- Unusual traffic patterns

### Quality Control:

- Manufacturing defects
- System monitoring
- Predictive maintenance

Unsupervised learning reveals hidden patterns and structures in data without labeled examples

## Part 4: Neural Networks and Deep Learning

### From Perceptrons to Modern Architectures

## Mathematical Model

### Linear Combination:

$$z = w_1x_1 + w_2x_2 + \dots + w_nx_n + b$$

$$z = \mathbf{w}^T \mathbf{x} + b$$

### Activation Function:

$$y = \sigma(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

### Decision Boundary:

$$\mathbf{w}^T \mathbf{x} + b = 0$$

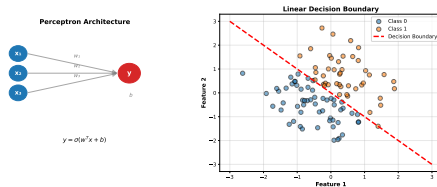
### Learning Rule (Perceptron Algorithm):

$$w_i := w_i + \eta(y - \hat{y})x_i$$

$$b := b + \eta(y - \hat{y})$$

where  $\eta$  is the learning rate

The Perceptron Model



### Perceptron Limitations:

- Can only learn linearly separable functions
- Cannot solve XOR problem
- Single decision boundary

### Historical Impact:

- First neural network model (1943)
- Led to “AI Winter” when limitations discovered
- Foundation for modern deep learning

## Architecture

### Forward Propagation:

$$z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

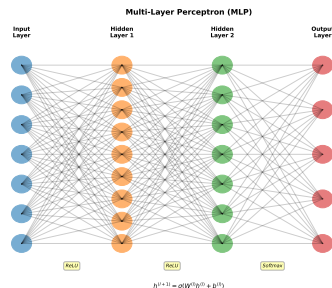
**Universal Approximation Theorem:** A neural network with:

- One hidden layer
- Finite number of neurons
- Non-linear activation function

can approximate any continuous function on a compact set to arbitrary accuracy.

**Key Insight:** Width vs depth tradeoff

- Wide shallow networks: Exponential width needed
- Deep narrow networks: Polynomial parameters



### Activation Functions:

- **Sigmoid:**  $\sigma(x) = \frac{1}{1+e^{-x}}$
- **Tanh:**  $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
- **ReLU:**  $\text{ReLU}(x) = \max(0, x)$
- **Leaky ReLU:**  $\text{LeakyReLU}(x) = \max(0.01x, x)$

## Algorithm

### Chain Rule Application:

$$\frac{\partial L}{\partial W^{[l]}} = \frac{\partial L}{\partial z^{[l]}} \frac{\partial z^{[l]}}{\partial W^{[l]}}$$

### Backward Pass:

$$\delta^{[l]} = \frac{\partial L}{\partial z^{[l]}}$$

$$\delta^{[l-1]} = (W^{[l]})^T \delta^{[l]} \odot g'(z^{[l-1]})$$

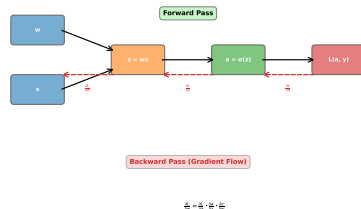
### Parameter Updates:

$$\frac{\partial L}{\partial W^{[l]}} = \delta^{[l]} (a^{[l-1]})^T$$

$$\frac{\partial L}{\partial b^{[l]}} = \delta^{[l]}$$

### Gradient Descent:

Backpropagation: Chain Rule in Action



### Computational Graph:

- Forward pass: Compute outputs
- Backward pass: Compute gradients
- Automatic differentiation
- Memory vs computation tradeoff

### Challenges:

- Vanishing gradients
- Exploding gradients



## Common Activations

**Sigmoid:**

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Range: (0, 1), smooth, vanishing gradients

**Tanh:**

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Range: (-1, 1), zero-centered, still vanishing gradients

**ReLU:**

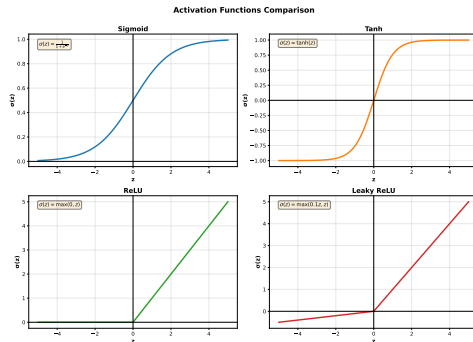
$$\text{ReLU}(x) = \max(0, x)$$

Simple, fast, sparse activations, dying ReLU problem

**Leaky ReLU:**

$$\text{LeakyReLU}(x) = \max(\alpha x, x)$$

Fixes dying ReLU,  $\alpha = 0.01$  typically



## Modern Activations:

- **ELU:**  $f(x) = \begin{cases} x & x > 0 \\ \alpha(e^x - 1) & x \leq 0 \end{cases}$
- **Swish:**  $f(x) = x \cdot \sigma(\beta x)$
- **GELU:**  $f(x) = x \cdot \Phi(x)$

## Choice Guidelines:

- Hidden layers: ReLU or variants

## Architecture Components

### Convolution Operation:

$$(I * K)_{ij} = \sum_m \sum_n I_{i+m, j+n} K_{m,n}$$

### Key Concepts:

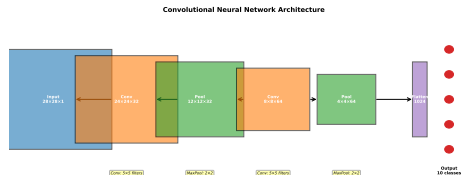
- **Local Connectivity:** Neurons connect to local regions
- **Parameter Sharing:** Same filter across all positions
- **Translation Invariance:** Features detected anywhere

### Typical CNN Architecture:

- 1 Convolution + ReLU
- 2 Pooling (max or average)
- 3 Repeat multiple times
- 4 Flatten and fully connected layers
- 5 Final classification layer

### Filter Parameters:

- Kernel size (3x3, 5x5, 7x7)
- Stride (step size)



### Pooling Operations:

- **Max Pooling:** Take maximum in region
- **Average Pooling:** Take average in region
- **Global Pooling:** Pool entire feature map

### Applications:

- Image classification
- Object detection
- Medical imaging
- Computer vision