Machine Learning in Finance: Theory & Applications

A Mathematical Foundation for Financial ML

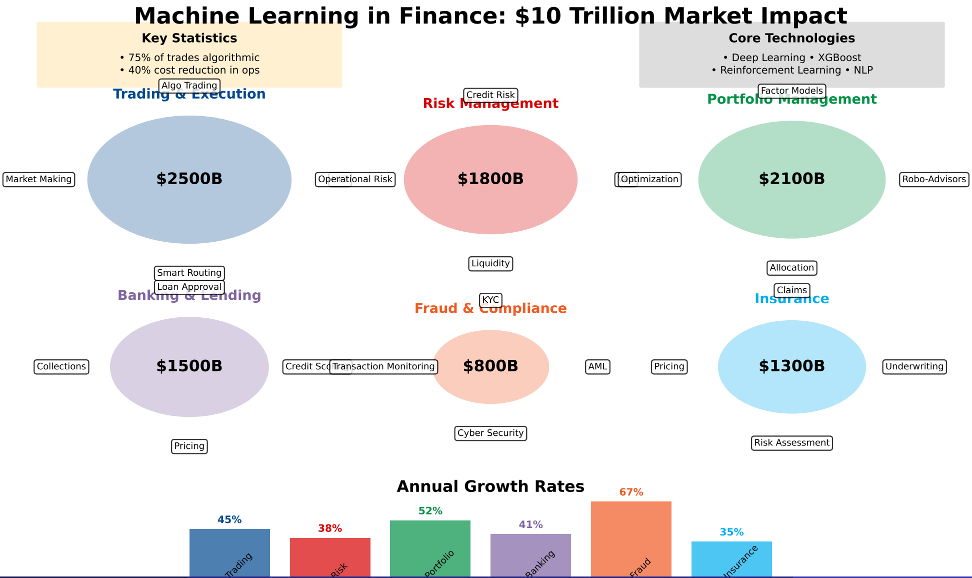
Advanced Course in Quantitative Finance

4-Hour Comprehensive Workshop

From Theory to Trading Floor

Part 1: Machine Learning Foundations

Theory, Mathematics, and Finance Applications


Machine Learning in Finance: $10 Trillion Market Impact
Key Statistics
• 75% of trades algorithmic
• 40% cost reduction in ops
Core Technologies
• Deep Learning • XGBoost
• Reinforcement Learning • NLP
Trading & Execution
$2500B
Algo Trading
Market Making
Smart Routing
Risk Management
$1800B
Credit Risk
Operational Risk
Liquidity
Portfolio Management
$2100B
Factor Models
Optimization
Robo-Advisors
Allocation
Banking & Lending
$1500B
Loan Approval
Collections
Credit Sc
Pricing
Fraud & Compliance
$800B
KYC
Transaction Monitoring
AML
Cyber Security
Insurance
$1300B
Claims
Pricing
Underwriting
Risk Assessment
Annual Growth Rates
45%
Trading
38%
Risk
52%
Portfolio
41%
Banking
67%
Fraud
35%
Insurance

Tom Mitchell's Definition (1997)

A computer program learns from:

- **Experience** E with respect to
- **Task** T and
- **Performance measure** P

if its performance at task T, as measured by P, improves with experience E.

Finance Example:
E: Historical stock prices
T: Predict tomorrow's return
P: Sharpe ratio of predictions

Mathematical View

Learn function $f : \mathcal{X} \to \mathcal{Y}$
Given training set:

$$\mathcal{D} = \{(x_i, y_i)\}_{i=1}^{n}$$

Find:

$$\hat{f} = \arg\min_{f \in \mathcal{F}} \sum_{i=1}^{n} L(y_i, f(x_i)) + \lambda R(f)$$

where:

- L: Loss function
- R: Regularization term
- λ: Regularization strength

Traditional Finance

Black-Scholes-Merton (1973)

$$C = S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2)$$

- Strong assumptions
- Closed-form solutions
- Model-driven
- Limited parameters
- Interpretable

Limitations:

- Assumes log-normal returns
- Constant volatility
- No transaction costs
- Perfect markets

Machine Learning

Neural Network Pricing

$$\hat{C} = NN(S_0, K, T, \sigma_{impl}, \text{Greeks}, \ldots)$$

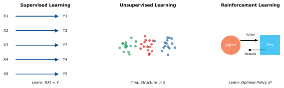
- Data-driven discovery
- Non-parametric
- Flexible architecture
- High-dimensional
- Accurate but opaque

Advantages:

- Captures market microstructure
- Adapts to regime changes
- Includes all observables
- Learns from anomalies

Three Paradigms of Machine Learning

Supervised



$$\{(x_i, y_i)\}_{i=1}^{n} \to \hat{f}$$

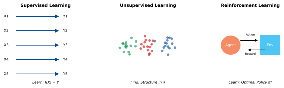
Finance Applications:

- Credit scoring
- Stock prediction
- Fraud detection
- Option pricing

Key Algorithms:

- Random Forest
- XGBoost
- Neural Networks

Unsupervised



$$\{x_i\}_{i=1}^{n} \to \text{Structure}$$

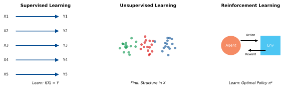
Finance Applications:

- Portfolio clustering
- Anomaly detection
- Risk factors
- Market regimes

Key Algorithms:

- K-means
- PCA/ICA
- Autoencoders

Reinforcement



$$(s_t, a_t, r_t, s_{t+1}) \to \pi^*$$

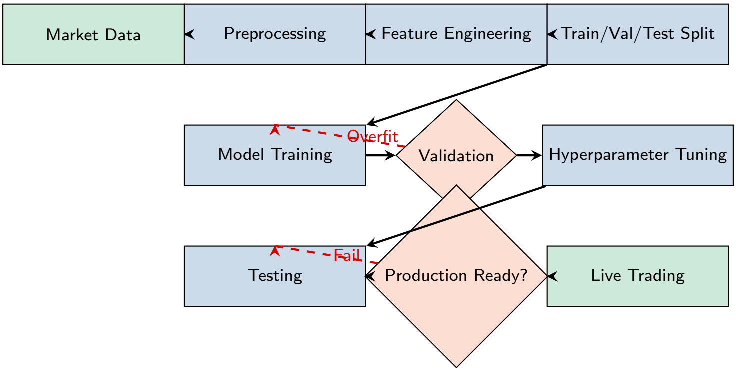
Finance Applications:

- Portfolio optimization
- Execution strategies
- Market making
- Hedging

Key Algorithms:

- Q-Learning
- PPO
- A3C

The Machine Learning Pipeline in Finance



Critical: 70/15/15 Split for Financial Time Series

Loss Functions: The Optimization Objective

Regression Losses

Mean Squared Error (MSE):

$$L_{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

Mean Absolute Error (MAE):

$$L_{MAE} = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

Huber Loss (Robust):

$$L_\delta(y, \hat{y}) = \begin{cases} \frac{1}{2}(y - \hat{y})^2 & \text{if } |y - \hat{y}| \leq \delta \\ \delta|y - \hat{y}| - \frac{1}{2}\delta^2 & \text{otherwise} \end{cases}$$

Quantile Loss (VaR):

$$L_\tau = \sum_i \rho_\tau(y_i - \hat{y}_i)$$

where $\rho_\tau(u) = u(\tau - \mathbb{1}_{u<0})$

Finance-Specific Losses

Sharpe Ratio Loss:

$$L_{Sharpe} = -\frac{\mathbb{E}[R_p]}{\sqrt{\text{Var}[R_p]}}$$

Maximum Drawdown:

$$L_{MDD} = \max_{t\in[0,T]}\left(1 - \frac{P_t}{\max_{s\in[0,t]} P_s}\right)$$

Directional Accuracy:

$$L_{DA} = -\frac{1}{n}\sum_{i=1}^{n}\mathbb{1}[\text{sign}(y_i) = \text{sign}(\hat{y}_i)]$$

Profit & Loss:

$$L_{PnL} = -\sum_{i=1}^{n}\hat{y}_i \cdot r_i$$

where r_i is actual return

Fundamental ML Theorem

For squared loss, the expected error decomposes as:

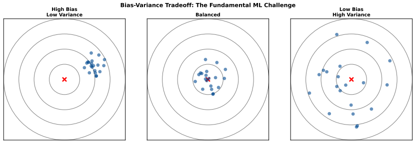
$$\mathbb{E}[(y - \hat{f}(x))^2] = \underbrace{\text{Bias}^2[\hat{f}(x)]}_{\text{Underfitting}} + \underbrace{\text{Var}[\hat{f}(x)]}_{\text{Overfitting}} + \underbrace{\sigma^2}_{\text{Irreducible}}$$

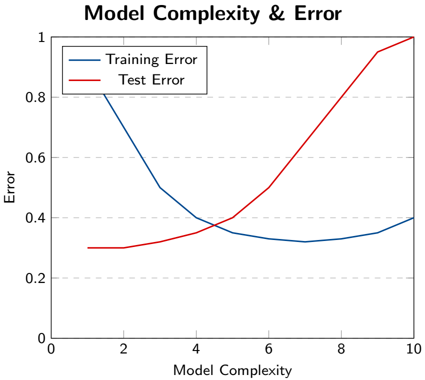
where:

$$\text{Bias}[\hat{f}(x)] = \mathbb{E}[\hat{f}(x)] - f(x)$$
$$\text{Var}[\hat{f}(x)] = \mathbb{E}[(\hat{f}(x) - \mathbb{E}[\hat{f}(x)])^2]$$
$$\sigma^2 = \text{Var}[\epsilon]$$

Finance Interpretation:

- **Bias**: Systematic pricing errors
- **Variance**: Model instability
- **Noise**: Market microstructure





Part 2: Statistical Learning Theory

Mathematical Foundations for Financial ML

Core Concepts

Random Variable:

$$X : \Omega \to \mathbb{R}$$

Expectation:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f_X(x) dx$$

Variance:

$$\mathrm{Var}[X] = \mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

Covariance:

$$\mathrm{Cov}(X, Y) = \mathbb{E}[(X - \mu_X)(Y - \mu_Y)]$$

Correlation:

$$\rho_{X,Y} = \frac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

Financial Applications

Return Distribution:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \sim \mathcal{N}(\mu, \sigma^2)$$

Portfolio Variance:

$$\sigma_p^2 = w^T \Sigma w$$

where Σ is covariance matrix
Value at Risk (95%):

$$\mathbb{P}(L > \mathrm{VaR}_{0.95}) = 0.05$$

Kelly Criterion:

$$f^* = \frac{p(b+1) - 1}{b}$$

where $f^* =$ optimal fraction to bet

Fundamental Formula

$$\boxed{P(H|D) = \frac{P(D|H) \cdot P(H)}{P(D)}}$$

Components:

- $P(H)$: Prior probability
- $P(D|H)$: Likelihood
- $P(H|D)$: Posterior probability
- $P(D)$: Evidence (normalizing constant)

Expanded Form:

$$P(H_i|D) = \frac{P(D|H_i)P(H_i)}{\sum_j P(D|H_j)P(H_j)}$$

Finance Example: Fraud Detection

Given:

- P(Fraud) $= 0.001$ (prior)
- P(Alert—Fraud) $= 0.95$
- P(Alert—Normal) $= 0.02$

Find: P(Fraud—Alert)

Solution:

$$P(F|A) = \frac{0.95 \times 0.001}{0.95 \times 0.001 + 0.02 \times 0.999}$$
$$= \frac{0.00095}{0.02093} = 0.045 = 4.5\%$$

Statistical Inference: From Sample to Population

Maximum Likelihood

Given data $\mathcal{D} = \{x_1, \ldots, x_n\}$
Likelihood Function:

$$\mathcal{L}(\theta|\mathcal{D}) = \prod_{i=1}^{n} p(x_i|\theta)$$

Log-Likelihood:

$$\ell(\theta) = \sum_{i=1}^{n} \log p(x_i|\theta)$$

MLE Estimate:

$$\hat{\theta}_{MLE} = \arg\max_{\theta} \ell(\theta)$$

Example: Normal Returns

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} r_i$$

Bayesian Inference

Posterior Distribution:

$$p(\theta|\mathcal{D}) \propto p(\mathcal{D}|\theta) \cdot p(\theta)$$

Conjugate Priors:

- Beta-Binomial
- Normal-Normal
- Gamma-Poisson

Black-Litterman Model: Combine market equilibrium (prior) with views (likelihood):

$$\mathbb{E}[R|\text{views}] = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1} \times \left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right]$$

Learning Guarantees

A learning algorithm is PAC if:
Given:

- Error parameter $\epsilon > 0$
- Confidence parameter $\delta > 0$
- Training sample size m

It outputs hypothesis h such that:

$$\mathbb{P}[\text{error}(h) \leq \epsilon] \geq 1 - \delta$$

Sample Complexity Bound:

$$m \geq \frac{1}{\epsilon}\left(\ln |\mathcal{H}| + \ln \frac{1}{\delta}\right)$$

where $|\mathcal{H}|$ is hypothesis space size

Finance Interpretation:

- ϵ: Maximum tolerable error (e.g., 5% mispricing)
- δ: Risk of failure (e.g., 1% chance)
- m: Required historical data

Example: Trading Strategy

- Want: 95% confidence
- Max error: 2%
- Hypothesis space: 1000 strategies
- Need: $m \geq \frac{1}{0.02}(\ln 1000 + \ln 100) \approx 689$ samples

Capacity of Learning Algorithms

Definition: The VC dimension of hypothesis class $\mathcal{H}$ is the maximum number of points that can be shattered (classified in all possible ways) by $\mathcal{H}$.

Examples:

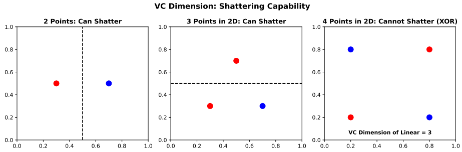
- Linear classifiers in $\mathbb{R}^d$: VC $= d + 1$
- Decision trees depth k: VC $\approx 2^k$
- Neural networks: VC $\propto$ #parameters

Generalization Bound: With probability $1 - \delta$:

$$R(h) \leq \hat{R}(h) + \sqrt{\frac{d(\ln(2m/d) + 1) + \ln(4/\delta)}{m}}$$

where:

- $R(h)$: True risk
- $\hat{R}(h)$: Empirical risk
- d: VC dimension



Trading Strategy Complexity:

- Simple MA crossover: VC ≈ 3
- Multi-factor model: VC ≈ 20
- Deep neural network: VC $\approx 10{,}000$

Warning: Higher VC = More overfitting risk!

Core Measures

Entropy (Uncertainty):

$$H(X) = -\sum_x p(x) \log_2 p(x)$$

Cross-Entropy (Loss):

$$H(p, q) = -\sum_x p(x) \log q(x)$$

KL Divergence:

$$D_{KL}(p||q) = \sum_x p(x) \log \frac{p(x)}{q(x)}$$

Mutual Information:

$$I(X; Y) = \sum_{x,y} p(x, y) \log \frac{p(x, y)}{p(x)p(y)}$$

Finance Applications

Portfolio Diversification:

$$H(\text{portfolio}) = -\sum_i w_i \log w_i$$

Maximum entropy = equal weights
Market Efficiency:

$$I(\text{Signal}; \text{Returns}) \approx 0$$

Efficient markets have low MI
Model Selection (AIC):

$$AIC = 2k - 2\ln(\mathcal{L})$$

where k = number of parameters
Active Information Ratio:

$$IR = \frac{\alpha}{\omega} = \sqrt{\text{Breadth} \times IC^2}$$

Part 3: Supervised Learning Methods

Prediction and Classification in Finance

Linear Regression Family

Ordinary Least Squares:

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

$$\min_{\beta} ||y - X\beta||_2^2$$

Ridge Regression (L2):

$$\hat{\beta}_{ridge} = (X^T X + \lambda I)^{-1} X^T y$$

$$\min_{\beta} ||y - X\beta||_2^2 + \lambda ||\beta||_2^2$$

LASSO (L1):

$$\min_{\beta} ||y - X\beta||_2^2 + \lambda ||\beta||_1$$

No closed form - use coordinate descent

Finance Applications:

Factor Models:

$$R_{i,t} = \alpha_i + \sum_{j=1}^{k} \beta_{i,j} F_{j,t} + \epsilon_{i,t}$$

Risk Premia Estimation: Fama-MacBeth regression

Elastic Net (Best of Both):

$$\min_{\beta} ||y - X\beta||_2^2 + \lambda_1 ||\beta||_1 + \lambda_2 ||\beta||_2^2$$

Useful for correlated predictors

Optimization Problem

Primal Form:

$$\min_{w,b} \frac{1}{2}||w||^2$$

$$\text{s.t. } y_i(w^T x_i + b) \geq 1, \forall i$$

Dual Form:

$$\max_{\alpha} \sum_i \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i \alpha_j y_i y_j x_i^T x_j$$

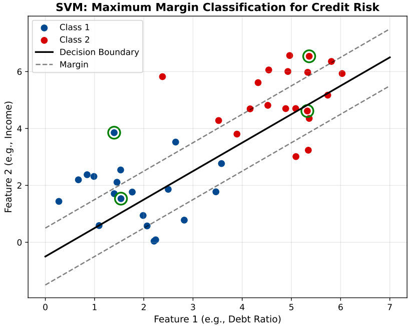
$$\text{s.t. } \alpha_i \geq 0, \sum_i \alpha_i y_i = 0$$

Kernel Trick:

$$K(x_i, x_j) = \phi(x_i)^T \phi(x_j)$$

Common kernels:

- RBF: $K(x, z) = e^{-\gamma||x-z||^2}$
- Polynomial: $K(x, z) = (x^T z + c)^d$



Credit Default Application:

- Features: Financial ratios
- Target: Default/No default
- Kernel: RBF for non-linearity
- Result: 92% accuracy

Tree-Based Methods: From Single Trees to Forests

Decision Trees

Splitting Criterion:
Gini Impurity:

$$G = \sum_{k=1}^{K} p_k(1 - p_k)$$

Information Gain:

$$IG = H(parent) - \sum_{j} \frac{n_j}{n} H(child_j)$$

CART Algorithm:

1. Find best split
2. Partition data
3. Recurse on children
4. Prune tree

Ensemble Methods

Random Forest:

- Bootstrap samples
- Random feature subsets
- Average predictions

Gradient Boosting:

$$F_m(x) = F_{m-1}(x) + \gamma_m h_m(x)$$

where h_m fits residuals
XGBoost Objective:

$$\mathcal{L} = \sum_{i} l(y_i, \hat{y}_i) + \sum_{k} \Omega(f_k)$$

$$\Omega(f) = \gamma T + \frac{1}{2}\lambda ||w||^2$$

Architecture and Learning

Forward Propagation:

$$z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

Backpropagation:

$$\frac{\partial \mathcal{L}}{\partial W^{[l]}} = \frac{\partial \mathcal{L}}{\partial z^{[l]}} \cdot a^{[l-1]T}$$

Update Rule (SGD):

$$W^{[l]} := W^{[l]} - \alpha \frac{\partial \mathcal{L}}{\partial W^{[l]}}$$

Activation Functions:

- ReLU: $\max(0, x)$
- Sigmoid: $\frac{1}{1+e^{-x}}$
- Tanh: $\frac{e^x - e^{-x}}{e^x + e^{-x}}$

Finance Applications:

Option Pricing NN:

- Input: S, K, T, r, σ
- Hidden: 3 layers, 100 units
- Output: Option price
- Loss: MSE vs Black-Scholes

Universal Approximation: Any continuous function on compact set can be approximated to arbitrary accuracy

Regularization:

- Dropout: $p = 0.5$
- L2 weight decay
- Early stopping

Part 4: Unsupervised Learning

Discovering Structure in Financial Data

Eigenportfolios and Risk Factors

Covariance Matrix Decomposition:

$$\Sigma = V\Lambda V^T$$

where V contains eigenvectors (factor loadings), Λ contains eigenvalues (factor variances).

Principal Components as Factors:

$$R_{i,t} = \sum_{j=1}^{k} \beta_{i,j} F_{j,t} + \epsilon_{i,t}$$

Variance Explained:

$$\text{Explained} = \frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{p} \lambda_i}$$

Typical Results:

- First 3 PCs explain 60-80% of equity return variance
- PC1: Market factor (beta)
- PC2: Size factor

Finance Applications:

Factor Models:

- Fama-French 3-factor model
- Statistical arbitrage
- Risk attribution
- Portfolio hedging

Advantages:

- Dimensionality reduction (1000 stocks -¿ 10 factors)
- Interpretable risk sources
- Computational efficiency
- Noise filtering

Limitations:

- Linear assumptions
- Unstable in crisis periods
- Ignores tail dependencies

Clustering-Based Portfolio Construction

Algorithm Steps:

1. Compute correlation matrix from returns
2. Hierarchical clustering of assets
3. Recursive bisection of dendrogram
4. Inverse-variance weighting within clusters

Weight Formula:

$$w_i = \frac{1/\sigma_i^2}{\sum_{j \in \text{cluster}} 1/\sigma_j^2}$$

Cluster Allocation:

$$w_{\text{cluster}} \propto \frac{1}{\sigma^2_{\text{cluster}}}$$

Key Properties:

- Stable to estimation error
- No matrix inversion required
- Accounts for hierarchical structure
- Robust to outliers

vs Traditional Markowitz:

Metric	MVO	HRP
Stability	Low	High
Turnover	High	Low
Out-sample Sharpe	0.4	0.6
Max Drawdown	-35%	-22%

Why HRP Works:

- Clustering identifies natural asset groups
- Hierarchical structure captures market organization
- Diversification across and within clusters
- Reduces estimation error impact

Applications:

- Multi-asset allocation
- Pension fund management
- Risk parity strategies

K-means for Market Regime Detection

Identifying Market States

Feature Engineering:

- Volatility: Rolling 20-day std
- Momentum: 60-day return
- Volume: Relative to 90-day average
- Correlation: Cross-asset correlation

Clustering Process:

$$\min_{\mu_1,\ldots,\mu_K} \sum_{k=1}^{K} \sum_{t \in C_k} ||x_t - \mu_k||^2$$

Typical Regimes Discovered:

1. **Bull Market:** Low vol, positive momentum, low correlation
2. **Bear Market:** High vol, negative momentum, high correlation
3. **Consolidation:** Medium vol, low momentum, medium correlation
4. **Crisis:** Extreme vol, negative momentum, correlation -¿ 1

Trading Applications:

Regime-Dependent Strategies:

- Bull: Long-only momentum
- Bear: Defensive hedging
- Consolidation: Mean reversion
- Crisis: Risk-off positioning

Performance Metrics:

Approach	Sharpe
Static strategy	0.5
Regime-aware	0.9

Implementation:

- sklearn.cluster.KMeans
- Weekly regime updates
- 3-5 regimes typical
- Combine with HMMs for transitions

Identifying Unusual Market Behavior

One-Class SVM Approach:

$$\min_{w,\rho} \frac{1}{2}||w||^2 - \rho + \frac{1}{\nu n}\sum_i \xi_i$$

$$\text{s.t. } w^T\phi(x_i) \geq \rho - \xi_i$$

Isolation Forest:

- Random feature selection
- Random split values
- Anomaly score: Average path length
- Faster than distance-based methods

Applications:

1. **Flash crashes:** Detect abnormal price movements
2. **Fat finger errors:** Identify erroneous orders
3. **Market manipulation:** Spot pump-and-dump schemes
4. **System failures:** Data feed anomalies

Real-World Example:
2010 Flash Crash Detection:

- Normal: SPY volatility 15-25%
- May 6, 2010: Volatility spike to 150%
- Anomaly score: 12 std deviations
- Duration: 5 minutes

Detection Metrics:

- Precision: 95% (few false alarms)
- Recall: 88% (catches most anomalies)
- Latency: ¡ 100ms detection time

ML Techniques:

- Autoencoders for reconstruction error
- LSTM for temporal anomalies
- Isolation Forest for multivariate
- Ensemble methods for robustness

Part 5: Risk & Portfolio Management

ML-Enhanced Risk Analytics

Risk Measurement Fundamentals

Definition:

$$\mathbb{P}(L > \text{VaR}_\alpha) = \alpha$$

α-quantile of loss distribution (typically $\alpha = 0.05$ or 0.01)

1. Historical VaR:

- Use empirical quantile from historical data
- $\text{VaR}_{0.05} =$ 5th percentile of returns
- Non-parametric, distribution-free
- Assumes past predicts future

2. Parametric VaR (Gaussian):

$$\text{VaR}_\alpha = \mu + \sigma\Phi^{-1}(\alpha)$$

where Φ^{-1} is inverse normal CDF

3. Monte Carlo VaR:

- Simulate 10,000+ scenarios
- Estimate empirical quantile
- Flexible for complex portfolios

ML-Enhanced VaR:

GARCH-based VaR:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Time-varying volatility improves accuracy

Quantile Regression:

$$\min_\beta \sum_i \rho_\alpha(r_i - x_i^T\beta)$$

where $\rho_\alpha(u) = u(\alpha - \mathbb{1}_{u<0})$

Directly estimates quantiles without distributional assumptions

Comparative Performance:

Method	Coverage	Violations
Historical	95.2%	48/1000
Gaussian	92.8%	72/1000
GARCH	94.8%	52/1000
Quantile RF	95.1%	49/1000

CVaR: Coherent Risk Measure

Definition:

$$\text{CVaR}_\alpha = \mathbb{E}[L|L > \text{VaR}_\alpha]$$

Average loss beyond VaR threshold

Mathematical Properties:

- **Sub-additivity:** $\text{CVaR}(X + Y) \leq \text{CVaR}(X) + \text{CVaR}(Y)$
- **Monotonicity:** $X \leq Y \Rightarrow \text{CVaR}(X) \leq \text{CVaR}(Y)$
- **Positive homogeneity:** $\text{CVaR}(\lambda X) = \lambda \text{CVaR}(X)$
- **Translation invariance:** $\text{CVaR}(X + c) = \text{CVaR}(X) + c$

Optimization Form:

$$\text{CVaR}_\alpha = \min_t \left\{ t + \frac{1}{\alpha} \mathbb{E}[\max(L - t, 0)] \right\}$$

Enables portfolio optimization with CVaR constraints

Why CVaR ¿ VaR:

Advantages:

- Captures tail risk severity
- Coherent risk measure
- Sub-additive (diversification)
- Optimization-friendly

VaR Limitations:

- Not sub-additive
- Ignores tail losses
- Multiple local minima

Example: Portfolio Loss

- $\text{VaR}_{0.05}$: -\$1M (95th percentile)
- $\text{CVaR}_{0.05}$: -\$2.5M (avg beyond VaR)
- Tail risk: CVaR captures 2.5x worse scenarios

Regulatory Adoption:

- Basel III prefers ES over VaR
- Better capital adequacy
- Captures tail dependencies

Scenario Generation and Analysis

Traditional Stress Testing:

- Historical scenarios (2008 crisis, COVID-19)
- Hypothetical shocks (+3 std vol, -20% market)
- Reverse stress testing (find breaking point)

ML-Enhanced Approaches:

1. VAE Scenario Generation:

$$z \sim \mathcal{N}(0, I), \quad x = \text{Decoder}(z)$$

Generate plausible but unseen market scenarios

2. GAN Stress Scenarios:

- Train on crisis periods
- Generate extreme but realistic scenarios
- Capture tail dependencies

3. Random Forest Sensitivity:

- Feature importance for risk drivers
- Partial dependence plots
- Interaction effects

Stress Testing Framework:

Steps:

1. Define risk factors (rates, spreads, FX)
2. Generate stressed scenarios
3. Revalue portfolio under stress
4. Measure impact on P&L, VaR, capital

Example Scenarios:

Scenario	Portfolio Loss
Rate +200bp	-\$15M
Equity -30%	-\$45M
Credit spread +150bp	-\$22M
Combined (ML)	-\$68M

ML Advantages:

- Discovers non-obvious correlations
- Generates tail scenarios
- Faster than Monte Carlo
- Learns from recent crises

Time-Varying Volatility Models

GARCH(1,1) Specification:

$$r_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t z_t, \quad z_t \sim \mathcal{N}(0,1)$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Parameters:

- ω: Baseline variance (long-run vol)
- α: ARCH effect (shock impact)
- β: GARCH effect (persistence)
- Stationarity: $\alpha + \beta < 1$

Extensions:

- **EGARCH:** Exponential (leverage effects)
- **GJR-GARCH:** Asymmetric (bad news ¿ good news)
- **DCC-GARCH:** Dynamic conditional correlation

Maximum Likelihood Estimation:

$$\ell(\theta) = \sum_{t=1}^{T}\left[-\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\sigma_t^2) - \frac{\epsilon_t^2}{2\sigma_t^2}\right]$$

ML Hybrid Approaches:

Neural Network GARCH:

$$\sigma_t^2 = NN(\epsilon_{t-1}, \ldots, \epsilon_{t-p}, \sigma_{t-1}, \ldots, \sigma_{t-q})$$

Learns nonlinear volatility dynamics

LSTM-GARCH:

- Captures long-memory effects
- Handles regime switches
- Outperforms standard GARCH

Forecast Performance (S&P 500):

Model	RMSE	MAE
GARCH(1,1)	2.8%	1.9%
EGARCH	2.6%	1.8%
LSTM-GARCH	2.2%	1.5%

Applications:

- Option pricing (implied vol)
- Risk management (VaR forecasting)
- Portfolio allocation (volatility timing)

Part 6: Algorithmic Trading

ML on the Trading Floor

Predictive Trading Signals

Signal Definition:

$$\alpha_{i,t} = \mathbb{E}[R_{i,t+1}|\mathcal{F}_t] - r_f$$

Excess return prediction conditional on information set

Feature Engineering:

- **Technical:** Moving averages, RSI, MACD, volume
- **Fundamental:** P/E, P/B, EPS growth, margins
- **Alternative:** Sentiment, news, satellite data
- **Market micro:** Bid-ask spread, order imbalance

ML Models for Alpha:

1. **Random Forest:** Feature interactions, non-linearity
2. **XGBoost:** State-of-art for tabular data
3. **LSTM:** Sequential dependencies, momentum
4. **Transformer:** Attention over time series

Signal Quality Metrics:

Information Coefficient (IC):

$$IC = \text{Corr}(\alpha_t, R_{t+1})$$

Information Ratio:

$$IR = \frac{\text{Mean}(\alpha)}{\text{Std}(\alpha)} = IC \times \sqrt{\text{Breadth}}$$

Typical Performance:

Metric	Value
IC (monthly)	0.05-0.08
IR (annual)	0.8-1.2
Sharpe	1.5-2.0
Hit rate	52-54%

Signal Combination:

$$\alpha_{\text{combined}} = \sum_i w_i \alpha_i$$

Ensemble of multiple signals improves robustness

Optimal Execution Algorithms

Minimizing Market Impact

Execution Cost Components:

$$\text{Total Cost} = \text{Spread} + \text{Impact} + \text{Timing Risk}$$

VWAP (Volume-Weighted Average Price):

$$\text{VWAP} = \frac{\sum_t P_t V_t}{\sum_t V_t}$$

Match historical volume profile

TWAP (Time-Weighted Average Price):

$$q_t = \frac{Q}{T}, \quad \text{for } t = 1, \ldots, T$$

Uniform time slicing

Implementation Shortfall (Almgren-Chriss):

$$\min_{\{q_t\}} \mathbb{E}[\text{Cost}] + \lambda \, \text{Var}[\text{Cost}]$$

Optimal Trade Schedule:

$$\sinh(\kappa(T - t))$$

ML-Enhanced Execution:

Deep Reinforcement Learning:

- State: Order book, inventory, time
- Action: Trade size at each time step
- Reward: -Cost - $\lambda \times$ Risk
- Policy: DDPG, PPO algorithms

Performance vs Benchmarks:

Strategy	Slippage (bps)
VWAP	8.2
TWAP	9.5
IS (Almgren-Chriss)	6.8
Deep RL	5.4

Advantages:

- Adapts to market conditions
- Learns optimal urgency
- Handles constraints dynamically

Providing Liquidity for Profit

Market Making Problem:

$$\max_{\delta^{bid},\delta^{ask}} \mathbb{E}[\text{PnL}] - \lambda\,\text{Var}[\text{PnL}]$$

subject to inventory constraints

Bid-Ask Spread Optimization:

$$\delta^{bid} = S_0 - P^{bid}, \quad \delta^{ask} = P^{ask} - S_0$$

Inventory Management:

$$q_t^* = -\frac{\alpha}{\gamma\sigma^2}$$

where α = alpha signal, γ = risk aversion

Order Book Forecasting:

$$P_{t+\Delta t} = f(\text{LOB}_t, \text{Trades}_t, \text{Features}_t)$$

ML Features:

- Order book imbalance
- Weighted mid-price

High-Frequency Strategies:

Statistical Arbitrage:

- Mean reversion at millisecond scale
- Cointegration pairs
- Lead-lag relationships
- Cross-venue arbitrage

ML Techniques:

- **LSTM:** Order flow prediction
- **CNN:** Limit order book patterns
- **RL:** Optimal quoting strategies
- **Transformers:** Multi-asset dependencies

Performance Metrics:

Metric	Value
Sharpe Ratio	5-15
Latency	¡1ms
Prediction R^2	0.15-0.25
Fill rate	85-95%

Part 7: Credit & Fraud

Protecting Financial Systems

Probability of Default Modeling

Traditional FICO Score:

- Payment history: 35%
- Amounts owed: 30%
- Length of history: 15%
- Credit mix: 10%
- New credit: 10%

Linear scorecard: 300-850 range

Logistic Regression Baseline:

$$PD = \frac{1}{1 + e^{-(\beta_0 + \beta^T X)}}$$

where X includes debt-to-income, payment history, utilization

ML Enhancements:

- **Random Forest:** Non-linear interactions
- **XGBoost:** Best predictive performance
- **Neural Networks:** Deep feature learning
- **Ensemble:** Combine multiple models

Model Comparison:

Model	AUC	Gini
FICO	0.72	0.44
Logistic	0.78	0.56
Random Forest	0.84	0.68
XGBoost	0.87	0.74
Deep NN	0.86	0.72

Alternative Data:

- Mobile phone usage patterns
- Social media activity
- Transaction history
- Geolocation data

Regulatory Considerations:

- Model explainability (GDPR, ECOA)
- Fairness constraints
- Adverse action notices
- Validation requirements

Expected Loss Framework

Expected Loss Decomposition:

$$\text{EL} = \text{EAD} \times \text{PD} \times \text{LGD}$$

- **EAD:** Exposure at Default
- **PD:** Probability of Default
- **LGD:** Loss Given Default (1 - Recovery Rate)

LGD Modeling Challenges:

- Bimodal distribution (full loss or high recovery)
- Limited default data
- Collateral valuation uncertainty
- Economic cycle dependencies

ML Approaches:

1. **Two-stage:** Logistic (default?) + Regression (LGD)
2. **Beta regression:** LGD $\in$ (0,1) naturally
3. **Quantile regression:** Estimate LGD distribution
4. **Survival analysis:** Time-to-recovery modeling

Feature Engineering:

Borrower Characteristics:

- Industry sector
- Firm size, leverage
- Credit rating history
- Management quality

Loan Characteristics:

- Seniority (secured vs unsecured)
- Collateral type and quality
- Loan-to-value ratio
- Covenants and protections

Macroeconomic Conditions:

- GDP growth, unemployment
- Interest rate environment
- Sectoral stress indicators

Typical LGD Estimates:

Loan Type	LGD
Senior secured	25-35%

Fraud Detection with ML

Real-Time Transaction Monitoring

Fraud Detection Pipeline:

1. Feature extraction from transaction
2. Real-time scoring (¡ 100ms)
3. Rule engine + ML model ensemble
4. Human review for high-risk cases

Key Features:

- **Transaction:** Amount, location, merchant, time
- **Behavioral:** Deviation from normal patterns
- **Historical:** Past fraud, dispute history
- **Network:** Graph-based (connected accounts)

Class Imbalance Problem:

- Fraud rate: 0.1-0.5% of transactions
- SMOTE oversampling for minority class
- Cost-sensitive learning (FP vs FN)
- Precision-recall optimization

ML Techniques:

Supervised Models:

- Random Forest: Feature interactions
- XGBoost: Best precision-recall
- Neural Networks: Deep patterns

Unsupervised Anomaly:

- Isolation Forest
- One-class SVM
- Autoencoders (reconstruction error)

Performance Metrics:

Model	Precision	Recall
Rules only	45%	62%
Logistic	68%	75%
XGBoost	82%	84%
Ensemble	85%	86%

Business Impact:

- Reduced false positives (better CX)
- Faster detection (lower losses)
- Adaptive to new fraud patterns

Part 8: Ethics & Regulation

Responsible AI in Finance

Model Risk Management: SR 11-7 Framework

Federal Reserve Supervisory Guidance

SR 11-7 Requirements:

1. **Model Development:** Documentation, assumptions, limitations
2. **Model Validation:** Independent review, backtesting
3. **Model Governance:** Approval, monitoring, controls

Three Lines of Defense:

- **1st Line:** Model developers and users
- **2nd Line:** Model risk management function
- **3rd Line:** Internal audit

Validation Components:

- Conceptual soundness review
- Ongoing monitoring (champion-challenger)
- Outcomes analysis (backtesting)
- Stress testing and scenario analysis

ML-Specific Challenges:

Black Box Problem:

- Deep networks lack interpretability
- Regulatory explainability requirements
- Need for model-agnostic explanations

Mitigation Strategies:

- SHAP values for feature importance
- LIME for local explanations
- Attention visualization
- Simplified surrogate models

Documentation Requirements:

- Training data provenance
- Hyperparameter selection rationale
- Performance metrics (in/out-sample)
- Limitations and failure modes
- Monitoring thresholds

Annual Review Mandate:

- Performance degradation checks

MiFID II, GDPR, and ECOA Compliance

Right to Explanation (GDPR Article 22):

- Automated decisions require human review
- Meaningful information about logic
- Consequences of processing
- Applies to EU customers

Equal Credit Opportunity Act (ECOA):

- Prohibits discrimination on protected attributes
- Adverse action notices required
- Must provide specific reasons for denial

Fairness Metrics:

$$\text{Demographic Parity: } P(\hat{Y} = 1|A = 0) = P(\hat{Y} = 1|A = 1)$$

$$\text{Equal Opportunity: } P(\hat{Y} = 1|Y = 1, A = 0) = P(\hat{Y} = 1|Y = 1, A = 1)$$

Trade-offs: Impossible to satisfy all fairness criteria simultaneously (Chouldechova impossibility theorem)

Explainability Techniques:

Global Explanations:

- Feature importance rankings
- Partial dependence plots
- Model-agnostic summaries

Local Explanations:

- SHAP values for individual predictions
- LIME: Local linear approximations
- Counterfactual examples

Bias Mitigation:

1. Pre-processing: Reweighting, resampling
2. In-processing: Fairness constraints
3. Post-processing: Threshold optimization

Model Cards:

- Intended use and limitations
- Training data characteristics
- Performance across subgroups
- Fairness metrics

MiFID II and Market Abuse Regulation

MiFID II Requirements:

- Algo trading registration and authorization
- Pre- and post-trade controls
- Kill switches and circuit breakers
- Audit trail for 5 years
- Annual self-assessment

Market Abuse Regulation (MAR):

- Prohibition of market manipulation
- Layering, spoofing illegal
- Insider trading detection
- Suspicious transaction reporting (STR)

Testing Requirements:

- Development environment testing
- Conformance testing (exchange rules)
- Stress testing under extreme conditions
- Kill switch functionality

ML Compliance Challenges:

Risks:

- Inadvertent market manipulation
- Flash crashes from feedback loops
- Unintended discrimination
- Model brittleness

Best Practices:

- Robust validation framework
- Continuous monitoring
- Human oversight requirements
- Graceful degradation

Regulatory Technology (RegTech):

- Automated compliance monitoring
- NLP for regulatory change detection
- ML for suspicious activity detection
- Real-time reporting automation

Future Trends:

- Global regulatory harmonization

Appendix: Mathematical Foundations

Proofs, Derivations, and Advanced Theory

From Stochastic Calculus to Option Pricing

Stock Price Dynamics:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

Ito's Lemma Application:

$$df = \left(\frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \right) dt + \sigma S \frac{\partial f}{\partial S} dW_t$$

Risk-Neutral Valuation:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV$$

Solution (European Call):

$$C = S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2)$$

where:

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

Karush-Kuhn-Tucker Conditions

For optimization problem:

$$\min_x f(x) \quad \text{s.t.} \quad g_i(x) \leq 0, \quad h_j(x) = 0$$

KKT conditions (necessary for optimality):

1. Stationarity: $\nabla f(x^*) + \sum_i \lambda_i \nabla g_i(x^*) + \sum_j \mu_j \nabla h_j(x^*) = 0$
2. Primal feasibility: $g_i(x^*) \leq 0, h_j(x^*) = 0$
3. Dual feasibility: $\lambda_i \geq 0$
4. Complementary slackness: $\lambda_i g_i(x^*) = 0$

Application: Portfolio Optimization with Constraints

Thank You

Questions & Discussion

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