Week 0d: Neural Networks

The Depth Challenge

Machine Learning for Smarter Innovation

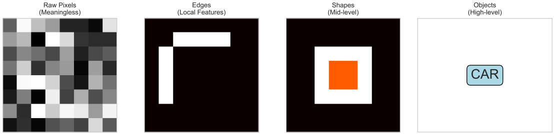
BSc Course - Theory Foundation

October 6, 2025

Course Outline

1. Image Recognition Needs Hierarchical Features

- Raw pixels are meaningless noise
- Vision builds up complexity:
  - Edges from pixel gradients
  - Shapes from edge combinations
  - Objects from shape patterns
- Traditional ML: Manual feature engineering
- Deep learning: Automatic feature hierarchy



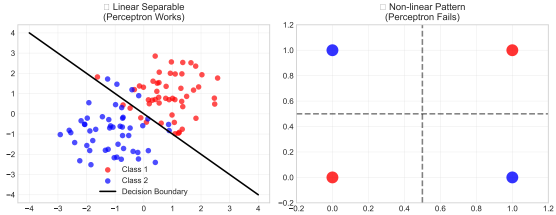
Neural networks must learn increasingly abstract representations

The Perceptron (1957)

$$y = \text{sign}(w_1 x_1 + w_2 x_2 + b) \quad (1)$$
$$= \text{sign}(\mathbf{w}^T \mathbf{x} + b) \quad (2)$$

Geometric Interpretation:

- Creates a linear decision boundary
- Hyperplane in n-dimensional space
- Cannot separate non-linear patterns



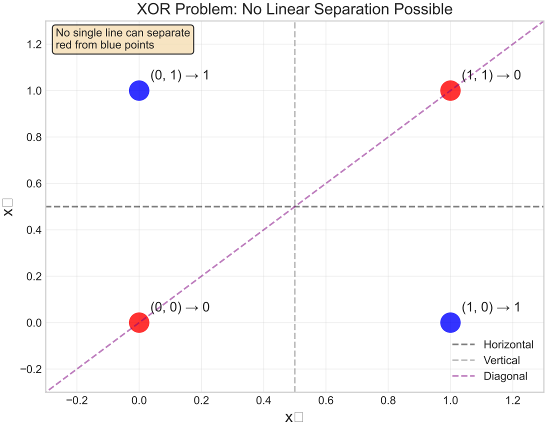
Single layer = single hyperplane = linear separation only

XOR Truth Table:

x_1	x_2	XOR
0	0	0
0	1	1
1	0	1
1	1	0

The Problem:

- No single line separates the classes
- Requires non-linear decision boundary
- Proof: Perceptron cannot solve XOR



XOR became the symbol of perceptron limitations (1969 AI winter)

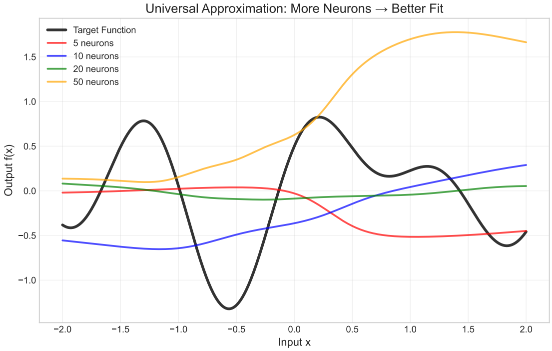
4. Universal Approximation Theorem

Theoretical Foundation:

- Any continuous function can be approximated
- Single hidden layer with enough neurons
- Activation: sigmoid, tanh, ReLU
- Arbitrarily small error possible

Mathematical Statement: For any $\epsilon > 0$ and continuous f on compact set K, there exists network N such that:

$$\sup_{x \in K} |f(x) - N(x)| < \epsilon$$

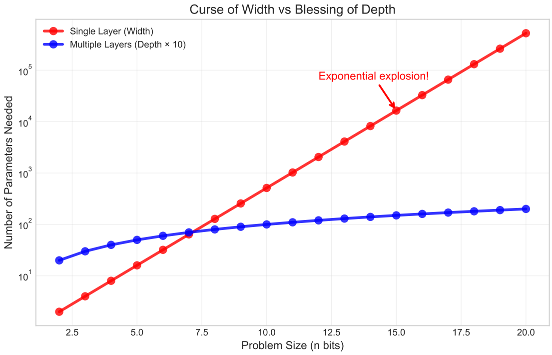


Theory says it's possible - but how many neurons do we actually need?

5. Quantify: How Many Neurons/Layers Needed?

Practical Reality:

- Theory: Single layer sufficient
- Practice: Exponentially many neurons
- Example: Parity function on n bits
  - 1 layer: 2^{n-1} neurons needed
  - 2 layers: $O(n)$ neurons sufficient
- Curse of width vs. blessing of depth



Depth provides exponential expressivity advantage over width

6. Add Hidden Layer Approach

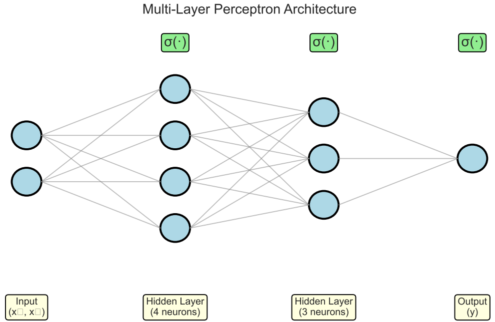
Multi-Layer Perceptron (MLP):

$$\mathbf{h} = \sigma(\mathbf{W}_1 \mathbf{x} + \mathbf{b}_1) \qquad (3)$$

$$y = \sigma(\mathbf{w}_2^T \mathbf{h} + b_2) \qquad (4)$$

Key Innovation:

- Hidden layer creates feature combinations
- Non-linear activation σ (sigmoid, tanh, ReLU)
- Each neuron = learned feature detector
- Output combines these features



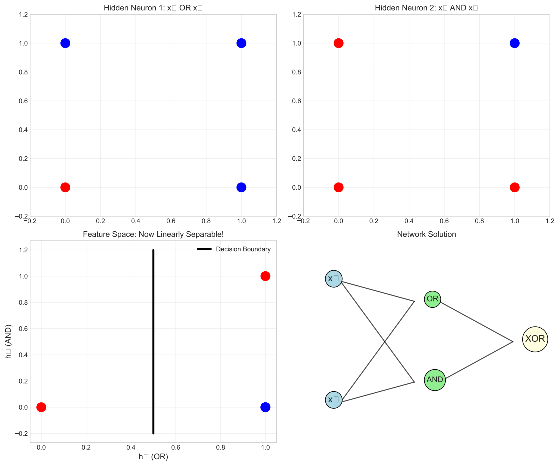
Hidden layer transforms input space to make linear separation possible

Network Architecture:

- Input: x_1, x_2
- Hidden: 2 neurons with sigmoid
- Output: 1 neuron with sigmoid

Solution Strategy:

- h_1: Detects x_1 OR x_2
- h_2: Detects x_1 AND x_2
- Output: h_1 AND NOT h_2



Actual Weights:

$$h_1 = \sigma(20x_1 + 20x_2 - 10) \quad (5)$$

$$h_2 = \sigma(20x_1 + 20x_2 - 30) \quad (6)$$

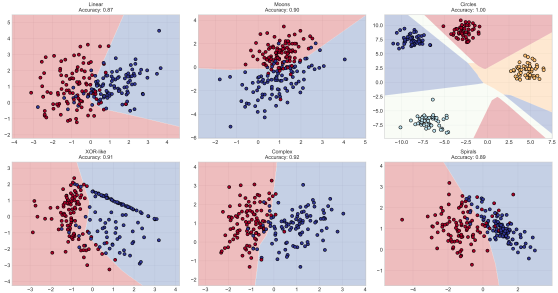
8. SUCCESS: Nonlinearity Achieved

What We Gained:

- Non-linear decision boundaries
- Universal approximation
- Automatic feature learning
- Backpropagation training

Applications Unlocked:

- Image classification (MNIST)
- Function approximation
- Pattern recognition
- Control systems

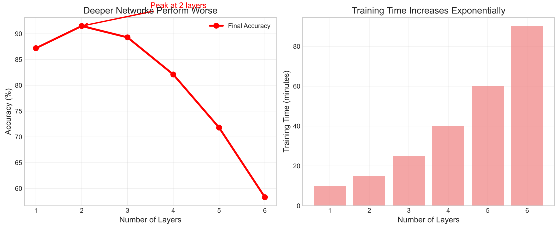


MLPs solved the non-linearity problem and enabled the neural network renaissance

9. FAILURE PATTERN: Vanishing Gradients in Deep Networks

The Problem with Depth:

Layers	Final Accuracy	Training Time
1	87.2%	10 min
2	91.5%	15 min
3	89.3%	25 min
4	82.1%	40 min
5	71.8%	60 min
6	58.3%	90 min



Observation: Deeper networks perform worse, not better!

Real data from 1990s experiments - deeper meant worse performance

10. Diagnosis: Gradient Multiplication -¿ Exponential Decay

Chain Rule in Deep Networks:

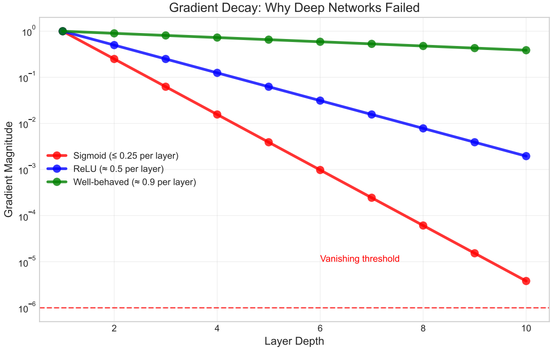
$$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial h_n} \cdot \ldots \cdot \frac{\partial h_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial W_1} \tag{8}$$

Sigmoid Derivative:

$$\sigma'(x) = \sigma(x)(1 - \sigma(x)) \tag{9}$$
$$\leq 0.25 \tag{10}$$

The Problem: Each layer multiplies by ≤ 0.25

- 5 layers: $(0.25)^5 = 0.001$
- 10 layers: $(0.25)^{10} = 0.000001$



Gradients vanish exponentially - early layers learn nothing

Mathematical Analysis:
For L-layer network with sigmoid activations:

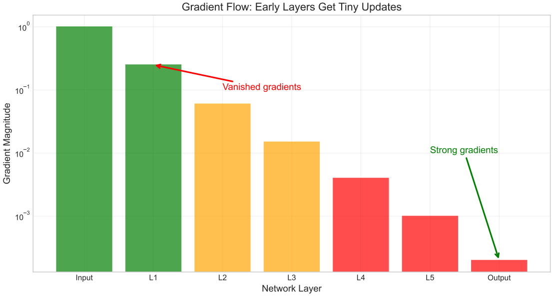
$$\left|\frac{\partial L}{\partial W_1}\right| \leq C \cdot (0.25)^{L-1}$$

Consequences:

- Early layers: Tiny gradients
- Late layers: Large gradients
- Gradient mismatch problem
- Training becomes impossible

Historical Impact:

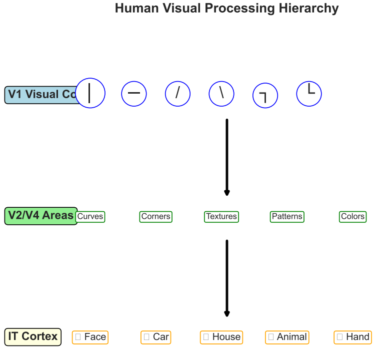
- 1990s: "Neural networks don't scale"
- SVMs and ensemble methods dominated
- Deep learning winter until 2006



Understanding this problem was crucial for the deep learning breakthrough

How Humans See:

- **Level 1**: Edge detection (V1 cortex)
  - Horizontal, vertical, diagonal lines
  - Local contrast detection
- **Level 2**: Texture & shape (V2, V4)
  - Curves, corners, textures
  - Spatial relationships
- **Level 3**: Objects (IT cortex)
  - Faces, cars, animals
  - Invariant recognition



Neuroscience insight: Vision builds complexity through specialized layers

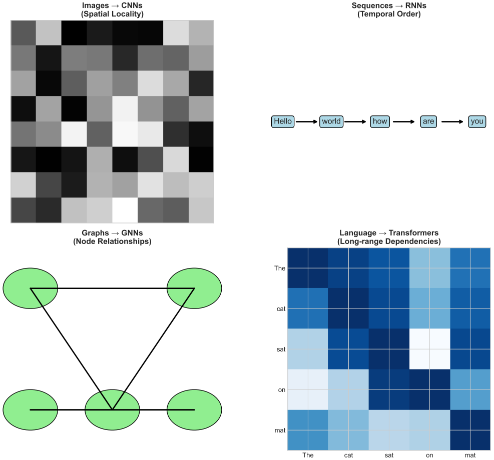
13. Hypothesis: Specialized Architectures Matching Data Structure

The Key Insight:

- Problem: Generic MLPs ignore data structure
- Solution: Architecture matches inductive bias

Examples:

- **Images**: Spatial locality -¿ CNNs
- **Sequences**: Temporal order -¿ RNNs
- **Graphs**: Node relationships -¿ GNNs
- **Language**: Long-range dependencies -¿ Transformers



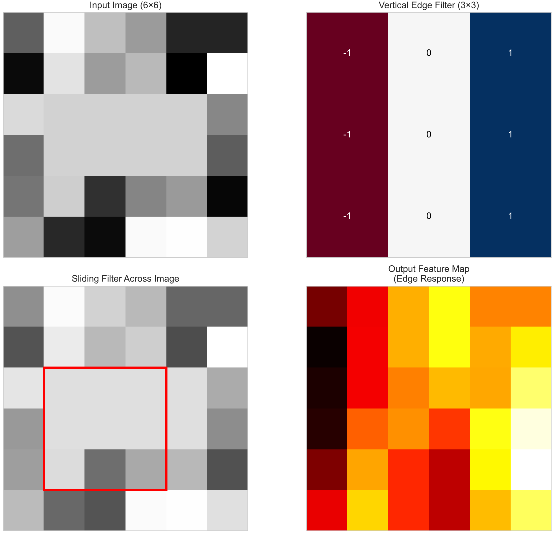
Right architecture = built-in prior knowledge about the problem domain

Convolution Intuition:

- Take a small "template" (3x3 filter)
- Slide it across the entire image
- At each position: compute similarity
- Result: "Where is this pattern?"

Example Filters:

- Edge detector: $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$
- Blur: $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$



Convolution = template matching with learnable templates

What Filters Learn:

Layer 1: Low-level features

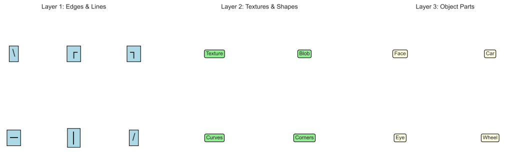
- Edges, corners, blobs
- Oriented lines at different angles
- Color gradients

Layer 2: Mid-level features

- Textures, patterns
- Simple shapes
- Motifs and repeating elements

Layer 3+: High-level features

- Object parts (eyes, wheels)
- Complex patterns



Each layer builds more complex features from simpler ones

16. CNN Architecture Details

Key Components:

1. Convolutional Layers

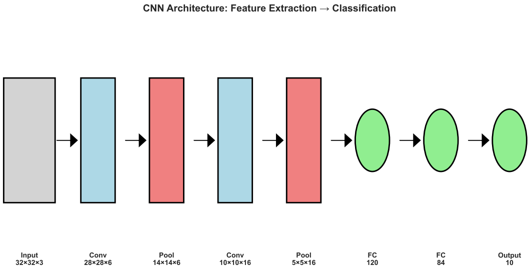
- Multiple filters per layer
- Shared weights across spatial locations
- Parameter sharing reduces overfitting

2. Pooling Layers

- Downsampling (max, average)
- Translation invariance
- Computational efficiency

3. Fully Connected

- Final classification
- Combines all learned features



CNN = Feature extraction (conv+pool) + Classification (FC)

17. Full Walkthrough: Convolve Filter with Actual Numbers

Example Calculation:

Input (3x3): $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

Filter (3x3): $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

Convolution (element-wise multiply + sum):

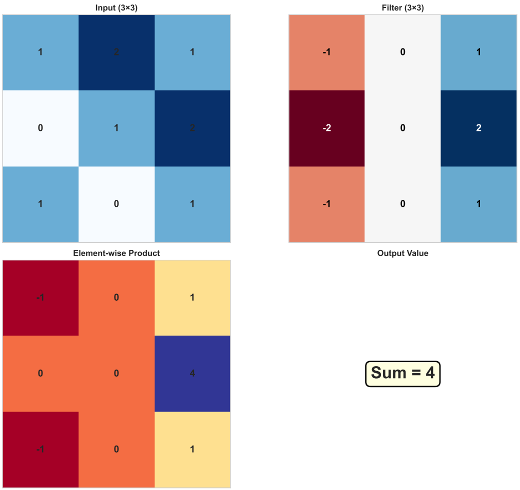
$$= (-1)(1) + (0)(2) + (1)(1) + \tag{11}$$

$$(-2)(0) + (0)(1) + (2)(2) + \tag{12}$$

$$(-1)(1) + (0)(0) + (1)(1) \tag{13}$$

$$= -1 + 0 + 1 - 0 + 0 + 4 - 1 + 0 + 1 \tag{14}$$

$$= 4 \tag{15}$$



High response (4) means vertical edge detected at this location

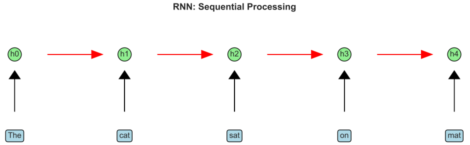
Recurrent Neural Networks (RNNs):

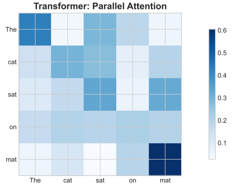
$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h) \tag{16}$$
$$y_t = W_{hy}h_t + b_y \tag{17}$$

- Hidden state carries memory
- Sequential processing
- Good for: Time series, NLP
- Problem: Vanishing gradients over time

Transformers (2017):

- Self-attention mechanism
- Parallel processing
- Long-range dependencies
- State-of-the-art for language





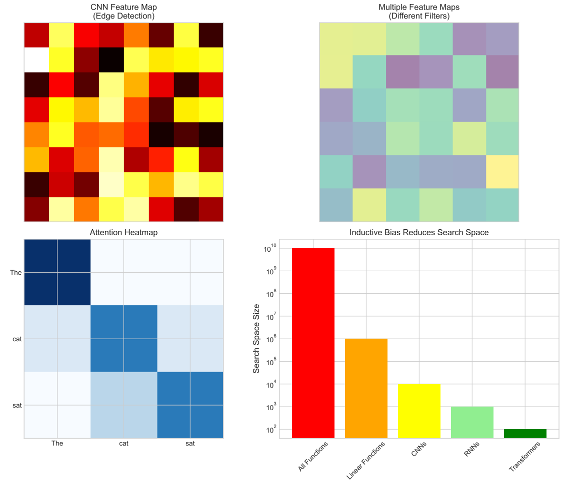
RNNs: Sequential memory, Transformers: Parallel attention

CNN Feature Maps:

- Each filter produces a feature map
- Bright areas = high activation
- Shows what the network “sees”
- Layer 1: Edges and textures
- Layer N: Complex patterns

Transformer Attention:

- Attention weights as heatmaps
- Shows which words influence others
- Different heads learn different patterns
- Interpretable relationships



Visualization reveals the internal representations learned by neural networks

20. Why It Works: Inductive Biases Reduce Search Space

The Core Principle:
Without Structure:

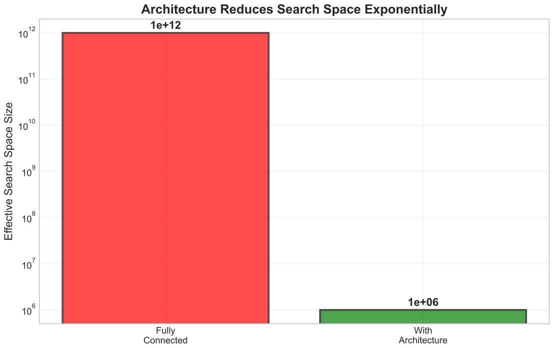
- Search space: All possible functions
- Size: Exponential in parameters
- Sample complexity: Intractable

With Architecture:

- Built-in assumptions about data
- Drastically reduced search space
- Faster learning, better generalization

Examples:

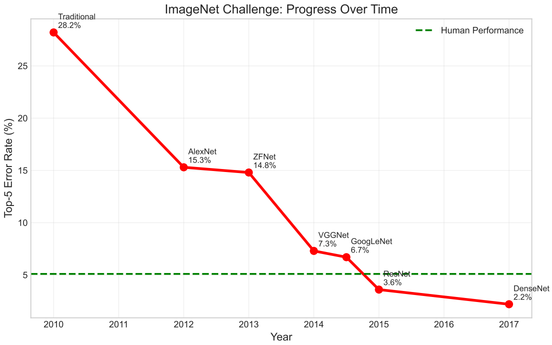
- CNNs assume translation invariance
- RNNs assume sequential dependence
- Transformers assume attention patterns



Architecture = built-in prior knowledge that guides learning

ImageNet Challenge Results:

Year	Model	Top-5 Error
2010	Traditional CV	28.2%
2012	AlexNet (CNN)	15.3%
2013	ZFNet	14.8%
2014	VGGNet	7.3%
2014	GoogLeNet	6.7%
2015	ResNet	3.6%
2017	DenseNet	2.2%
-	Human Performance	5.1%



CNNs achieved superhuman performance in just 5 years

23. Deep Learning Evolution Timeline

Key Milestones:

2012 - AlexNet:

- CNNs + ImageNet breakthrough
- 8-layer network, ReLU activation
- GPU acceleration

2014 - Sequence-to-Sequence:

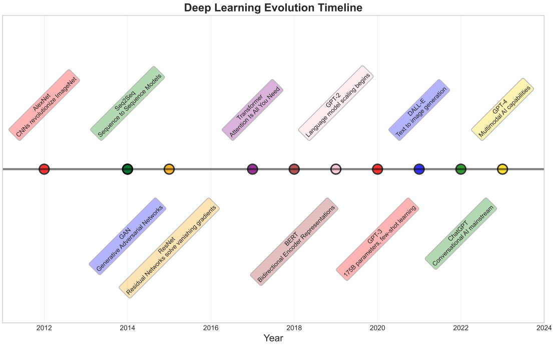
- RNNs for machine translation
- Encoder-decoder architecture

2017 - Transformers:

- “Attention Is All You Need”
- Self-attention mechanism
- Foundation for GPT, BERT

2022 - GPT-4:

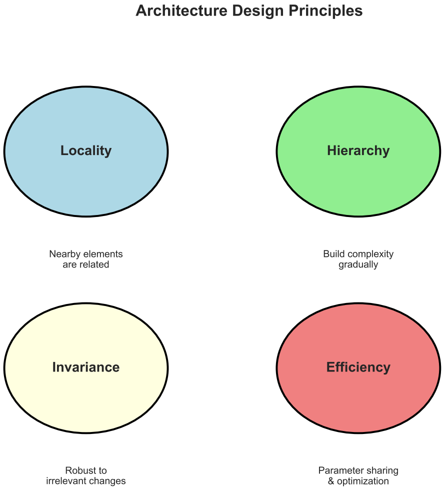
- 1.7 trillion parameters
- Multimodal capabilities
- Human-level performance



From 8 layers to 1000+ layers in just one decade

Universal Design Principles:

1. **Locality:**
   - Nearby elements are related
   - CNNs: Spatial locality
   - RNNs: Temporal locality
2. **Hierarchy:**
   - Build complexity gradually
   - Low-level -¿ High-level features
   - Mirrors human cognition
3. **Invariance:**
   - Robust to irrelevant changes
   - Translation, rotation, scale
   - Attention: Permutation invariance
4. **Efficiency:**
   - Parameter sharing
   - Computational optimization
   - Memory constraints



Good architectures encode the right inductive biases for the domain

Computer Vision:

- Object detection (YOLO, R-CNN)
- Image segmentation
- Medical imaging diagnosis
- Autonomous driving

Natural Language Processing:

- Machine translation (95% human quality)
- Text generation (GPT family)
- Question answering
- Code generation

Multimodal AI:

- Image captioning
- Visual question answering
- Video understanding
- Robotics integration

Neural networks now match or exceed human performance in many domains

26. Summary & Preview: Generative AI

What We Learned:

- Perceptrons: Linear limitations
- MLPs: Non-linear but shallow
- Deep networks: Vanishing gradients
- Modern architectures: Structured solutions

Key Insight: Architecture matters more than size
Next: Generative AI

- From recognition -¿ generation
- VAEs, GANs, Diffusion models
- Large language models
- Applications in innovation

Neural networks: From solving XOR to generating Shakespeare