Proof of Work Consensus

BSc Blockchain, Crypto Economy & NFTs

Course Instructor

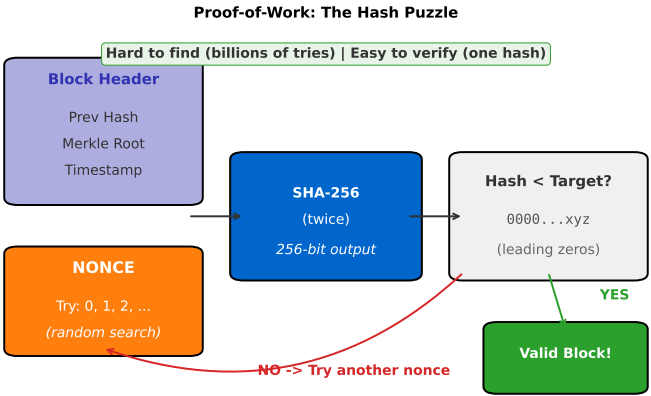
Module A: Blockchain Foundations

Learning Objectives

By the end of this lesson, you will be able to:

- Explain the proof-of-work consensus mechanism
- Describe the mining process and nonce searching
- Understand difficulty adjustment and its purpose
- Calculate mining profitability and hash rate economics
- Recognize the security guarantees and vulnerabilities of PoW
- Evaluate the 51% attack threat model
- Discuss energy consumption and environmental impact

What is Proof of Work?



Core Concept: Find a nonce such that the hash of block header is below the target. No shortcut: must try nonces randomly until one works.

Proof-of-Work Properties

Key Properties:

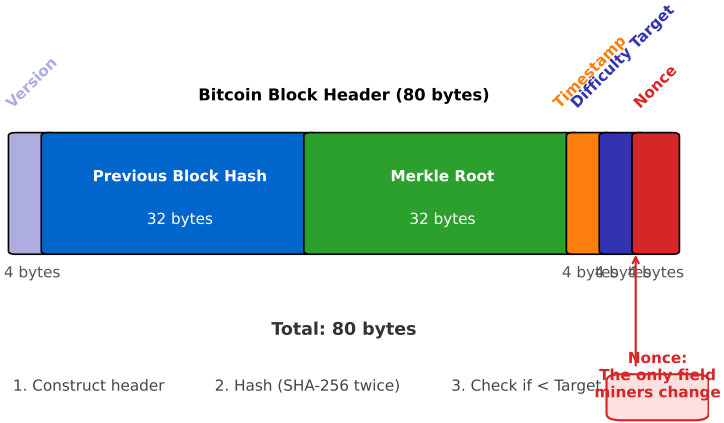
1. **Asymmetry:** Hard to find, easy to verify
2. **Probabilistic:** Expected time to find solution, no guarantee
3. **Adjustable difficulty:** Target changes to maintain block time
4. **Progress-free:** Past attempts do not help future attempts

Mathematical Formulation:

$$\text{SHA-256}(\text{SHA-256}(\text{BlockHeader})) < \text{Target}$$

Analogy: Rolling dice until you get 10 sixes in a row. Each roll is independent. Expected number of attempts: 6^{10}. Verification: just look at the result.

Bitcoin Block Header Structure



Mining Process: Construct block → Set timestamp/difficulty → Try nonces → If hash $<$ target: success, else repeat.

Nonce Space and Extra Nonce

Nonce Space Exhaustion:

- 4 bytes = 2^{32} = 4.3 billion possible nonces
- Modern ASICs exceed this in milliseconds
- Solution: modify coinbase transaction (extra nonce), recompute Merkle root

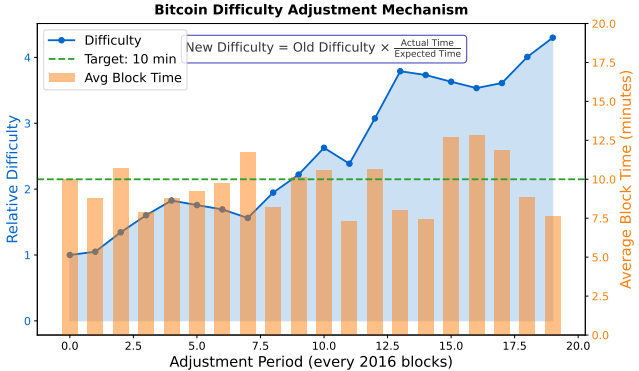
Merkle Trees:

- Commit to all transactions with single 32-byte hash
- Tree height: $\log_2(n)$ for n transactions
- Proof size: $\log_2(n)$ hashes to prove inclusion
- Example: 1000 transactions $\rightarrow$ 10 hashes ($\sim$320 bytes proof)

Extra Nonce Trick:

- Miners modify coinbase transaction (includes extra nonce field)
- Recompute Merkle root (different root for each extra nonce)
- Expands search space beyond 2^{32} nonces

Difficulty Adjustment



Adjustment Rule (Every 2016 Blocks):

$$\text{New Target} = \text{Old Target} \times \frac{\text{Actual Time}}{\text{Expected Time (20,160 min)}}$$

Clamped to $[T/4, T \times 4]$ to prevent extreme changes.

Hash Rate and Mining Probability

Hash Rate:

- Number of hashes computed per second
- Units: H/s, KH/s, MH/s, GH/s, TH/s, PH/s, EH/s
- Bitcoin network (late 2024): $\sim$700 EH/s (all-time high)

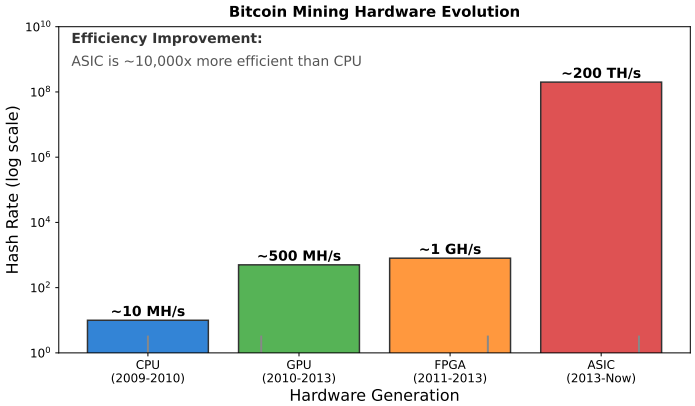
Mining Probability:

$$P(\text{find block}) = \frac{\text{Your Hash Rate}}{\text{Network Hash Rate}}$$

Example:

- Miner: 100 TH/s, Network: 500 EH/s
- Probability: $0.0000002 = 0.00002\%$
- Expected time to find block: $\sim$100 years solo mining
- Solution: Join mining pools for steady income

Mining Hardware Evolution



Implications: Mining centralization, high barrier to entry, geographic concentration in low-electricity regions.

Mining Profitability

Revenue:

$$\text{Daily Revenue} = \frac{\text{Hash Rate}}{\text{Network Hash Rate}} \times 144 \times (\text{Reward} + \text{Fees})$$

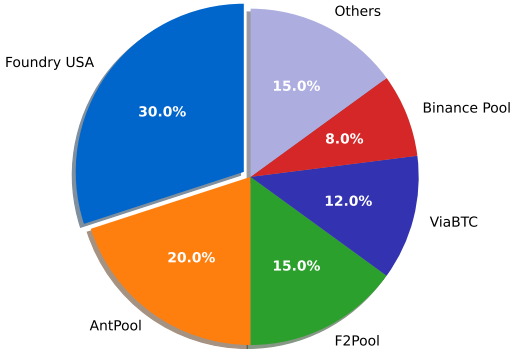
Example (Antminer S19 Pro):

- Hash rate: 110 TH/s, Power: 3250 W = 78 kWh/day
- Electricity: \$0.05/kWh $\times$ 78 = \$3.90/day
- Revenue (@\$40k BTC): $\sim$\$7.92/day
- Profit: \$4.02/day, Payback: $\sim$2 years

Risk Factors: BTC price volatility, difficulty increases, hardware obsolescence, electricity cost changes.

Mining Pools

Bitcoin Mining Pool Distribution (2024)



Top 5 pools control ~85% of hash rate

(Miners can switch pools - pools do not own hardware)

Payout Schemes: PPS (fixed/share), PPLNS (share recent blocks), FPPS (PPS + fees).

Mining Pool Operation

Why Pools Exist:

- Solo mining: high variance (might wait years for block)
- Pooled mining: steady income (proportional to hash rate)

Pool Operation:

1. Pool coordinator distributes mining tasks (shares)
2. Miners submit partial solutions (lower difficulty)
3. Pool tracks contribution of each miner
4. When pool finds block, reward distributed proportionally
5. Pool takes fee (1-3%)

Centralization Concern:

- Pools do not own hardware (miners can switch pools)
- Mitigation: decentralized pool protocols (P2Pool, Stratum V2)

Block Rewards and Halving

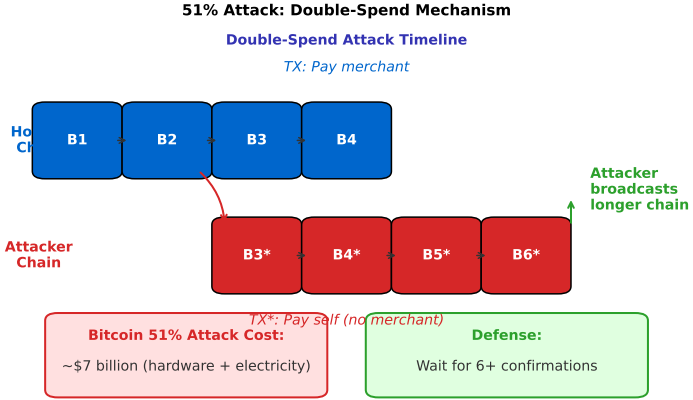
Block Reward Components:

$$\text{Total Reward} = \text{Block Subsidy} + \text{Transaction Fees}$$

Period	Reward	Cumulative Supply
2009-2012	50 BTC	10.5M BTC
2012-2016	25 BTC	15.75M BTC
2016-2020	12.5 BTC	18.375M BTC
2020-2024	6.25 BTC	19.6875M BTC
2024-2028 (current)	3.125 BTC	20.34M BTC

Future: Transaction fees must eventually sustain mining as block subsidy approaches zero ($\sim$2140).

The 51% Attack



CAN do: Double-spend, censor transactions.
CANNOT do: Steal coins without keys, create coins out of thin air.

51% Attack Economics

Cost of Attack (Bitcoin):

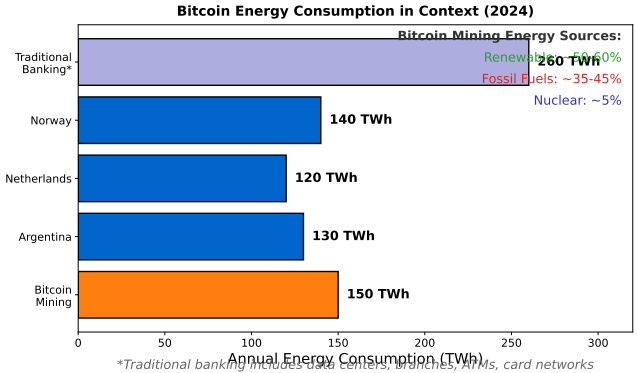
- Need 255 EH/s (>50% of network)
- Hardware: ∼2.3 million ASICs = $6.9 billion
- Electricity (1 week): ∼$63 million
- Total: ∼$7 billion

Consequences:

- Attack becomes public immediately
- Bitcoin price crashes (attacker's hardware worthless)
- Community may hard fork (bricks attacker's ASICs)

Vulnerable Chains: Small PoW chains, shared mining algorithms. Historical: Bitcoin Gold, Ethereum Classic, Verge.

Energy Consumption



Counterarguments: Incentivizes renewable development, facilitates grid balancing, security proportional to value.

ASIC Resistance Alternatives

ASIC-Resistant Algorithms:

- **Scrypt (Litecoin):** Memory-hard hashing (ASICs developed 2014)
- **Ethash (Ethereum pre-merge):** Memory-hard with large DAG
- **RandomX (Monero):** CPU-optimized, frequently updated

Trade-offs:

- ASIC resistance $\rightarrow$ lower security per watt
- Easier for botnets to attack (commodity hardware)
- Algorithm changes create hard fork risks
- Debate: specialization increases security investment

Key Takeaways

- Proof-of-work provides Sybil resistance via computational cost
- Mining searches for nonces to produce valid block hashes
- Difficulty adjusts every 2016 blocks to maintain 10-minute block time
- Mining profitability depends on hash rate, electricity cost, and BTC price
- 51% attacks are economically infeasible for large PoW chains
- Block rewards halve every 4 years, shifting incentives toward fees
- Energy consumption is significant but incentivizes renewable energy

Core Insight: Proof-of-work converts energy into cryptographic security. The cost of attacking equals cumulative computational work by honest miners.

Discussion Questions

1. Why is proof-of-work described as "progress-free"?
2. How does difficulty adjustment make Bitcoin resilient to hash rate fluctuations?
3. What would happen if block rewards fell to zero but fees remained low?
4. Is ASIC mining centralization a threat to Bitcoin's decentralization?
5. How does mining pool concentration differ from miner concentration?
6. Can proof-of-work be justified from an environmental perspective?

Next Lesson Preview: L08 Lab - Wallet Setup

Lab activities:

- Install and configure MetaMask wallet
- Understand seed phrase security and backup
- Connect to Ethereum testnet (Sepolia)
- Obtain testnet ETH from faucets
- Execute first testnet transaction

Preparation:

- Install a modern web browser (Chrome, Firefox, Brave)
- Review public-private key concepts from Lesson 5
- Prepare a secure location for seed phrase backup