

# L34: AMM Mechanics

## Module E: DeFi Ecosystem

Blockchain & Cryptocurrency

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- Understand the constant product formula ( $x \cdot y = k$ )
- Analyze how liquidity provision works in AMMs
- Calculate impermanent loss and its implications
- Understand slippage and price impact
- Compare AMMs to traditional order book exchanges

# Traditional Order Book Exchanges

## How They Work:

- **Buyers** place bids at various prices
- **Sellers** place asks at various prices
- **Matching engine** pairs buy/sell orders
- Trade executes when bid meets ask

## Example Order Book:

	Price	Size
<b>Bids (Buy)</b>	\$1,999	5 ETH
	\$1,998	10 ETH
	\$1,997	15 ETH

	Price	Size
<b>Asks (Sell)</b>	\$2,000	8 ETH
	\$2,001	12 ETH
	\$2,002	20 ETH

## Challenges on Blockchain:

- Gas costs for every order update
- Slow block times (not real-time)
- Front-running and MEV

# Automated Market Makers (AMMs)

**Key Idea:** Replace order books with liquidity pools governed by mathematical formulas.

## How It Works:

- Liquidity Providers (LPs) deposit token pairs into a pool
- Algorithm sets price based on pool ratio
- Users trade directly against the pool
- No matching engine or counterparty needed

## Advantages:

- Always available liquidity (no need to wait for orders)
- Passive income for LPs (earn trading fees)
- Simple smart contract implementation
- Gas efficient (fewer transactions)

**Trade-off:** Price determined by formula, not market consensus.

## Uniswap V2 Model:

$$x \cdot y = k$$

where:

- $x$  = quantity of token A in pool
- $y$  = quantity of token B in pool
- $k$  = constant product (invariant)

**Key Property:** The product  $k$  remains constant before and after trades (ignoring fees).

## Example Pool:

- 100 ETH and 200,000 USDC
- $k = 100 \times 200,000 = 20,000,000$
- Implied price: 1 ETH = 2,000 USDC

**Price is the ratio of reserves:**

$$P = \frac{y}{x}$$

**Example:**

- Pool: 100 ETH, 200,000 USDC
- Price of 1 ETH:  $\frac{200,000}{100} = 2,000$  USDC

**After Trade:** User buys 1 ETH with USDC:

- New reserves: 99 ETH,  $y'$  USDC
- Constant product:  $99 \cdot y' = 20,000,000$
- Solve:  $y' = \frac{20,000,000}{99} \approx 202,020$  USDC
- USDC added:  $202,020 - 200,000 = 2,020$  USDC
- **Cost:** 2,020 USDC for 1 ETH (effective price: \$2,020)

**Observation:** Price moves unfavorably with trade size (slippage).

# Trade Calculation Example

## Pool State:

- 100 ETH, 200,000 USDC
- $k = 20,000,000$

## User wants to buy 10 ETH:

### Step 1: Calculate new ETH reserve

$$x' = 100 - 10 = 90 \text{ ETH}$$

### Step 2: Calculate required USDC reserve

$$y' = \frac{k}{x'} = \frac{20,000,000}{90} \approx 222,222 \text{ USDC}$$

### Step 3: USDC to pay

$$\Delta y = 222,222 - 200,000 = 22,222 \text{ USDC}$$

**Average price:**  $\frac{22,222}{10} = 2,222 \text{ USDC per ETH}$  (vs. 2,000 initially).

**Price impact:** 11% higher than starting price.

**Definition:** The difference between expected price and executed price due to trade size.

## Why Slippage Occurs:

- AMM formula moves price as reserves change
- Larger trades = larger price impact
- Smaller pools = more slippage

## Slippage Formula:

$$\text{Slippage} = \frac{\text{Executed Price} - \text{Initial Price}}{\text{Initial Price}} \times 100\%$$

## Example from Previous Slide:

$$\text{Slippage} = \frac{2,222 - 2,000}{2,000} \times 100\% = 11\%$$

**Slippage Tolerance:** Users set maximum acceptable slippage (e.g., 0.5%, 1%, 5%). Transaction reverts if exceeded.



## How to Become an LP:

- 1 Deposit equal value of both tokens (e.g., 1 ETH + 2,000 USDC)
- 2 Receive LP tokens representing pool share
- 3 Earn trading fees proportional to share
- 4 Withdraw anytime (burn LP tokens, receive reserves back)

## Example:

- Pool has 100 ETH + 200,000 USDC
- You deposit 10 ETH + 20,000 USDC
- Total pool now: 110 ETH + 220,000 USDC
- Your share:  $\frac{10}{110} = 9.09\%$
- You receive 9.09% of LP tokens

## Fee Earnings:

- Uniswap charges 0.3% per trade
- Fees added to pool reserves
- LPs earn pro-rata share

**Purpose:** Represent ownership share of liquidity pool.

**Properties:**

- Fungible ERC-20 tokens
- Can be transferred or sold
- Redeemable for underlying assets
- Value appreciates with fee accumulation

**Withdrawal Process:**

- 1 Burn LP tokens
- 2 Receive pro-rata share of current pool reserves
- 3 May be different ratio than deposit (due to trades)

**Example:**

- You hold 9.09% of LP tokens
- Pool now has 105 ETH + 210,000 USDC (after trades and fees)
- You withdraw:  $0.0909 \times 105 = 9.54$  ETH and  $0.0909 \times 210,000 = 19,089$  USDC

# Impermanent Loss: Concept

**Definition:** The opportunity cost of providing liquidity compared to simply holding tokens.

**Occurs when:**

- Token prices diverge from deposit ratio
- Arbitrageurs rebalance pool to match external prices
- LPs end up with more of the depreciated token

**Why “Impermanent”?**

- Loss is only realized upon withdrawal
- If prices return to original ratio, loss disappears
- Trading fees may offset the loss over time

**Key Insight:** LPs effectively become market makers who buy low and sell high (but miss out on holding gains).

# Impermanent Loss: Example

## Initial Deposit:

- 1 ETH + 2,000 USDC (ETH price = \$2,000)
- Total value: \$4,000

## Scenario: ETH doubles to \$4,000

### If you just held:

- 1 ETH now worth \$4,000
- 2,000 USDC still worth \$2,000
- **Total: \$6,000**

### If you provided liquidity:

- Arbitrageurs rebalance pool:  $x \cdot y = k$
- New reserves: 0.707 ETH + 2,828 USDC
- Value:  $(0.707 \times \$4,000) + \$2,828 = \$5,656$
- **Impermanent Loss:  $\$6,000 - \$5,656 = \$344$  (5.7%)**

## General Formula:

$$IL = \frac{2\sqrt{r}}{1+r} - 1$$

where  $r$  is the price ratio change.

## Common Scenarios:

- 1.25x price change: -0.6% IL
- 1.5x price change: -2.0% IL
- 2x price change: -5.7% IL
- 3x price change: -13.4% IL
- 4x price change: -20.0% IL
- 5x price change: -25.5% IL

**Observation:** IL accelerates with larger price movements (non-linear).

## Strategies:

### 1. Choose Stable Pairs

- Provide liquidity for correlated assets (e.g., USDC/DAI, ETH/stETH)
- Minimal price divergence = minimal IL

### 2. High Trading Volume Pools

- More fees to offset IL
- Example: ETH/USDC on Uniswap (high volume)

### 3. Concentrated Liquidity (Uniswap V3)

- Provide liquidity in narrow price range
- Higher fee efficiency but more active management

### 4. Liquidity Mining Rewards

- Extra token incentives may exceed IL

### 5. Short-Term Provision

- Withdraw before large price movements

## How Arbitrage Works:

- 1 External market price deviates from AMM price
- 2 Arbitrageur buys cheaper asset, sells expensive one
- 3 Profits from price difference
- 4 AMM pool rebalances to match external price

## Example:

- Centralized exchange (CEX):  $1 \text{ ETH} = \$2,100$
- Uniswap pool implies:  $1 \text{ ETH} = \$2,000$
- Arbitrageur: Buy ETH on Uniswap (\$2,000), sell on CEX (\$2,100)
- Profit: \$100 per ETH
- Pool adjusts: ETH reserve decreases, USDC increases, price rises

**Benefit:** Arbitrage keeps AMM prices aligned with global markets.

**Cost:** LPs experience impermanent loss from price adjustments.

## Uniswap Fee Tiers:

- **0.01%:** Stablecoin pairs (low volatility)
- **0.05%:** Correlated pairs (e.g., ETH/stETH)
- **0.3%:** Most pairs (standard)
- **1%:** Exotic/volatile pairs

## Fee Distribution:

- 100% to LPs (Uniswap governance can enable protocol fee)
- Fees compound in pool reserves
- LPs earn proportional to liquidity share and duration

## APY Calculation:

$$\text{APY} \approx \frac{\text{Daily Fees} \times 365}{\text{Pool TVL}} - \text{Impermanent Loss}$$

## Example:

- Pool TVL: \$10M, Daily fees: \$10,000
- APY:  $\frac{10,000 \times 365}{10,000,000} = 36.5\%$  (before IL)



## Problem with Constant Product:

- Inefficient for assets that should trade 1:1 (stablecoins)
- High slippage even for small trades

## Curve's Solution: Stableswap Invariant

- Hybrid of constant product and constant sum
- Flat curve near 1:1 price (low slippage)
- Reverts to constant product at extremes (prevents pool drain)

## Benefits:

- Trade millions of USDC/DAI with  $<0.01\%$  slippage
- Capital efficient for stablecoin swaps
- Minimal impermanent loss (prices stay near 1:1)

**Use Case:** Dominant DEX for stablecoin trading.

## Generalization:

$$\prod_i x_i^{w_i} = k$$

where  $w_i$  are weights (must sum to 1).

## Example: 80/20 Pool

- 80% token A, 20% token B by value
- Less impermanent loss than 50/50 pool
- Still earn fees from trading

## Advantages:

- Customizable exposure (e.g., 80% ETH, 20% USDC)
- Multi-token pools (up to 8 tokens)
- Index fund functionality

**Use Case:** LPs wanting concentrated exposure to one asset while earning fees.

**Problem:** In constant product AMMs, most liquidity is unused.

**Example:**

- ETH/USDC pool with 100 ETH and 200,000 USDC
- Most trades happen near current price (\$2,000)
- Liquidity far from current price (e.g., \$1,000 or \$4,000) rarely used

**Solution: Concentrated Liquidity (Uniswap V3)**

- LPs choose specific price range
- Liquidity only active within range
- More capital efficient (same liquidity with less capital)

**Trade-off:**

- Higher returns if price stays in range
- Zero fees if price moves outside range
- Requires active management (rebalancing)

## AMM (Uniswap)

- Always available liquidity
- Passive LP income
- Slippage on large trades
- Price discovery via arbitrage
- Impermanent loss risk
- Simple to implement

## Order Book (Binance)

- Liquidity depends on makers
- Active market making
- Better for large trades (limit orders)
- Real-time price discovery
- No impermanent loss
- Complex infrastructure

**Trend:** Hybrid models emerging (e.g., dYdX order book on Cosmos, Uniswap X).

## Maximal Extractable Value (MEV):

- Profit from reordering/inserting/censoring transactions
- Particularly prevalent in AMM trades

## Common MEV Strategies:

- 1 **Front-Running:** See large buy, buy first, sell after price moves
- 2 **Sandwich Attacks:** Buy before user, sell after (profit from their slippage)
- 3 **Arbitrage:** Exploit price differences between AMMs/CEXs

## Impact on Users:

- Worse execution prices
- Hidden cost (in addition to explicit fees)

## Mitigation:

- Private mempools (Flashbots Protect)
- MEV-aware routers
- Batch auctions (CoW Swap)

## Key Takeaways:

- AMMs use  $x \cdot y = k$  to provide algorithmic liquidity
- Price determined by reserve ratio, trades move price
- Slippage increases with trade size and decreases with pool depth
- LPs earn fees but face impermanent loss when prices diverge
- IL formula:  $\frac{2\sqrt{r}}{1+r} - 1$  where  $r$  is price change ratio
- Arbitrage keeps AMM prices aligned with external markets
- Variants (Curve, Balancer, Uniswap V3) optimize for specific use cases
- MEV is a hidden cost for AMM traders

**Next Lecture:** Uniswap Deep Dive - Evolution from V1 to V4, concentrated liquidity, governance.

- 1 Calculate the cost to buy 5 ETH from a pool with 100 ETH and 200,000 USDC.
- 2 Why does slippage increase non-linearly with trade size?
- 3 How do trading fees help offset impermanent loss for LPs?
- 4 Why is Curve more suitable for stablecoin trading than Uniswap V2?
- 5 What are the trade-offs of concentrated liquidity in Uniswap V3?