L34: AMM Mechanics

Module E: DeFi Ecosystem

Blockchain & Cryptocurrency

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- Understand the constant product formula ($x \cdot y = k$)
- Analyze how liquidity provision works in AMMs
- Calculate impermanent loss and its implications
- Understand slippage and price impact
- Compare AMMs to traditional order book exchanges

How They Work:

- **Buyers** place bids at various prices
- **Sellers** place asks at various prices
- **Matching engine** pairs buy/sell orders
- Trade executes when bid meets ask

Example Order Book:

	Price	Size			Price	Size
Bids (Buy)	$1,999	5 ETH		**Asks (Sell)**	$2,000	8 ETH
	$1,998	10 ETH			$2,001	12 ETH
	$1,997	15 ETH			$2,002	20 ETH

Challenges on Blockchain:

- Gas costs for every order update
- Slow block times (not real-time)
- Front-running and MEV

Automated Market Makers (AMMs)

Key Idea: Replace order books with liquidity pools governed by mathematical formulas.

How It Works:

- Liquidity Providers (LPs) deposit token pairs into a pool
- Algorithm sets price based on pool ratio
- Users trade directly against the pool
- No matching engine or counterparty needed

Advantages:

- Always available liquidity (no need to wait for orders)
- Passive income for LPs (earn trading fees)
- Simple smart contract implementation
- Gas efficient (fewer transactions)

Trade-off: Price determined by formula, not market consensus.

Uniswap V2 Model:

$$x \cdot y = k$$

where:

- x = quantity of token A in pool
- y = quantity of token B in pool
- k = constant product (invariant)

Key Property: The product k remains constant before and after trades (ignoring fees).

Example Pool:

- 100 ETH and 200,000 USDC
- $k = 100 \times 200{,}000 = 20{,}000{,}000$
- Implied price: 1 ETH = 2,000 USDC

Price Determination in AMMs

Price is the ratio of reserves:

$$P = \frac{y}{x}$$

Example:
- Pool: 100 ETH, 200,000 USDC
- Price of 1 ETH: $\frac{200,000}{100} = 2,000$ USDC

After Trade: User buys 1 ETH with USDC:
- New reserves: 99 ETH, y' USDC
- Constant product: $99 \cdot y' = 20,000,000$
- Solve: $y' = \frac{20,000,000}{99} \approx 202,020$ USDC
- USDC added: $202,020 - 200,000 = 2,020$ USDC
- **Cost:** 2,020 USDC for 1 ETH (effective price: \$2,020)

Observation: Price moves unfavorably with trade size (slippage).

Pool State:

- 100 ETH, 200,000 USDC
- $k = 20{,}000{,}000$

User wants to buy 10 ETH:

Step 1: Calculate new ETH reserve

$$x' = 100 - 10 = 90 \text{ ETH}$$

Step 2: Calculate required USDC reserve

$$y' = \frac{k}{x'} = \frac{20{,}000{,}000}{90} \approx 222{,}222 \text{ USDC}$$

Step 3: USDC to pay

$$\Delta y = 222{,}222 - 200{,}000 = 22{,}222 \text{ USDC}$$

Average price: $\frac{22{,}222}{10} = 2{,}222$ USDC per ETH (vs. 2,000 initially).

Price impact: 11% higher than starting price.

Slippage

Definition: The difference between expected price and executed price due to trade size.

Why Slippage Occurs:
- AMM formula moves price as reserves change
- Larger trades = larger price impact
- Smaller pools = more slippage

Slippage Formula:

$$\text{Slippage} = \frac{\text{Executed Price} - \text{Initial Price}}{\text{Initial Price}} \times 100\%$$

Example from Previous Slide:

$$\text{Slippage} = \frac{2{,}222 - 2{,}000}{2{,}000} \times 100\% = 11\%$$

Slippage Tolerance: Users set maximum acceptable slippage (e.g., 0.5%, 1%, 5%). Transaction reverts if exceeded.

How to Become an LP:

1. Deposit equal value of both tokens (e.g., 1 ETH + 2,000 USDC)
2. Receive LP tokens representing pool share
3. Earn trading fees proportional to share
4. Withdraw anytime (burn LP tokens, receive reserves back)

Example:

- Pool has 100 ETH + 200,000 USDC
- You deposit 10 ETH + 20,000 USDC
- Total pool now: 110 ETH + 220,000 USDC
- Your share: $\frac{10}{110} = 9.09\%$
- You receive 9.09% of LP tokens

Fee Earnings:

- Uniswap charges 0.3% per trade
- Fees added to pool reserves
- LPs earn pro-rata share

LP Token Mechanics

Purpose: Represent ownership share of liquidity pool.

Properties:
- Fungible ERC-20 tokens
- Can be transferred or sold
- Redeemable for underlying assets
- Value appreciates with fee accumulation

Withdrawal Process:
1. Burn LP tokens
2. Receive pro-rata share of current pool reserves
3. May be different ratio than deposit (due to trades)

Example:
- You hold 9.09% of LP tokens
- Pool now has 105 ETH + 210,000 USDC (after trades and fees)
- You withdraw: $0.0909 \times 105 = 9.54$ ETH and $0.0909 \times 210,000 = 19,089$ USDC

Definition: The opportunity cost of providing liquidity compared to simply holding tokens.

Occurs when:
- Token prices diverge from deposit ratio
- Arbitrageurs rebalance pool to match external prices
- LPs end up with more of the depreciated token

Why "Impermanent"?
- Loss is only realized upon withdrawal
- If prices return to original ratio, loss disappears
- Trading fees may offset the loss over time

Key Insight: LPs effectively become market makers who buy low and sell high (but miss out on holding gains).

Initial Deposit:
- 1 ETH + 2,000 USDC (ETH price = $2,000)
- Total value: $4,000

Scenario: ETH doubles to $4,000
If you just held:
- 1 ETH now worth $4,000
- 2,000 USDC still worth $2,000
- **Total: $6,000**

If you provided liquidity:
- Arbitrageurs rebalance pool: $x \cdot y = k$
- New reserves: 0.707 ETH + 2,828 USDC
- Value: $(0.707 \times \$4,000) + \$2,828 = \$5,656$
- **Impermanent Loss: $6,000 - $5,656 = $344 (5.7%)**

General Formula:

$$IL = \frac{2\sqrt{r}}{1 + r} - 1$$

where r is the price ratio change.

Common Scenarios:

- 1.25x price change: -0.6% IL
- 1.5x price change: -2.0% IL
- 2x price change: -5.7% IL
- 3x price change: -13.4% IL
- 4x price change: -20.0% IL
- 5x price change: -25.5% IL

Observation: IL accelerates with larger price movements (non-linear).

Mitigating Impermanent Loss

Strategies:

1. Choose Stable Pairs
- Provide liquidity for correlated assets (e.g., USDC/DAI, ETH/stETH)
- Minimal price divergence = minimal IL

2. High Trading Volume Pools
- More fees to offset IL
- Example: ETH/USDC on Uniswap (high volume)

3. Concentrated Liquidity (Uniswap V3)
- Provide liquidity in narrow price range
- Higher fee efficiency but more active management

4. Liquidity Mining Rewards
- Extra token incentives may exceed IL

5. Short-Term Provision
- Withdraw before large price movements

Arbitrage in AMMs

How Arbitrage Works:

1. External market price deviates from AMM price
2. Arbitrageur buys cheaper asset, sells expensive one
3. Profits from price difference
4. AMM pool rebalances to match external price

Example:

- Centralized exchange (CEX): 1 ETH = $2,100
- Uniswap pool implies: 1 ETH = $2,000
- Arbitrageur: Buy ETH on Uniswap ($2,000), sell on CEX ($2,100)
- Profit: $100 per ETH
- Pool adjusts: ETH reserve decreases, USDC increases, price rises

Benefit: Arbitrage keeps AMM prices aligned with global markets.
Cost: LPs experience impermanent loss from price adjustments.

Uniswap Fee Tiers:

- **0.01%:** Stablecoin pairs (low volatility)
- **0.05%:** Correlated pairs (e.g., ETH/stETH)
- **0.3%:** Most pairs (standard)
- **1%:** Exotic/volatile pairs

Fee Distribution:

- 100% to LPs (Uniswap governance can enable protocol fee)
- Fees compound in pool reserves
- LPs earn proportional to liquidity share and duration

APY Calculation:

$$\text{APY} \approx \frac{\text{Daily Fees} \times 365}{\text{Pool TVL}} - \text{Impermanent Loss}$$

Example:

- Pool TVL: \$10M, Daily fees: \$10,000
- APY: $\frac{10,000 \times 365}{10,000,000} = 36.5\%$ (before IL)

Problem with Constant Product:

- Inefficient for assets that should trade 1:1 (stablecoins)
- High slippage even for small trades

Curve's Solution: Stableswap Invariant

- Hybrid of constant product and constant sum
- Flat curve near 1:1 price (low slippage)
- Reverts to constant product at extremes (prevents pool drain)

Benefits:

- Trade millions of USDC/DAI with ¡0.01% slippage
- Capital efficient for stablecoin swaps
- Minimal impermanent loss (prices stay near 1:1)

Use Case: Dominant DEX for stablecoin trading.

Generalization:

$$\prod_i x_i^{w_i} = k$$

where w_i are weights (must sum to 1).

Example: 80/20 Pool
- 80% token A, 20% token B by value
- Less impermanent loss than 50/50 pool
- Still earn fees from trading

Advantages:
- Customizable exposure (e.g., 80% ETH, 20% USDC)
- Multi-token pools (up to 8 tokens)
- Index fund functionality

Use Case: LPs wanting concentrated exposure to one asset while earning fees.

Capital Efficiency

Problem: In constant product AMMs, most liquidity is unused.

Example:
- ETH/USDC pool with 100 ETH and 200,000 USDC
- Most trades happen near current price ($2,000)
- Liquidity far from current price (e.g., $1,000 or $4,000) rarely used

Solution: Concentrated Liquidity (Uniswap V3)
- LPs choose specific price range
- Liquidity only active within range
- More capital efficient (same liquidity with less capital)

Trade-off:
- Higher returns if price stays in range
- Zero fees if price moves outside range
- Requires active management (rebalancing)

AMM (Uniswap)

- Always available liquidity
- Passive LP income
- Slippage on large trades
- Price discovery via arbitrage
- Impermanent loss risk
- Simple to implement

Order Book (Binance)

- Liquidity depends on makers
- Active market making
- Better for large trades (limit orders)
- Real-time price discovery
- No impermanent loss
- Complex infrastructure

Trend: Hybrid models emerging (e.g., dYdX order book on Cosmos, Uniswap X).

MEV and Front-Running in AMMs

Maximal Extractable Value (MEV):
- Profit from reordering/inserting/censoring transactions
- Particularly prevalent in AMM trades

Common MEV Strategies:
1. **Front-Running:** See large buy, buy first, sell after price moves
2. **Sandwich Attacks:** Buy before user, sell after (profit from their slippage)
3. **Arbitrage:** Exploit price differences between AMMs/CEXs

Impact on Users:
- Worse execution prices
- Hidden cost (in addition to explicit fees)

Mitigation:
- Private mempools (Flashbots Protect)
- MEV-aware routers
- Batch auctions (CoW Swap)

Summary

Key Takeaways:
- AMMs use $x \cdot y = k$ to provide algorithmic liquidity
- Price determined by reserve ratio, trades move price
- Slippage increases with trade size and decreases with pool depth
- LPs earn fees but face impermanent loss when prices diverge
- IL formula: $\frac{2\sqrt{r}}{1+r} - 1$ where r is price change ratio
- Arbitrage keeps AMM prices aligned with external markets
- Variants (Curve, Balancer, Uniswap V3) optimize for specific use cases
- MEV is a hidden cost for AMM traders

Next Lecture: Uniswap Deep Dive - Evolution from V1 to V4, concentrated liquidity, governance.

Questions for Reflection

1. Calculate the cost to buy 5 ETH from a pool with 100 ETH and 200,000 USDC.
2. Why does slippage increase non-linearly with trade size?
3. How do trading fees help offset impermanent loss for LPs?
4. Why is Curve more suitable for stablecoin trading than Uniswap V2?
5. What are the trade-offs of concentrated liquidity in Uniswap V3?