

ECDSA: Elliptic Curve Digital Signature Algorithm

SIGNING

1. Hash message

$$z = \text{SHA256}(\text{message})$$

2. Random k

$$k = \text{random}[1, n-1]$$

3. Calculate R

$$R = k * G, \quad r = R.x \bmod n$$

4. Calculate s

$$s = k^{-1}(z + r * d) \bmod n$$

5. Signature

$$\text{sig} = (r, s)$$

VERIFICATION

1. Hash message

$$z = \text{SHA256}(\text{message})$$

2. Calculate u1

$$u1 = z * s^{-1} \bmod n$$

3. Calculate u2

$$u2 = r * s^{-1} \bmod n$$

4. Calculate P

$$P = u1 * G + u2 * Q$$

5. Check

$$\text{Valid if } P.x \bmod n = r$$

Variables: G=generator, n=curve order, d=private key, Q=public key, k=nonce

Bitcoin uses secp256k1 curve with 256-bit keys and SHA-256 hashing