Lesson 15: Public Key Cryptography & Digital Signatures

Module 2: Blockchain Fundamentals

Digital Finance

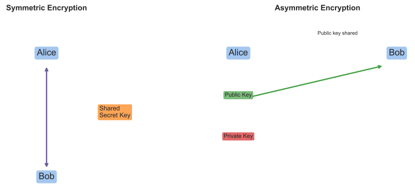
The Problem: Secure Communication Over Insecure Channels

Challenge:

- How do two parties communicate securely without meeting?
- How do you verify someone's identity online?
- How do you prove authorship of a message?

Traditional Solution:

- Symmetric cryptography (shared secret)
- Problem: Key distribution
- Requires secure channel to share key



Symmetric vs Asymmetric Cryptography

charts/lesson_15/crypto_comparison.pdf

Public Key Cryptography: Revolutionary Idea

Key Pair Structure:

- Public Key: Shared openly
- Private Key: Kept secret
- Mathematical relationship
- One-way function (easy to compute, hard to reverse)

Properties:

- Encrypt with public $\longrightarrow$ decrypt with private
- Sign with private $\longrightarrow$ verify with public
- Cannot derive private from public

charts/lesson_15/key_pair_generation.pdf

Mathematical Foundation: Trapdoor Functions

One-Way Function:

$$y = f(x) \quad \text{(easy)}$$

$$x = f^{-1}(y) \quad \text{(hard)}$$

Examples:

- Factoring large primes (RSA)
- Discrete logarithm (Diffie-Hellman)
- Elliptic curve discrete log (ECDSA)

charts/lesson_15/trapdoor_function.pdf

Trapdoor: Secret information (private key) makes inverse easy

RSA Cryptography (Classic Approach)

Key Generation:

1. Choose two large primes: p, q
2. Compute $n = p \times q$ (modulus)
3. Compute $\phi(n) = (p-1)(q-1)$
4. Choose public exponent e (commonly 65537)
5. Compute private exponent $d \equiv e^{-1} \pmod{\phi(n)}$

Encryption/Decryption:

$$\text{Ciphertext: } c = m^e \bmod n \quad | \quad \text{Plaintext: } m = c^d \bmod n$$

Example: $p = 61, q = 53, n = 3233, e = 17, d = 2753$

- Message $m = 123$: $c = 123^{17} \bmod 3233 = 855$
- Decrypt: $m = 855^{2753} \bmod 3233 = 123$

Elliptic Curve Cryptography (ECC)

Why ECC?

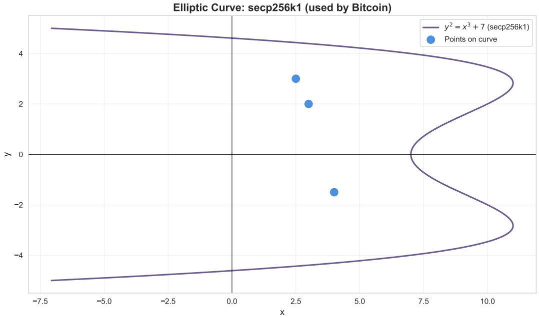
- Smaller key sizes (256-bit ECC $\approx$ 3072-bit RSA)
- Faster computations
- Lower bandwidth
- Standard in Bitcoin/Ethereum

Curve Equation:

$$y^2 = x^3 + ax + b$$

Bitcoin uses: secp256k1

$$y^2 = x^3 + 7$$



ECC Point Addition: The Core Operation

charts/lesson_15/ecc_point_addition.pdf

ECC Security: Discrete Logarithm Problem

Easy Problem:

- Given point G and scalar k
- Compute $P = k \cdot G$
- Fast using double-and-add

Hard Problem (ECDLP):

- Given points G and P
- Find scalar k such that $P = k \cdot G$
- No efficient algorithm known
- This is the private key

charts/lesson_15/ecdlp_visualization.pdf

Security: Best attack takes $O(\sqrt{n})$ operations for n-bit key

Digital Signatures: Proving Authorship

Purpose:

- Prove message was created by you
- Ensure message wasn't altered
- Non-repudiation (can't deny)

Process:

1. Hash the message
2. Encrypt hash with private key
3. Attach signature to message
4. Verify: Decrypt with public key, compare hash

charts/lesson_15/digital_signature_flow.pdf

ECDSA: Elliptic Curve Digital Signature Algorithm

Signature Generation:

1. Message m, private key d, public key $Q = d \cdot G$
2. Hash message: $z = \text{hash}(m)$
3. Choose random k, compute $R = k \cdot G = (x_R, y_R)$
4. Compute $r = x_R \bmod n$
5. Compute $s = k^{-1}(z + rd) \bmod n$
6. Signature: (r, s)

Signature Verification:

1. Compute $w = s^{-1} \bmod n$
2. Compute $u_1 = zw \bmod n$, $u_2 = rw \bmod n$
3. Compute $P = u_1 \cdot G + u_2 \cdot Q$
4. Valid if $x_P \equiv r \pmod{n}$

ECDSA Visualization

charts/lesson_15/ecdsa_process.pdf

Cryptocurrency Wallets: Key Management

Wallet Components:

- Private key (spend authority)
- Public key (derived from private)
- Address (hash of public key)

Key Derivation:

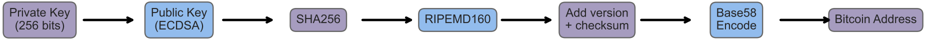
$$\text{Private Key} \xrightarrow{\text{ECC}} \text{Public Key} \xrightarrow{\text{Hash}} \text{Address}$$

One-way: Cannot derive private from address

charts/lesson_15/wallet_key_hierarchy.pdf

Bitcoin Address Generation

Bitcoin Address Generation Pipeline



Steps:

1. Generate 256-bit private key (random number)
2. Compute public key: PubKey $=$ PrivKey $\times$ G (secp256k1)
3. Hash public key: SHA256 then RIPEMD160

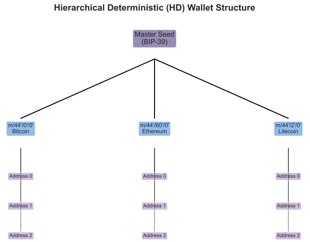
Hierarchical Deterministic (HD) Wallets

Problem:

- Managing multiple random keys
- Backup complexity
- Privacy (address reuse)

Solution (BIP-32/44):

- Master seed (12/24 words)
- Derive infinite keys deterministically
- Hierarchical tree structure
- One backup for all keys



Mnemonic Seed Phrases (BIP-39)

charts/lesson_15/mnemonic_generation.pdf

Wallet Types: Hot vs Cold

charts/lesson_15/wallet_types_comparison.pdf

Security Best Practices

Never Share:

- Private keys
- Seed phrases
- Wallet files without encryption

Recommendations:

- Use hardware wallets for large amounts (Ledger, Trezor)
- Backup seed phrase offline (metal, paper in safe)
- Multi-signature for institutional custody
- Never store keys on cloud services
- Verify addresses carefully (malware can swap addresses)

Warning: Lost private key = Lost funds permanently

Benefits:

- Self-custody (be your own bank)
- No intermediaries
- Censorship resistance
- Programmable ownership

Challenges:

- User error (lost keys)
- No password reset
- Irreversible transactions
- Phishing attacks

charts/lesson_15/key_security_threats.pdf

Summary

- **Public Key Cryptography**: Two keys (public/private), trapdoor functions
- **ECC**: Efficient, smaller keys, basis for Bitcoin/Ethereum signatures
- **ECDSA**: Digital signatures prove transaction authorship
- **Wallets**: Manage private keys, addresses derived from public keys
- **HD Wallets**: One seed generates infinite keys (BIP-32/39/44)
- **Security**: Private key = ownership, loss is permanent

Next Lesson: Proof of Work – how cryptography secures the blockchain