Lesson 16: Proof of Work

Module 2: Blockchain Fundamentals

Digital Finance

The Double-Spending Problem

Digital Money Challenge:

- Digital files are copyable
- How to prevent spending same coin twice?
- Traditional solution: Central authority (bank)

Decentralized Challenge:

- No central ledger
- Network latency
- Conflicting transactions
- Malicious actors

charts/lesson_16/double_spending_scenario.pdf

Consensus Problem: Agreeing on Transaction Order

charts/lesson_16/consensus_problem.pdf

Proof of Work: The Solution

Core Idea:

- Make block creation expensive
- Require computational work
- Probability-based selection
- Longest chain wins

Properties:

- Sybil resistance (one CPU = one vote)
- Objective chain selection rule
- Economic security
- No coordination needed

charts/lesson_16/pow_concept.pdf

Mining Goal: Find nonce such that block hash is below target

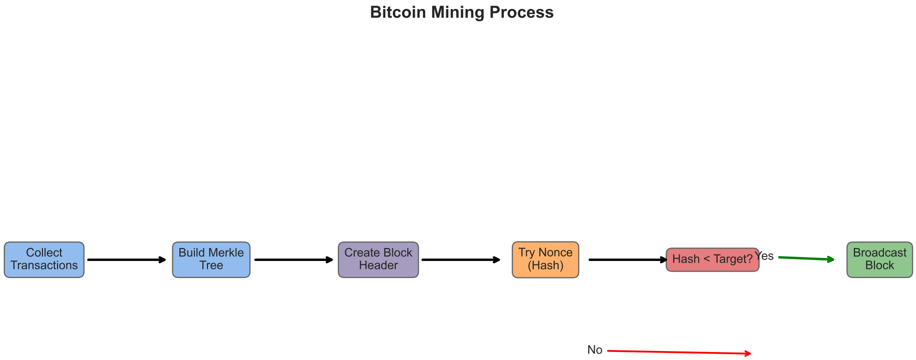
$$\text{SHA256(Block Header)} < \text{Target}$$

Block Header Contains:

- Previous block hash
- Merkle root (transaction summary)
- Timestamp
- Difficulty target
- Nonce (number to vary)

Example:

$$\begin{aligned} \text{Target:} &\quad 0000000000000000000f1a2b3c4d\ldots \\ \text{Hash attempt 1:} &\quad 8a3f2e1d9c\ldots \quad \text{(too high)} \\ \text{Hash attempt 2:} &\quad 0000000000000000000a1b2c3d\ldots \quad \text{(success!)} \end{aligned}$$



Steps:

1. Collect transactions from mempool
2. Build Merkle tree, create block header

Difficulty Target: Controlling Block Time

Target Representation:

$$\text{Target} = \text{coefficient} \times 2^{8(\text{exponent}-3)}$$

Difficulty:

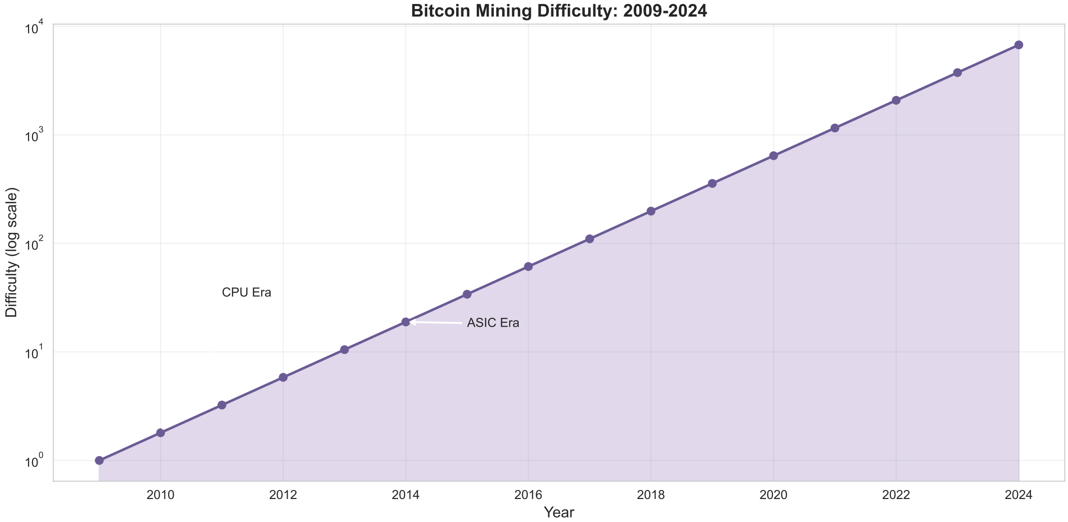
$$\text{Difficulty} = \frac{\text{Max Target}}{\text{Current Target}}$$

Bitcoin:

- Target block time: 10 minutes
- Adjusts every 2016 blocks (2 weeks)
- Difficulty $\propto$ total hashrate

charts/lesson_16/difficulty_adjustment.pdf

Difficulty Over Time: Bitcoin Example



Observations:

- Exponential growth from 2009 to 2024

Mining Probability and Expected Time

Probability of Success per Hash:

$$P(\text{success}) = \frac{\text{Target}}{2^{256}}$$

Expected Number of Hashes:

$$E[\text{hashes}] = \frac{2^{256}}{\text{Target}} = \text{Difficulty} \times 2^{32}$$

Expected Time to Find Block:

$$T = \frac{\text{Difficulty} \times 2^{32}}{\text{Hashrate}}$$

Example: Difficulty = 50 trillion, Hashrate = 100 TH/s

$$T = \frac{50 \times 10^{12} \times 2^{32}}{100 \times 10^{12}} \approx 2147 \text{ seconds} \approx 36 \text{ minutes}$$

Mining Difficulty vs Hashrate

charts/lesson_16/hashrate_vs_difficulty.pdf

Blockchain Security: The 51% Attack

Attack Scenario:

- Attacker controls >50% hashrate
- Can create longest chain
- Rewrite transaction history
- Double-spend attack

Limitations:

- Cannot steal others' coins
- Cannot create coins from nothing
- Cannot change protocol rules

charts/lesson_16/51_percent_attack.pdf

Confirmation Depth: Security Over Time

charts/lesson_16/confirmation_depth.pdf

Mining Economics: Costs and Rewards

Revenue:

- Block reward: 3.125 BTC (as of 2024, halves every 4 years)
- Transaction fees: Variable (0.1–2 BTC per block)

Costs:

- Hardware (ASICs): \$3,000–\$15,000 per unit
- Electricity: 3–6 cents per kWh (industrial rates)
- Cooling, maintenance, facility

Profitability Equation:

$$\text{Profit} = (\text{Block Reward} + \text{Fees}) \times \text{BTC Price} - \text{Electricity Cost}$$

Break-even: Electricity cost $\approx$ 40–60% of revenue at scale

Mining Hardware Evolution

charts/lesson_16/mining_hardware_evolution.pdf

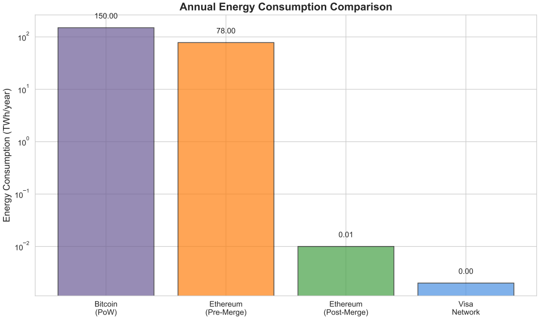
Energy Consumption: The Elephant in the Room

Bitcoin Network (2024):

- Total hashrate: $\sim$600 EH/s
- Power consumption: $\sim$150 TWh/year
- Comparable to Argentina or Netherlands

Per Transaction:

- $\sim$700 kWh per transaction
- vs Visa: $\sim$0.001 kWh
- But: Bitcoin = settlement layer



Energy Sources: Renewable Debate

charts/lesson_16/mining_energy_sources.pdf

Environmental Criticisms and Counterarguments

Criticisms:

- Massive carbon footprint
- E-waste from obsolete ASICs
- Inefficient compared to databases
- Competes with useful computing

Counterarguments:

- Energy = security (makes attacks expensive)
- Incentivizes renewable buildout
- Banking system also energy-intensive
- Enables censorship-resistant money

Trade-off: Security vs energy efficiency (Proof of Stake addresses this)

Mining Centralization Risks

charts/lesson_16/mining_pool_distribution.pdf

Selfish Mining Attack

Strategy:

1. Miner finds block, keeps secret
2. Continues mining on private chain
3. Reveals when ahead by 2+ blocks
4. Honest chain orphaned

Result:

- Unfair revenue (more than hashrate share)
- Effective with >25% hashrate
- Wastes other miners' work

charts/lesson_16/selfish_mining.pdf

Alternatives to Proof of Work

Mechanism	Selection	Pros	Cons
Proof of Work	Computational power	Proven security, decentralized	Energy intensive
Proof of Stake	Staked capital	Energy efficient	Rich get richer, slashing risk
Proof of Authority	Approved validators	Fast, low energy	Centralized, permissioned
Proof of Space	Disk storage	Lower energy than PoW	New, unproven security

Note: Ethereum switched from PoW to PoS in 2022 (The Merge)

Summary

- **Double-Spending Problem:** Solved by probabilistic consensus via PoW
- **Mining:** Find nonce making block hash $<$ target (SHA256 puzzle)
- **Difficulty:** Auto-adjusts to maintain constant block time (10 min for Bitcoin)
- **Security:** 51% attack possible but expensive; confirmation depth increases safety
- **Economics:** Revenue (block reward $+$ fees) vs costs (hardware $+$ electricity)
- **Energy Debate:** $\sim$150 TWh/year, trade-off between security and efficiency

Next Lesson: Proof of Stake – energy-efficient alternative consensus