Digital Finance 3: Technology in Finance

Lesson 27: Supervised Learning - Regression

FHGR

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Summary of key concepts presented above.

Learning Objectives

By the end of this lesson, you will be able to:

- Explain the supervised learning paradigm (features, labels, training)
- Understand simple and multiple linear regression
- Interpret regression coefficients in financial contexts
- Evaluate model performance using R-squared and related metrics
- Recognize overfitting and apply regularization techniques
- Identify finance applications of regression models

Summary of key concepts presented above.

Supervised Learning: The Framework

Core Idea:

- Learn from labeled examples
- **Training data:** $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$
- X = features (inputs, predictors)
- Y = label (output, target)
- Goal: Learn function $f : X \rightarrow Y$

Two Types:

1. **Regression:** Predict continuous Y (today's lesson)
2. **Classification:** Predict discrete Y (next lesson)

Finance Example (Regression):

- Features X: Company financials (P/E, ROE, Size)
- Label Y: Next-month stock return
- Training: Historical data (2000-2020)
- Test: Predict 2021 returns

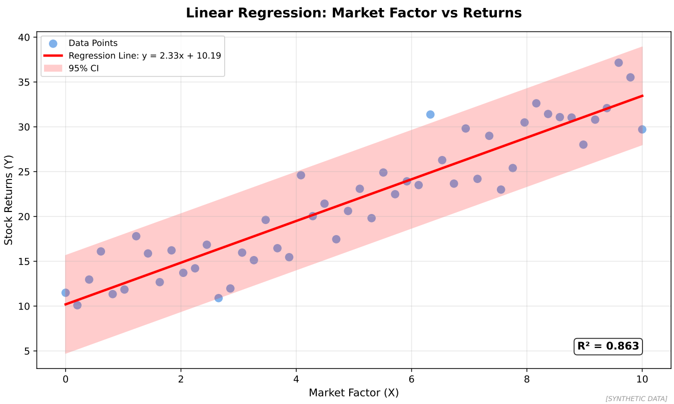
Key Steps:

1. Collect labeled data
2. Split: Train (70%), Validation (15%), Test (15%)
3. Train model on training set
4. Tune on validation set
5. Evaluate on test set (never seen before)

Golden Rule: Never use test data until final evaluation (avoid overfitting).

Summary of key concepts presented above.

Simple Linear Regression: Visual Example



OLS finds the line that minimizes the sum of squared vertical distances (residuals).

Regression models predict continuous outcomes based on input features.

Multiple Linear Regression

Model:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon$$

- Multiple features: $X_1, X_2, \ldots, X_p$
- Each β_j: Partial effect (holding others constant)
- OLS still minimizes squared errors

Matrix Form:

$$Y = X\beta + \epsilon$$
$$\hat{\beta} = (X^T X)^{-1} X^T Y$$

Finance Example:
Predict stock return from:

- X_1: P/E ratio
- X_2: Debt/Equity
- X_3: Market cap (log)
- X_4: Past 12-month return (momentum)

Estimated model:

$$\begin{aligned} \text{Return} = 0.03 &- 0.001 \times \text{P/E} \\ &- 0.015 \times \text{D/E} \\ &+ 0.002 \times \log(\text{Size}) \\ &+ 0.12 \times \text{Mom} \end{aligned}$$

Interpretation:

- Momentum (0.12): Strongest predictor
- Debt (-0.015): Financial risk reduces returns
- Size (+0.002): Weak positive effect

Regression models predict continuous outcomes based on input features.

Assumptions of Linear Regression

Five Key Assumptions:

1. **Linearity:** Relationship is linear
2. **Independence:** Observations are independent
3. **Homoscedasticity:** Constant error variance
4. **Normality:** Errors normally distributed
5. **No multicollinearity:** Features not perfectly correlated

Diagnostics:

- Residual plots (linearity, homoscedasticity)
- QQ plots (normality)
- Variance Inflation Factor (VIF) for multicollinearity

Violations in Finance:

- **Non-linearity:** Returns vs. ratios often non-linear
- **Heteroscedasticity:** Volatility clustering (GARCH effects)
- **Autocorrelation:** Time series dependence
- **Multicollinearity:** Related accounting ratios

Remedies:

- Transformations (log, square root)
- Robust standard errors (White, Newey-West)
- Feature selection (remove correlated vars)
- Non-linear models (polynomial, GAM)

Bottom Line: Regression is robust, but severe violations reduce reliability.

Regression models predict continuous outcomes based on input features.

Evaluating Regression Models: R-squared

R-squared (R^2):

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} = 1 - \frac{\sum(Y_i - \hat{Y}_i)^2}{\sum(Y_i - \bar{Y})^2}$$

- Proportion of variance explained
- Range: $[0, 1]$ (higher is better)
- $R^2 = 0$: Model no better than mean
- $R^2 = 1$: Perfect fit (suspicious!)

Interpretation:

- $R^2 = 0.25$: Model explains 25% of variance
- Remaining 75%: Unexplained (noise, other factors)

Adjusted R-squared:

$$R^2_{adj} = 1 - (1 - R^2)\frac{n-1}{n-p-1}$$

- Penalizes adding features
- Use when comparing models with different p

Typical R^2 in Finance:

- Stock return prediction: 0.02-0.10 (very noisy)
- Bond yield modeling: 0.70-0.95 (more predictable)
- Credit spreads: 0.40-0.60

Warning:

- High R^2 doesn't mean good out-of-sample performance
- Can overfit to training data

Regression models predict continuous outcomes based on input features.

Other Regression Metrics

Mean Absolute Error (MAE):

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|Y_i - \hat{Y}_i|$$

- Average absolute prediction error
- Same units as Y (interpretable)
- Robust to outliers

Root Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}$$

- Penalizes large errors more (squared)
- Same units as Y
- Most common in ML

Mean Absolute Percentage Error (MAPE):

$$MAPE = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{Y_i - \hat{Y}_i}{Y_i}\right|$$

- Percentage error (scale-free)
- Problematic if Y_i near zero

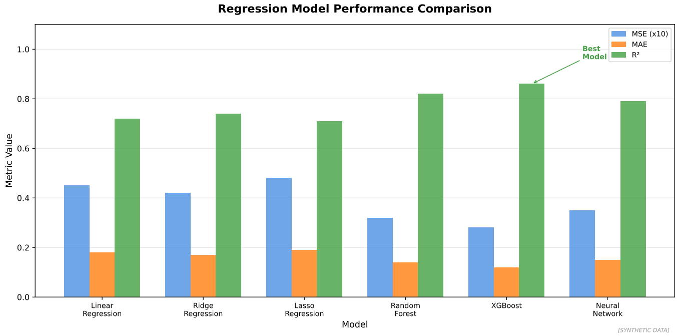
Which to Use?

- RMSE: Standard choice (differentiable, penalizes outliers)
- MAE: If outliers less important
- MAPE: Comparing models across different scales
- R^2: Variance explanation (interpretability)

Key: Always evaluate on held-out test set.

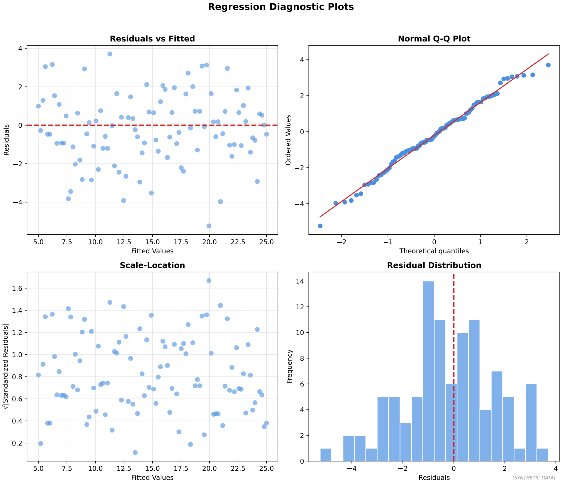
Network metrics provide objective measures of adoption and ecosystem health.

Regression Metrics: Comparison



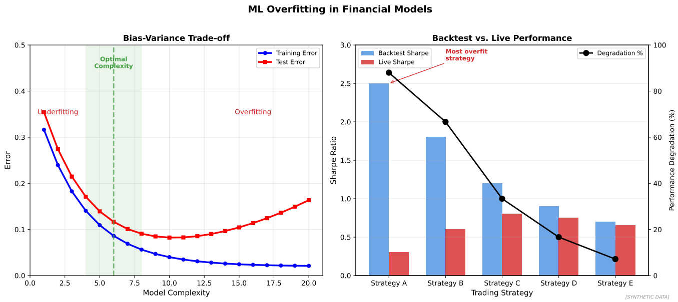
Different metrics emphasize different aspects of model performance.

Residual Analysis



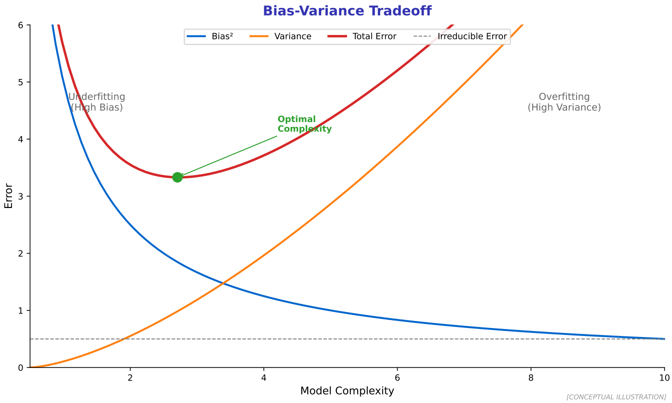
Residual plots help diagnose violations of regression assumptions.

Overfitting Example



The overly complex model fits training noise perfectly but generalizes poorly.

Overfitting is the central challenge in machine learning.



Sweet spot balances underfitting (high bias) and overfitting (high variance).

Finding the right model complexity is key to good generalization.

Regularization: Ridge and Lasso

Idea: Penalize large coefficients to reduce overfitting.

Ridge Regression (L2):

$$\min_{\beta} \sum_{i=1}^{n} (Y_i - X_i\beta)^2 + \lambda \sum_{j=1}^{p} \beta_j^2$$

- Penalty: Sum of squared coefficients
- Shrinks coefficients toward zero
- All features retained (no feature selection)
- λ: Regularization strength (tune via CV)

Effect:

- Reduces variance (less overfitting)
- Increases bias (slightly)
- Handles multicollinearity well

Lasso Regression (L1):

$$\min_{\beta} \sum_{i=1}^{n} (Y_i - X_i\beta)^2 + \lambda \sum_{j=1}^{p} |\beta_j|$$

- Penalty: Sum of absolute coefficients
- Sets some β_j exactly to zero (feature selection)
- Sparse solutions (interpretable)

Elastic Net: Combines L1+L2 penalties for grouped selection

Ridge shrinks coefficients; Lasso enables feature selection; choose lambda via cross-validation.

Cross-Validation

Why Cross-Validation?

- Estimate out-of-sample performance
- Tune hyperparameters (e.g., λ)
- Maximize use of limited data

K-Fold Cross-Validation:

1. Split data into K folds (typically 5 or 10)
2. For each fold k:
   - Train on $K-1$ folds
   - Validate on fold k
3. Average performance across folds

Advantages:

- All data used for training and validation
- Reduces variance of performance estimate

Leave-One-Out CV (LOOCV):

- $K = n$ (extreme case)
- Train on $n-1$, test on 1
- Computationally expensive
- Low bias, high variance

Time Series CV:

- Cannot randomly shuffle (temporal order)
- Use expanding or rolling windows
- Example: Train on 2000-2010, test on 2011; train on 2000-2011, test on 2012; etc.

Best Practice:

- Use CV for model selection
- Reserve separate test set for final evaluation
- Never use test set for tuning

Key concepts from this slide inform practical applications in finance.

Feature Engineering for Regression

Transformations:

- **Log:** $\log(Y)$ or $\log(X)$ (skewed distributions)
- **Polynomial:** X, X^2, X^3 (capture non-linearity)
- **Interactions:** $X_1 \times X_2$ (joint effects)
- **Binning:** Convert continuous to categorical

Finance-Specific:

- Ratios: P/E, P/B, ROE, Debt/Equity
- Momentum: Past returns (1-month, 12-month)
- Volatility: Rolling standard deviation
- Technical indicators: MA, RSI, MACD

Normalization:

- **Standardization:** $(X - \mu)/\sigma$ (mean 0, std 1)
- **Min-Max:** $(X - X_{min})/(X_{max} - X_{min})$ (range [0,1])
- Important for regularization (features on same scale)

Lag Variables (Time Series):

- $Y_{t-1}, Y_{t-2}, \ldots$ (autoregressive)
- Moving averages
- Seasonal indicators

Avoid:

- Leakage (using future info)
- Perfectly correlated features
- Too many features (curse of dimensionality)

Regression models predict continuous outcomes based on input features.

Finance Application: Stock Return Prediction

Problem Setup:

- Target: Next-month stock return
- Features: Fundamentals, technical, macro
- Data: Monthly, 1990-2020
- Universe: S&P 500 stocks

Feature Categories:

1. **Value:** P/E, P/B, dividend yield
2. **Momentum:** Past 12-month return
3. **Quality:** ROE, profit margin, accruals
4. **Size:** Market cap (log)
5. **Volatility:** 60-day std dev

Model Comparison:

- OLS: $R^2 = 0.04$ (test)
- Ridge ($\lambda = 10$): $R^2 = 0.06$
- Lasso ($\lambda = 0.1$): $R^2 = 0.07$ (selected 12/30 features)

Key Findings:

- Momentum strongest predictor ($\beta = 0.15$)
- P/B negative ($\beta = -0.03$) - value effect
- Low overall R^2 (markets are noisy)
- Lasso improves via feature selection

Reality Check:

- Transaction costs erode small edges
- Out-of-sample performance lower
- Regime shifts (models break in crises)

Real-world applications demonstrate the practical value of blockchain technology.

Finance Application: Bond Yield Curve Modeling

Problem:

- Predict bond yields at various maturities
- Features: Macro variables, term structure factors
- More predictable than stocks

Nelson-Siegel Model (Parametric):

$$Y(m) = \beta_1 + \beta_2 e^{-m/\tau} + \beta_3 \frac{m}{\tau} e^{-m/\tau}$$

- β_1: Long-term level
- β_2: Short-term component
- β_3: Curvature
- m: Maturity, τ: Decay parameter

ML Approach (Non-Parametric):

- Features: GDP growth, inflation, Fed Funds rate, VIX, yield spreads
- Target: 10-year Treasury yield
- Ridge regression: $R^2 = 0.82$ (test)

Key Predictors:

- Fed Funds rate ($\beta = 0.65$)
- Inflation expectations ($\beta = 0.42$)
- 2-year yield ($\beta = 0.58$)

Use Cases:

- Portfolio allocation
- Hedging interest rate risk
- Trading strategies (carry, curve)

Real-world applications demonstrate the practical value of blockchain technology.

Finance Application: Real Estate Valuation

Problem: Predict property sale price using hedonic pricing

Key Features:

- Size: Square footage, beds, baths
- Location: Zip code, school quality
- Property: Age, lot size, amenities

Model: $\log(\text{Price}) = \beta_0 + \beta_1 \log(\text{SqFt}) + \cdots$

Results (typical $R^2 \approx 0.75$):

- SqFt: 10% larger $\rightarrow$ 6% higher price
- Extra bedroom: $+\$20$k
- Ridge handles location multicollinearity

Applications: AVMs (Zillow Zestimate), mortgage underwriting, investment analysis

Regression models power automated property valuation across real estate markets.

Limitations of Linear Regression

When Regression Fails:

- Non-linear relationships
- High-dimensional data ($p \gg n$)
- Heavy-tailed distributions (outliers)
- Non-stationary data (regime changes)

Finance Example: Stock returns vs. market cap show small-cap premium (non-linear), January effects, crisis breaks.

Alternatives:

- Polynomial regression: Add X^2, X^3
- GAMs: Smooth non-linear functions
- Tree methods: Random Forests, XGBoost
- Neural networks: Deep learning

Best Practice: Start linear, try non-linear if poor fit

Linear regression assumes constant relationships; use alternatives for non-linear patterns.

Practical Tips for Regression in Finance

Data Preparation:

- Winsorize outliers (1st-99th percentile)
- Check for multicollinearity (VIF ¡ 10)
- Normalize features (especially for regularization)
- Handle missing data carefully

Model Selection:

- Start simple (OLS)
- Add regularization (Ridge/Lasso)
- Use cross-validation for λ
- Check residual diagnostics

Avoid Common Mistakes:

- Look-ahead bias (using future data)
- Overfitting (too many features)
- Ignoring transaction costs
- Extrapolation beyond data range

Interpretation:

- Report coefficients with confidence intervals
- Economic significance ¿ statistical significance
- Explain magnitude in practical terms

Validation:

- Out-of-sample testing (time series: walk-forward)
- Robustness checks (different time periods, subsamples)
- Benchmark against simple models (mean, random walk)

Communication:

- Visualize predictions vs. actuals
- Report multiple metrics (R^2, RMSE, MAE)
- Acknowledge limitations

Regression models predict continuous outcomes based on input features.

Summary and Key Takeaways

Core Concepts:

- Supervised learning: Learn from labeled data
- Linear regression: $Y = X\beta + \epsilon$
- OLS minimizes squared errors
- Multiple regression: Multiple predictors

Evaluation:

- R^2: Variance explained
- RMSE, MAE: Prediction error
- Always test out-of-sample

Overfitting:

- Central problem in ML
- Regularization (Ridge, Lasso) helps
- Cross-validation for tuning

Finance Applications:

- Stock returns (low R^2, noisy)
- Bond yields (higher R^2, predictable)
- Real estate (moderate R^2)
- Limitations: Non-linearity, regime changes

Summary of key concepts presented above.

Next Lesson Preview

Lesson 28: Supervised Learning - Classification

Topics to be covered:

- Logistic regression (binary classification)
- Decision boundaries and probabilities
- Confusion matrix (TP, FP, TN, FN)
- Accuracy, precision, recall, F1-score
- ROC curves and AUC
- Applications: Credit default, fraud detection

Preparation:

- Review probability basics (odds, log-odds)
- Think: What financial problems involve yes/no predictions?

Summary of key concepts presented above.