Methods and Algorithms

Spring 2026

From Rules to Trees

- Decision trees encode if-then-else rules
- Each node splits data based on a feature threshold
- Leaves contain predictions (class or value)

Key Questions

- How to choose the best split?
- When to stop splitting?
- How to make predictions?

Decision trees: the building blocks of Random Forests

Gini Impurity measures class mixture at a node:

$$G = 1 - \sum_{k=1}^{K} p_k^2$$

where p_k is the proportion of class k samples.

Properties:

- $G = 0$: pure node (all samples same class)
- $G = 0.5$: maximum impurity for binary classification
- Lower Gini = better split

Gini impurity: probability of misclassifying a random sample

Entropy measures disorder:

$$H = -\sum_{k=1}^{K} p_k \log_2(p_k)$$

Information Gain:

$$IG = H(\text{parent}) - \sum_{j} \frac{n_j}{n} H(\text{child}_j)$$

Comparison:

- Gini: faster to compute, tends to isolate most frequent class
- Entropy: more balanced trees, slightly slower
- In practice: similar performance

Both criteria aim to create pure child nodes

For regression, use Mean Squared Error:

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

Split quality:

$$\mathrm{Reduction} = \mathrm{MSE}(\mathrm{parent}) - \sum_{j}\frac{n_j}{n}\mathrm{MSE}(\mathrm{child}_j)$$

Leaf prediction: mean of samples in leaf

Trees can handle both classification and regression tasks

Recursive Partitioning:

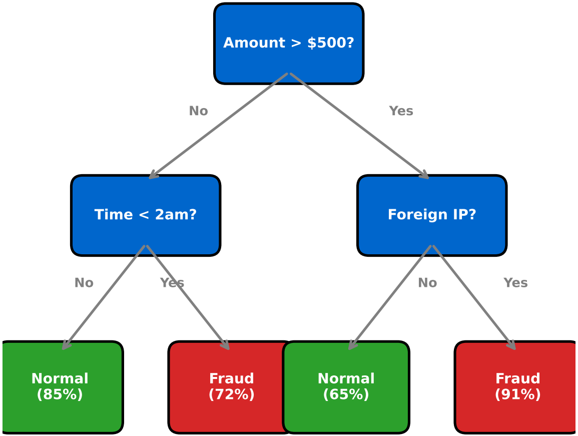
1. Start with all samples at root
2. For each feature and threshold:
   - Calculate impurity reduction
   - Select split with maximum reduction
3. Create child nodes with split samples
4. Recurse until stopping criterion met

Stopping Criteria:

- Maximum depth reached
- Minimum samples per leaf
- No improvement in impurity

Greedy algorithm: locally optimal splits at each step

Decision Tree for Fraud Detection



https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L04_Random_Forests/01_decision_tree

Each path through the tree represents a fraud detection rule

Problem with Single Trees:

- High variance: small data changes → very different trees
- Prone to overfitting
- Unstable predictions

Ensemble Solution:

- Train multiple diverse models
- Combine predictions
- Reduce variance while maintaining low bias

"Wisdom of crowds": aggregate many weak learners into strong learner

Bootstrap: sample with replacement from original data

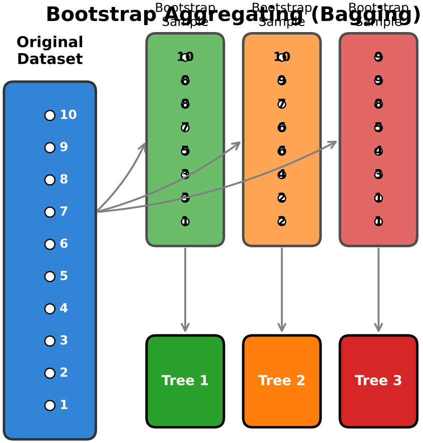
Properties:

- Each sample: same size as original (n observations)
- Expected unique samples: $\approx 63.2\%$ (probability $1 - (1 - 1/n)^n$)
- Remaining $\sim 37\%$: out-of-bag (OOB) samples

Effect:

- Each tree sees different data subset
- Creates diversity among trees
- OOB samples provide validation

Bootstrap: key ingredient for reducing variance through aggregation



Each tree trained on ~63% unique samples (with replacement)

https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L04_Random_Forests/03_bootstrap

Bootstrap Aggregating: train on random samples, aggregate predictions

Variance Reduction by Averaging

For B independent predictions with variance σ^2:

$$\text{Var}\left(\frac{1}{B}\sum_{b=1}^{B}\hat{f}_b(x)\right) = \frac{\sigma^2}{B}$$

With correlation ρ:

$$\text{Var} = \rho\sigma^2 + \frac{1-\rho}{B}\sigma^2$$

Key insight: Reduce correlation between trees to maximize variance reduction

Lower correlation between trees = greater ensemble benefit

Two Sources of Randomness:

1. **Bootstrap sampling**: each tree trained on random sample
2. **Feature randomization**: each split considers random subset

Feature Subset Size (at each split):

- Classification: $\sqrt{p}$ features (default)
- Regression: $p/3$ features (default)
- Decorrelates trees more than bagging alone

Feature randomization: Breiman's key innovation over bagging

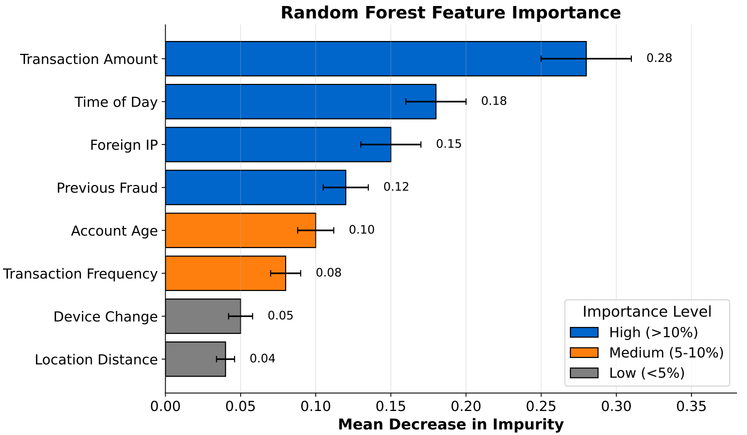
Training:

1. For $b = 1$ to B trees:
    - Draw bootstrap sample of size n
    - Grow tree:
        - At each node, select m features randomly
        - Find best split among m features
        - Split until stopping criterion

Prediction:

- Classification: majority vote across trees
- Regression: average predictions

Typical: 100-500 trees, but more trees never hurts (just slower)

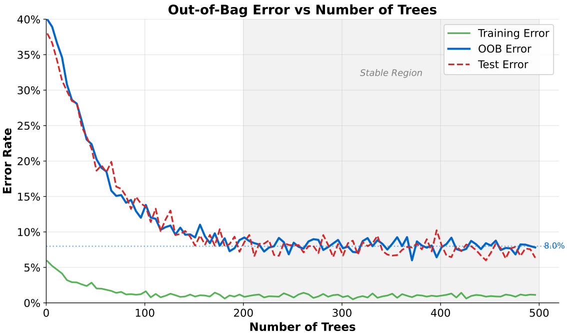


https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L04_Random_Forests/02_feature_importance

Mean Decrease in Impurity: sum of impurity reductions from splits on feature

1. **Mean Decrease in Impurity (MDI):**
   - Sum of Gini/entropy reductions from splits on feature
   - Fast to compute (comes free from training)
   - Bias toward high-cardinality features
2. **Permutation Importance:**
   - Permute feature values, measure accuracy drop
   - More reliable, less biased
   - Slower (requires re-evaluation)

Permutation importance preferred for final feature selection



https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L04_Random_Forests/04_oob_error

OOB error: free cross-validation using samples not in bootstrap

For each observation *i*:

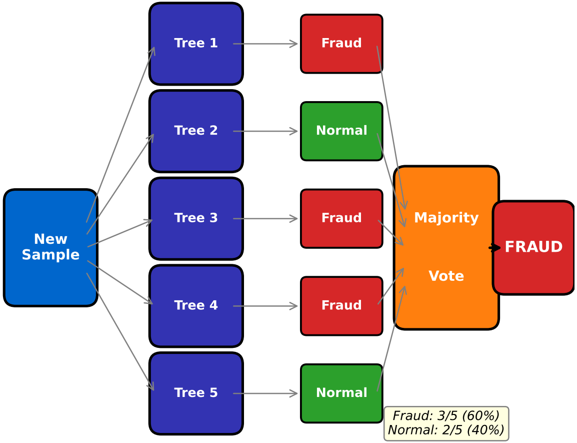
1. Identify trees where *i* was OOB (not in bootstrap sample)
2. Aggregate predictions from only those trees
3. Compare to true label

Benefits:

- No separate validation set needed
- Uses $\sim 37\%$ of trees per sample
- Unbiased estimate of generalization error

OOB error converges to leave-one-out cross-validation error

Ensemble Voting (Classification)



New Sample

Tree 1

Tree 2

Tree 3

Tree 4

Tree 5

Fraud

Normal

Fraud

Fraud

Normal

Majority Vote

FRAUD

Fraud: 3/5 (60%)
Normal: 2/5 (40%)

https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L04_Random_Forests/05_ensemble_voting

Classification: majority vote. Regression: average prediction

Expected Prediction Error:

$$\mathbb{E}[(y - \hat{f}(x))^2] = \text{Bias}^2 + \text{Variance} + \text{Irreducible Error}$$

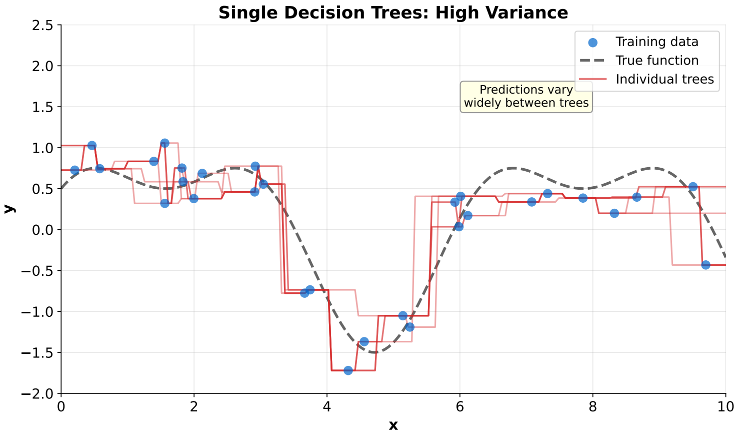
Single Tree:

- Low bias (can fit complex patterns)
- High variance (sensitive to training data)

Random Forest:

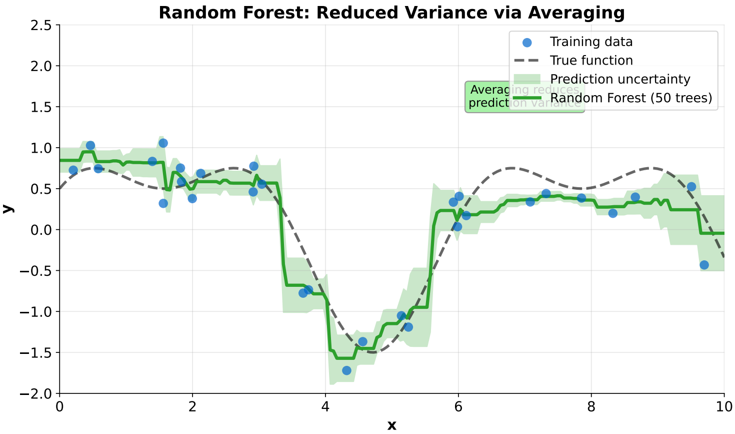
- Bias: similar to single tree
- Variance: reduced by $\approx$ factor of $1/B$ (with decorrelation)

Ensembles reduce variance without increasing bias



https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L04_Random_Forests/06a_single_tree_variance

Individual trees overfit to their bootstrap samples, producing erratic predictions



https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L04_Random_Forests/06b_random_forest_variance

Averaging decorrelated trees dramatically reduces prediction variance

n_estimators (number of trees):

- More trees = lower variance, never overfits
- Diminishing returns after 100-500 trees
- Cost: linear increase in training/prediction time

Guidelines:

- Start with 100, increase if OOB error still decreasing
- For production: balance accuracy vs. latency
- More trees always better (if time permits)

Unlike most hyperparameters, more trees cannot hurt accuracy

max_depth: Maximum tree depth

- Deeper = more complex patterns, higher variance
- Default: unlimited (grow full trees)

min_samples_split: Minimum samples to split

- Higher = simpler trees, more regularization
- Default: 2 (full trees)

min_samples_leaf: Minimum samples in leaf

- Higher = smoother predictions
- Default: 1 (full trees)

Full trees (default) often work well due to bagging's variance reduction

max_features: Features considered at each split

- Lower = more decorrelated trees, higher bias
- Higher = less decorrelated, lower bias

Defaults:

- Classification: $\sqrt{p}$ (e.g., 10 features → 3)
- Regression: $p/3$ (e.g., 30 features → 10)

Tuning:

- Try: $\sqrt{p}$, $\log_2(p)$, $p/3$
- Cross-validate to find optimal

Feature randomization is key differentiator from bagged trees

Advantages:

- Excellent accuracy out-of-the-box
- Handles mixed feature types
- Built-in feature importance
- Robust to outliers and missing values
- Parallelizable (trees independent)

Limitations:

- Less interpretable than single tree
- Memory intensive (stores all trees)
- Slower prediction than linear models
- Cannot extrapolate beyond training range

Random Forests: excellent default choice for tabular data

Aspect	**RF**	**Single Tree**	**Logistic**	**KNN**
Accuracy	High	Medium	Medium	Medium
Interpretability	Medium	High	High	Low
Training Speed	Medium	Fast	Fast	Fast
Feature Importance	Yes	Yes	Yes	No
Non-linear	Yes	Yes	No	Yes

RF trades some interpretability for significant accuracy gains

Use When:

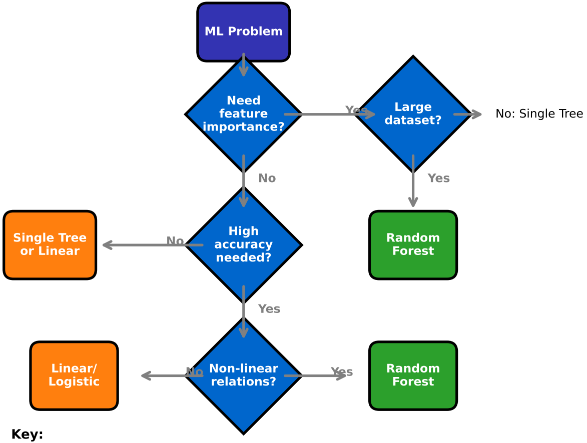
- Tabular data with mixed feature types
- Non-linear relationships expected
- Feature importance needed
- Out-of-the-box performance matters

Consider Alternatives When:

- Need fully interpretable model (use single tree)
- Very high-dimensional sparse data (use linear models)
- Extrapolation required (use parametric models)
- Need fastest prediction (use linear/shallow tree)

Random Forest: often the first model to try on tabular data

When to Use Random Forests



Key:

Random Forest: Best for accuracy + feature importance

Alternative: When interpretability is paramount

https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L04_Random_Forests/07_decision_flowchart

Start with Random Forest; switch if specific constraints require it

Open the Colab Notebook

- Exercise 1: Train a decision tree on credit data
- Exercise 2: Build a random forest and analyze feature importance
- Exercise 3: Tune hyperparameters with cross-validation

Link: `https://colab.research.google.com/` [TBD]

Classification:

```
from sklearn.ensemble import RandomForestClassifier
rf = RandomForestClassifier(n_estimators=100, max_features='sqrt',
                            oob_score=True, random_state=42)
rf.fit(X_train, y_train)
print(f"OOB Score: {rf.oob_score_:.3f}")
```

Key Parameters:

- `n_estimators`: number of trees
- `max_features`: features per split
- `oob_score`: compute OOB error
- `n_jobs`: parallel trees (-1 for all cores)

Set random_state for reproducibility

Core Concepts:

- Ensemble of decision trees with bootstrap + feature randomization
- Reduces variance while maintaining low bias
- OOB error provides free cross-validation

Practical Takeaways:

- Excellent default for tabular data
- Feature importance aids interpretation
- More trees never hurts (just slower)
- Hyperparameter tuning usually optional

Next: PCA and t-SNE for dimensionality reduction

Textbooks:

- James et al. (2021). *ISLR*, Chapter 8: Tree-Based Methods
- Hastie et al. (2009). *ESL*, Chapter 15: Random Forests

Original Papers:

- Breiman, L. (2001). Random Forests. *Machine Learning*, 45(1), 5-32.
- Breiman, L. (1996). Bagging Predictors. *Machine Learning*, 24(2), 123-140.

Documentation:

- scikit-learn: `sklearn.ensemble.RandomForestClassifier`

Breiman's 2001 paper: one of the most cited in ML