L02: Logistic Regression
Classification with Probability Estimates

Methods and Algorithms – MSc Data Science

Learning Objectives

By the end of this lecture, you will be able to:

1. Explain how logistic regression models binary outcomes
2. Derive the maximum likelihood estimation for logistic regression
3. Interpret classification metrics (precision, recall, AUC)
4. Apply logistic regression for credit scoring decisions

Finance Application: Credit default prediction

These objectives span Bloom's levels: Understand, Apply, Analyze

Why Logistic Regression?

The Business Problem

- Banks must decide: approve or reject loan applications
- Need probability of default, not just yes/no prediction
- Regulatory requirement: interpretable, auditable models

Why Not Linear Regression?

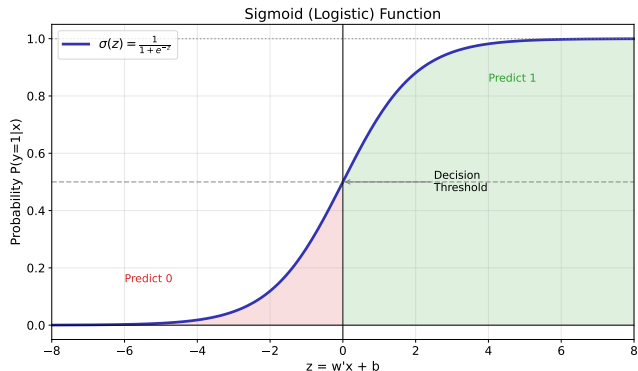- Linear regression can predict values outside [0,1]
- Binary outcomes need probability-based approach
- Logistic regression outputs calibrated probabilities

Logistic regression: the industry standard for credit scoring since 1980s
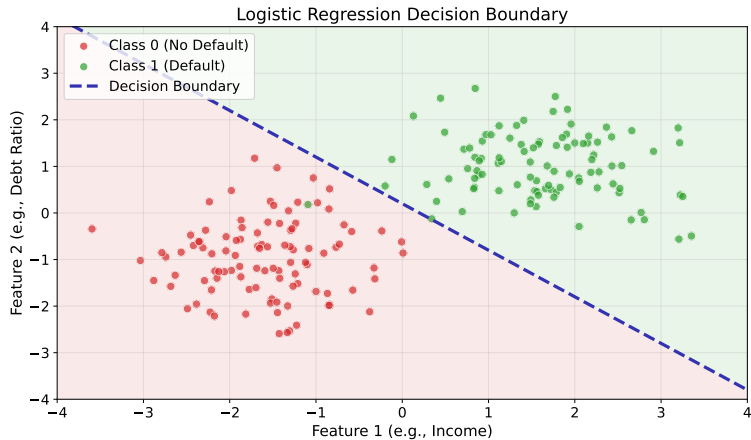
The Sigmoid Function

From Linear to Probability

- Maps any real number to (0, 1) range
- Smooth, differentiable, interpretable

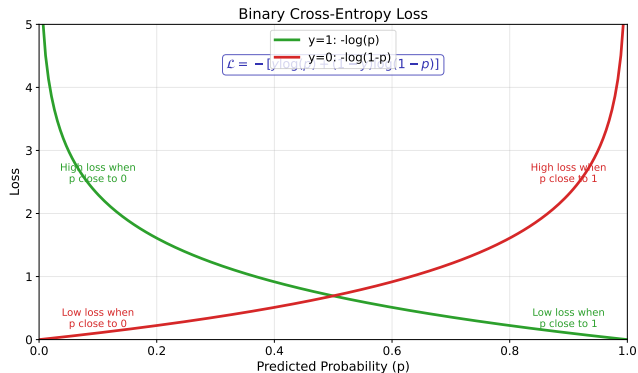$\sigma(z) = 1/(1 + e^{-z})$ transforms linear combination to probability

Decision Boundary

Logistic Regression Decision Boundary

The decision boundary is where $P(y = 1|x) = 0.5$, i.e., $w'x + b = 0$

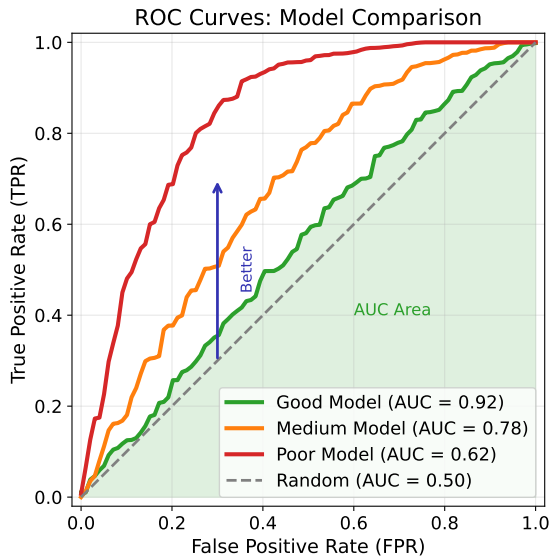# Binary Cross-Entropy Loss

Why Not MSE?

- MSE with sigmoid creates non-convex loss landscape
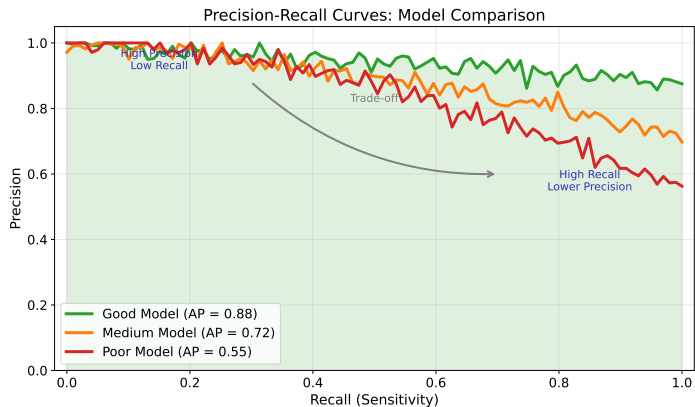- Cross-entropy is convex, guarantees global optimum

Binary Cross-Entropy Loss

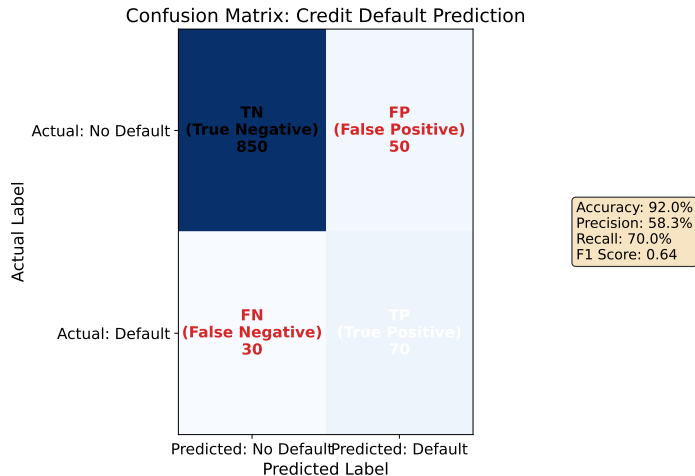$$\mathcal{L} = -[y \log(p) + (1-y) \log(1-p)]$$

Heavily penalizes confident wrong predictions

ROC Curves: Model Comparison

AUC = probability random positive ranks higher than random negative

Precision-Recall Curves: Model Comparison

Legend:
- Good Model (AP = 0.88)
- Medium Model (AP = 0.72)
- Poor Model (AP = 0.55)

Use PR curve when classes are imbalanced (common in fraud detection)
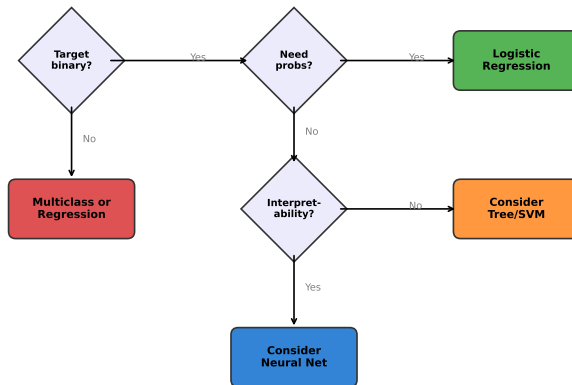
Confusion Matrix: Credit Default Prediction

FP = approve bad loans (costly), FN = reject good customers (lost revenue)

Logistic Regression Decision Guide

Key strengths: interpretable coefficients, probability outputs, fast training