Methods and Algorithms

Spring 2026

The Problem with One-Hot Encoding

- Vocabulary of 10,000 words $\rightarrow$ 10,000-dim sparse vectors
- No semantic similarity: "king" and "queen" equally distant from "car"
- Curse of dimensionality

Solution: Dense Embeddings

- Map words to dense vectors (50-300 dimensions)
- Similar words $\rightarrow$ similar vectors
- Learn from context (distributional hypothesis)

"You shall know a word by the company it keeps" – Firth, 1957

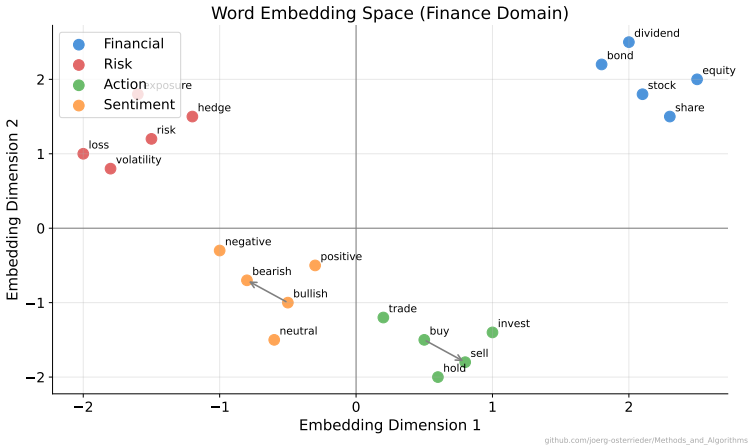
Objective: Predict context words given target word

$$P(w_{context}|w_{target}) = \frac{\exp(v_{context}^T v_{target})}{\sum_{w \in V} \exp(v_w^T v_{target})}$$

Training:

- Slide window over text corpus
- For each word, predict surrounding words
- Update embeddings via gradient descent

Skip-gram works well for rare words; CBOW better for frequent words



Finance terms cluster by semantic category in embedding space

Famous Example:

$$\vec{king} - \vec{man} + \vec{woman} \approx \vec{queen}$$

Finance Examples:

- $\vec{stock} - \vec{equity} + \vec{debt} \approx \vec{bond}$
- $\vec{CEO} - \vec{company} + \vec{country} \approx \vec{president}$

How it works:

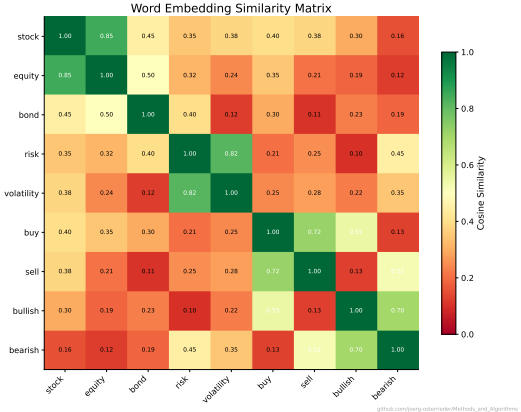
- Vector arithmetic in embedding space
- Relationships encoded as directions

Embeddings capture relational structure, not just similarity

Cosine Similarity:

$$\text{sim}(u, v) = \frac{u \cdot v}{||u|| \cdot ||v||} = \cos(\theta)$$

- Range: $[-1, 1]$; 1=identical, 0=orthogonal, -1=opposite



Cosine similarity ignores magnitude, focuses on direction

Popular Options:

- **Word2Vec**: Google, 300-dim, 3M words
- **GloVe**: Stanford, trained on Wikipedia + Common Crawl
- **FastText**: Facebook, handles subwords (OOV robust)

Domain-Specific:

- FinBERT: Pre-trained on financial text
- BioBERT: Biomedical domain

Fine-tuning pre-trained embeddings usually outperforms training from scratch

Applications:

- **Sentiment Analysis**: News → embedding → positive/negative
- **Document Similarity**: Find similar SEC filings
- **Named Entity Recognition**: Extract company names
- **Event Detection**: Identify earnings announcements

Sentence Embeddings:

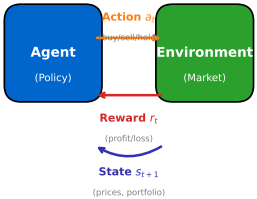
- Average word vectors (simple baseline)
- Doc2Vec (paragraph vectors)
- Sentence-BERT (state-of-the-art)

Aggregate word embeddings to represent documents

Key Components:

- **Agent**: Learner/decision-maker
- **Environment**: What agent interacts with
- **State** s: Current situation
- **Action** a: What agent can do
- **Reward** r: Feedback signal

Reinforcement Learning: Agent-Environment Interaction



At each time step t:

Agent observes state, takes action, receives reward

github.com/joerg-osterrieder/Methods_and_Algorithms

RL: Learning from interaction, not from labeled examples

MDP Tuple: (S, A, P, R, γ)

- S: Set of states
- A: Set of actions
- $P(s'|s, a)$: Transition probability
- $R(s, a, s')$: Reward function
- $\gamma \in [0, 1]$: Discount factor

Markov Property:

$$P(s_{t+1}|s_t, a_t, s_{t-1}, ...) = P(s_{t+1}|s_t, a_t)$$

Future depends only on current state, not history

Policy: $\pi(a|s) = P(A_t = a|S_t = s)$

- Maps states to action probabilities
- Goal: Find optimal policy π^*

Value Function:

$$V^\pi(s) = \mathbb{E}_\pi \left[\sum_{t=0}^{\infty} \gamma^t R_{t+1} | S_0 = s \right]$$

Q-Function (Action-Value):

$$Q^\pi(s, a) = \mathbb{E}_\pi \left[\sum_{t=0}^{\infty} \gamma^t R_{t+1} | S_0 = s, A_0 = a \right]$$

Q-function: expected return starting from state s, taking action a

Optimal Q-Function:

$$Q^*(s, a) = \mathbb{E}\left[R + \gamma \max_{a'} Q^*(s', a')\right]$$

Interpretation:

- Value = immediate reward + discounted future value
- Recursive definition enables dynamic programming

Optimal Policy:

$$\pi^*(s) = \arg\max_a Q^*(s, a)$$

Bellman equation: foundation of all value-based RL methods

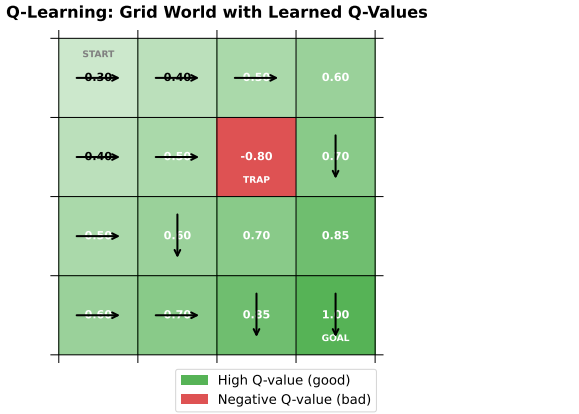
Update Rule:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \left[r + \gamma \max_{a'} Q(s',a') - Q(s,a) \right]$$

Algorithm:

1. Initialize $Q(s,a)$ arbitrarily
2. For each episode:
   - Observe state s
   - Choose action a (ϵ-greedy)
   - Execute a, observe r, s'
   - Update $Q(s,a)$

Q-learning is off-policy: learns optimal Q regardless of policy followed



Arrows show policy; colors show Q-values (green=high, red=negative)

The Dilemma:

- **Exploit**: Choose best known action (greedy)
- **Explore**: Try new actions (discover better options)

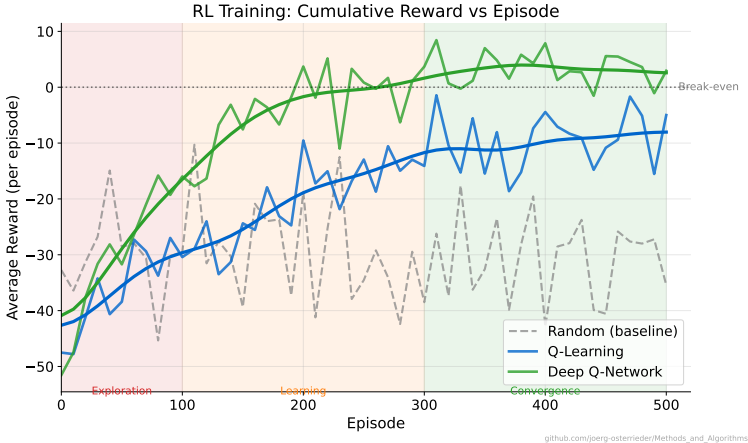
ϵ-Greedy Strategy:

$$a = \begin{cases} \arg\max_a Q(s,a) & \text{with probability } 1-\epsilon \\ \text{random action} & \text{with probability } \epsilon \end{cases}$$

Decay Schedule:

- Start with high ϵ (explore more)
- Decay ϵ over time (exploit more)

Balance: too much exploration wastes time; too little misses optima



Reward improves as agent learns; DQN often outperforms tabular Q-learning

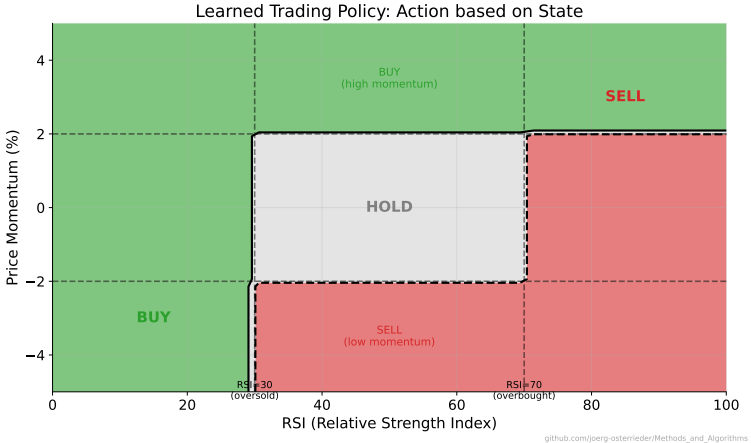
Formulation:

- **State**: Price history, portfolio, technical indicators
- **Action**: Buy, sell, hold (+ position size)
- **Reward**: Profit/loss, risk-adjusted return

Challenges:

- Non-stationary environment
- High noise, low signal-to-noise ratio
- Transaction costs
- Partial observability

RL for trading is active research area; not solved problem



Learned policy: buy when oversold/high momentum, sell when overbought

Idea: Use neural network to approximate Q-function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

Key Innovations:

- **Experience Replay**: Store transitions, sample randomly
- **Target Network**: Separate network for stability
- **Function Approximation**: Handle large state spaces

DQN: breakthrough for RL on Atari games (Mnih et al., 2015)

Direct Policy Optimization:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} \left[\nabla_\theta \log \pi_\theta(a|s) \cdot Q^{\pi_\theta}(s, a) \right]$$

Algorithms:

- REINFORCE (vanilla policy gradient)
- Actor-Critic (combine value and policy)
- PPO (Proximal Policy Optimization)
- A3C (Asynchronous Advantage Actor-Critic)

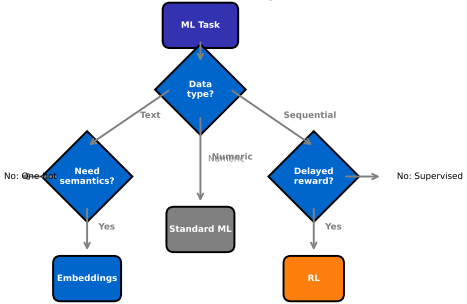
Policy gradient: directly optimize policy, can handle continuous actions

Open the Colab Notebook

- Exercise 1: Explore word embeddings with Word2Vec
- Exercise 2: Implement basic Q-learning
- Exercise 3: Apply RL to a simple trading environment

Link: `https://colab.research.google.com/` [TBD]

When to Use Embeddings vs RL



Embeddings: Text, categorical -> dense vectors (Word2Vec, BERT)

RL: Sequential decisions with delayed rewards (trading, games)

github.com/joerg-osterrieder/Methods_and_Algorithms

Embeddings for text/categorical; RL for sequential decisions

Aspect	Embeddings	RL
Input	Text, categorical	State sequence
Output	Dense vectors	Actions/policy
Learning	Unsupervised/supervised	Trial and error
Signal	Context (words)	Rewards
Key challenge	Semantics	Credit assignment
Finance use	Sentiment	Trading

Both transform complex inputs into learnable representations

Embeddings in Python:

- `gensim.models.Word2Vec`: Train your own
- `gensim.downloader.load('glove-wiki-gigaword-100')`: Pre-trained
- `transformers.BertModel`: BERT embeddings

RL Libraries:

- `gymnasium`: Environment interface (formerly OpenAI Gym)
- `stable-baselines3`: Pre-implemented algorithms
- `ray[rllib]`: Scalable RL

Start with pre-trained embeddings; use stable-baselines3 for RL

Embeddings:

- Start with pre-trained, fine-tune if needed
- Check domain match (general vs financial)
- Visualize with t-SNE/UMAP to verify quality

RL:

- Start simple (tabular Q-learning before DQN)
- Reward shaping is crucial (sparse rewards are hard)
- Normalize observations
- Use established environments first (Gym, FinRL)

Both domains: start simple, iterate, validate thoroughly

Embeddings:

- Dense vector representations of text/categories
- Capture semantic similarity
- Use pre-trained (Word2Vec, GloVe, BERT)

Reinforcement Learning:

- Agent learns from environment interaction
- Q-learning: value-based, tabular or deep (DQN)
- Applications: trading, portfolio optimization

Key Takeaway: Different tools for different problems

Course complete! Apply these methods in your capstone project

Embeddings:

- Mikolov et al. (2013). Word2Vec
- Pennington et al. (2014). GloVe
- Devlin et al. (2019). BERT

Reinforcement Learning:

- Sutton & Barto (2018). RL: An Introduction (free online)
- Mnih et al. (2015). DQN (Atari)
- Schulman et al. (2017). PPO

Finance Applications:

- Liu et al. (2021). FinRL: Deep RL for Trading
- Araci (2019). FinBERT

Sutton & Barto: the definitive RL textbook (free at incompleteideas.net)