L06: Embeddings & RL

Deep Dive: Theory, Implementation, and Applications

Methods and Algorithms

MSc Data Science

Spring 2026

Outline

1. Problem
2. Method
3. Solution
4. Practice
5. Decision Framework
6. Summary

Part 1: Word Embeddings Introduction

The Problem with One-Hot Encoding

- Vocabulary of 10,000 words → 10,000-dim sparse vectors
- No semantic similarity: “king” and “queen” equally distant from “car”
- Curse of dimensionality

Solution: Dense Embeddings

- Map words to dense vectors (50-300 dimensions)
- Similar words → similar vectors
- Learn from context (distributional hypothesis)

“You shall know a word by the company it keeps” – Firth, 1957

Word2Vec: Skip-gram

Objective: Predict context words given target word

$$P(w_{context}|w_{target}) = \frac{\exp(v_{context}^T v_{target})}{\sum_{w \in V} \exp(v_w^T v_{target})}$$

Training:

- Slide window over text corpus
- For each word, predict surrounding words
- Update embeddings via gradient descent

Skip-gram works well for rare words; CBOW better for frequent words

Skip-gram: Computational Challenge

Problem: The softmax denominator sums over **entire vocabulary**:

$$\sum_{w \in V} \exp(v_w^T v_{target}) \quad \text{— } O(|V|) \text{ per update!}$$

For $|V| = 100{,}000$ words, this is computationally intractable.

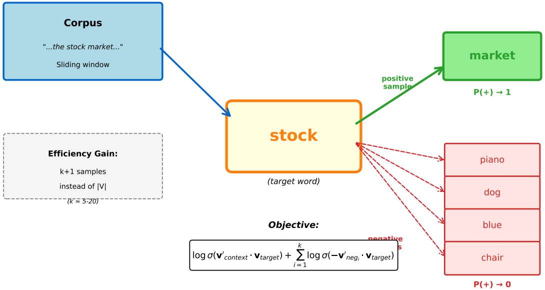
Solution: Negative Sampling (Mikolov et al., 2013b)
Replace full softmax with binary classification:

$$\log \sigma({v'_{w_O}}^T v_{w_I}) + \sum_{i=1}^{k} \mathbb{E}_{w_i \sim P_n} \left[\log \sigma(-{v'_{w_i}}^T v_{w_I}) \right]$$

- Positive pair: (target, true context) → predict 1
- k negative pairs: (target, random word) → predict 0
- Reduces $O(|V|)$ to $O(k)$ where $k = 5$–20

Negative sampling: the key innovation that made Word2Vec practical

Negative Sampling Illustrated

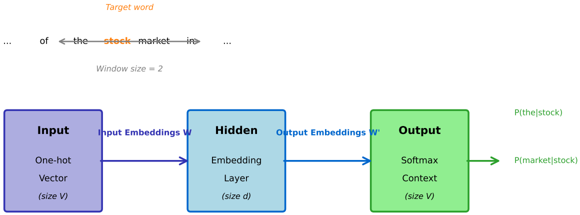


$$\log \sigma(\mathbf{v}'_{context} \cdot \mathbf{v}_{target}) + \sum_{i=1}^{k} \log \sigma(-\mathbf{v}'_{neg_i} \cdot \mathbf{v}_{target})$$

Negative Sampling: Binary Classification Instead of Full Softmax

Binary classification: distinguish true context words from random “noise” words

Skip-gram Architecture



Skip-gram Architecture: Predict Context from Target

Two embedding matrices: input W (word vectors) and output W' (context vectors)

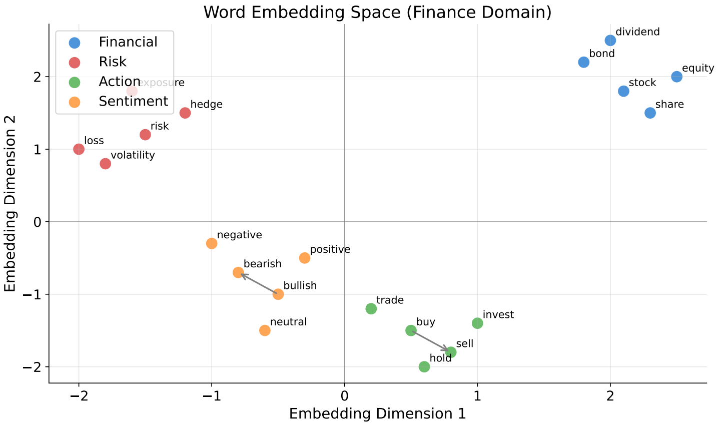
Skip-Gram with Negative Sampling: Algorithm

```
Require: corpus, embedding dim $d$, negatives $k$, window size, epochs
Initialize $W, W' \in \mathbb{R}^{|V| \times d}$ randomly
for each epoch do
    for each word $w_t$ in corpus do
        for each context word $w_c$ within window do
            Positive: update $(w_t, w_c)$ to increase $\sigma(v_{w_t}^\top v'_{w_c})$
            for $i = 1, \ldots, k$ do
                {$k$ negative samples}

                Sample $w_n \sim P_n(w) \propto f(w)^{3/4}$
                Negative: update $(w_t, w_n)$ to decrease $\sigma(v_{w_t}^\top v'_{w_n})$
            end for
        end for
    end for
end for
return $W$ (word embeddings)
```

Mikolov et al. (2013). Distributed representations of words and phrases. NeurIPS, 3111–3119.

Word Embedding Space



https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L06_Embeddings_RL/01_word_embedding_space

Finance terms cluster by semantic category in embedding space

Word Analogies

Famous Example:

$$\vec{king} - \vec{man} + \vec{woman} \approx \vec{queen}$$

Finance Examples:

- $\vec{stock} - \vec{equity} + \vec{debt} \approx \vec{bond}$
- $\vec{CEO} - \vec{company} + \vec{country} \approx \vec{president}$

How it works:

- Vector arithmetic in embedding space
- Relationships encoded as directions

Embeddings capture relational structure, not just similarity

Word Analogy Limitations

Known Issues:

- Success rates typically 40–70%, not near 100% (Levy & Goldberg, 2014)
- Evaluation methodology inflates accuracy (nearest-neighbor dominance)
- Finance-domain analogies ($\vec{stock} - \vec{equity} + \vec{debt} \approx \vec{bond}$) not empirically validated

Bias in Embeddings:

- Embeddings encode societal biases from training data (Bolukbasi et al., 2016)
- Example: man:programmer :: woman:homemaker
- **Finance concern**: Biased embeddings in credit scoring or hiring tools

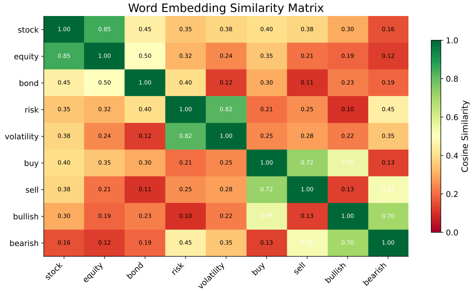
Critical thinking: embeddings capture statistical patterns, including harmful ones

Similarity Measures

Cosine Similarity:

$$\text{sim}(u, v) = \frac{u \cdot v}{||u|| \cdot ||v||} = \cos(\theta)$$

- Range: $[-1, 1]$; 1=same direction, 0=orthogonal, -1=opposite



	stock	equity	bond	risk	volatility	buy	sell	bullish	bearish
stock	1.00	0.85	0.45	0.35	0.38	0.40	0.38	0.30	0.16
equity	0.85	1.00	0.50	0.32	0.24	0.35	0.21	0.19	0.12
bond	0.45	0.50	1.00	0.40	0.12	0.30	0.11	0.23	0.19
risk	0.35	0.32	0.40	1.00	0.82	0.21	0.25	0.10	0.45
volatility	0.38	0.24	0.12	0.82	1.00	0.25	0.28	0.22	0.35
buy	0.40	0.35	0.30	0.21	0.25	1.00	0.72	0.55	0.13
sell	0.38	0.21	0.11	0.25	0.28	0.72	1.00	0.13	0.52
bullish	0.30	0.19	0.23	0.10	0.22	0.55	0.13	1.00	0.70
bearish	0.16	0.12	0.19	0.45	0.35	0.13	0.52	0.70	1.00

https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L06_Embeddings_RL/02_similarity_heatmap

Cosine similarity ignores magnitude, focuses on direction

Pre-trained Embeddings

Popular Options:

- **Word2Vec**: Google, 300-dim, 3M words
- **GloVe**: Stanford, trained on Wikipedia + Common Crawl
- **FastText**: Facebook, handles subwords (OOV robust)

Domain-Specific:

- **FinBERT**: BERT further pre-trained on financial corpora (Araci, 2019)
- BioBERT: Biomedical domain

Fine-tuning pre-trained embeddings usually outperforms training from scratch

Static vs Contextual Embeddings

Static Embeddings (Word2Vec, GloVe, FastText):

- ONE fixed vector per word, regardless of context
- "bank" in "river bank" = "bank" in "bank account"
- Fast, simple, good baseline

Contextual Embeddings (BERT, GPT, FinBERT):

- DIFFERENT vector per occurrence based on surrounding context
- "bank" gets different representations in different sentences
- State-of-the-art for most NLP tasks

Key Insight: Contextual models solve polysemy (multiple word senses)

Static: one meaning per word. Contextual: meaning depends on context.

Embeddings in Finance

Applications:

- **Sentiment Analysis**: News → embedding → positive/negative
- **Document Similarity**: Find similar SEC filings
- **Named Entity Recognition**: Extract company names
- **Event Detection**: Identify earnings announcements

Sentence Embeddings:

- Average word vectors (simple but loses word order: “bank robber” = “robber bank”)
- Doc2Vec (paragraph vectors)
- Sentence-BERT (state-of-the-art)

Aggregate word embeddings to represent documents

Finance Example: Embedding-Based Sentiment

Task: Classify "Fed signals rate hike" as positive or negative

Step 1: Average word embeddings (simplified 3-dim vectors):

$$\vec{v}_{\text{sentence}} = \frac{1}{4}(\vec{v}_{\text{Fed}} + \vec{v}_{\text{signals}} + \vec{v}_{\text{rate}} + \vec{v}_{\text{hike}}) = [0.12, -0.31, 0.45]$$

Step 2: Compare to sentiment anchors via cosine similarity:

- $\text{sim}(\vec{v}_{\text{sentence}}, \vec{v}_{\text{positive}}) = 0.23$
- $\text{sim}(\vec{v}_{\text{sentence}}, \vec{v}_{\text{negative}}) = 0.61$

Step 3: Classify: **Negative sentiment** (rate hikes → tighter policy)

Real-world: Use FinBERT for production sentiment (up to 87% accuracy on financial text; Araci, 2019)

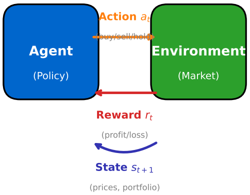
Simplified example — real embeddings are 300-768 dimensions with learned sentiment structure

Part 2: Reinforcement Learning Framework

Key Components:

- **Agent**: Learner/decision-maker
- **Environment**: What agent interacts with
- **State** s: Current situation
- **Action** a: What agent can do
- **Reward** r: Feedback signal

Reinforcement Learning: Agent-Environment Interaction



At each time step t:

Agent observes state, takes action, receives reward

https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L06_Embeddings_RL/03_rl_loop

RL: Learning from interaction, not from labeled examples

Markov Decision Process

MDP Tuple: (S, A, P, R, γ)

- S: Set of states
- A: Set of actions
- $P(s'|s, a)$: Transition probability
- $R(s, a, s')$: Reward function
- $\gamma \in [0, 1)$: Discount factor (or $\gamma \in [0, 1]$ for episodic tasks)

Markov Property:

$$P(s_{t+1}|s_t, a_t, s_{t-1}, ...) = P(s_{t+1}|s_t, a_t)$$

Future depends only on current state, not history

Policy and Value Functions

Policy: $\pi(a|s) = P(A_t = a|S_t = s)$

- Maps states to action probabilities
- Goal: Find optimal policy π^*

Value Function:

$$V^\pi(s) = \mathbb{E}_\pi\left[\sum_{t=0}^{\infty} \gamma^t R_{t+1} | S_0 = s\right]$$

Q-Function (Action-Value):

$$Q^\pi(s, a) = \mathbb{E}_\pi\left[\sum_{t=0}^{\infty} \gamma^t R_{t+1} | S_0 = s, A_0 = a\right]$$

Q-function: expected return starting from state s, taking action a

Bellman Equation

Optimal Q-Function:

$$Q^*(s,a) = \mathbb{E}\left[R + \gamma \max_{a'} Q^*(s',a')\right]$$

Interpretation:

- Value = immediate reward + discounted future value
- Recursive definition enables dynamic programming

Optimal Policy:

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

Bellman equation: foundation of all value-based RL methods

Temporal Difference Learning

TD(0) Update Rule — learn from each transition:

$$V(s) \leftarrow V(s) + \alpha \left[r + \gamma V(s') - V(s)\right]$$

TD Error: $\delta_t = r + \gamma V(s') - V(s)$ (surprise signal)

- **vs Monte Carlo**: MC waits for episode end; TD updates every step
- **vs Dynamic Programming**: DP requires model $P(s'|s, a)$; TD is model-free
- **Q-learning**: TD applied to Q-function with max over actions

TD learning: the theoretical foundation connecting DP, MC, and Q-learning (Sutton, 1988)

Q-Learning Algorithm

Update Rule:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \left[r + \gamma \max_{a'} Q(s', a') - Q(s,a) \right]$$

Algorithm:

1. Initialize $Q(s,a)$ arbitrarily
2. For each episode:
    - Observe state s
    - Choose action a (ϵ-greedy)
    - Execute a, observe r, s'
    - Update $Q(s,a)$

Q-learning is off-policy: converges to Q^* given sufficient exploration and decaying α (Watkins & Dayan, 1992)

Q-Learning: Worked Example

Trading scenario: State s_1 = [RSI=25, position=none]

Current Q-values: $Q(s_1, \text{buy}) = 3.2$, $Q(s_1, \text{hold}) = 1.0$

Agent takes action **buy**, observes:

- Reward $r = -0.5$ (transaction cost)
- New state s_2 = [RSI=35, position=long]
- Best future: $\max_{a'} Q(s_2, a') = 4.0$

Update ($\alpha = 0.1$, $\gamma = 0.9$):

$$\underbrace{r + \gamma \max_{a'} Q(s_2, a')}_{\text{TD target}} - \underbrace{Q(s_1, \text{buy})}_{\text{current}} = -0.5 + 0.9 \times 4.0 - 3.2 = -0.1$$

$$Q(s_1, \text{buy}) \leftarrow 3.2 + 0.1 \times (-0.1) = \mathbf{3.19}$$

Each update moves Q toward the "better" estimate: immediate reward + discounted future

Q-Learning Algorithm: Pseudocode

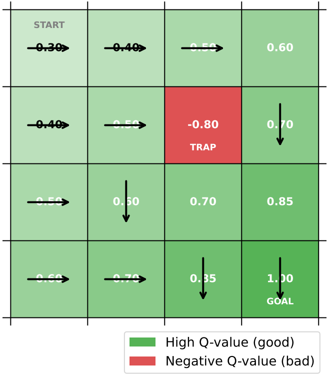
```
**Require:** environment, $\alpha$, $\gamma$, $\epsilon$, episodes
Initialize $Q(s, a) \leftarrow 0$ for all $s \in \mathcal{S}$, $a \in \mathcal{A}$
**for** episode $= 1, \ldots,$ episodes **do**
    $s \leftarrow$ initial state
    **while** $s$ is not terminal **do**
        $a \leftarrow \begin{cases} \text{random } a \in \mathcal{A} & \text{with prob. } \epsilon \\ \arg\max_{a'} Q(s, a') & \text{otherwise} \end{cases}$ $\{\epsilon\text{-greedy}\}$
        Take action $a$, observe reward $r$ and next state $s'$
        $Q(s, a) \leftarrow Q(s, a) + \alpha\left[r + \gamma \max_{a'} Q(s', a') - Q(s, a)\right]$
        $s \leftarrow s'$
    **end while**
**end for**
**return** $Q$
```

Key: The $\max_{a'}$ makes Q-learning **off-policy** — it learns the optimal policy regardless of the exploration strategy used.

Watkins & Dayan (1992). Q-learning. Machine Learning, 8(3-4), 279–292.

Q-Values Visualization

Q-Learning: Grid World with Learned Q-Values



https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L06_Embeddings_RL/04_q_learning_grid

Arrows show policy; colors show Q-values (green=high, red=negative)

Exploration vs Exploitation

The Dilemma:

- **Exploit**: Choose best known action (greedy)
- **Explore**: Try new actions (discover better options)

ϵ-**Greedy Strategy:**

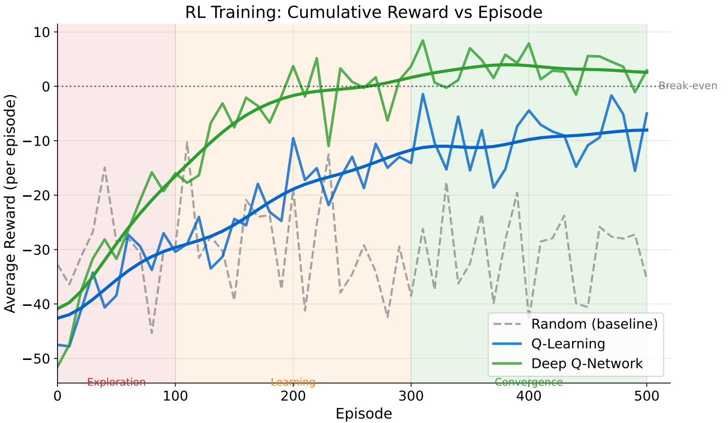
$$a = \begin{cases} \arg\max_a Q(s,a) & \text{with probability } 1-\epsilon \\ \text{random action} & \text{with probability } \epsilon \end{cases}$$

Decay Schedule:

- Start with high ϵ (explore more)
- Decay ϵ over time (exploit more)

Balance: too much exploration wastes time; too little misses optima

Learning Curves



https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L06_Embeddings_RL/05_reward_curves

Reward improves as agent learns; DQN often outperforms tabular Q-learning

Part 3: RL for Trading

Formulation:

- **State**: Price history, portfolio, technical indicators
- **Action**: Buy, sell, hold (+ position size)
- **Reward**: Profit/loss, risk-adjusted return

Challenges:

- Non-stationary environment
- High noise, low signal-to-noise ratio
- Transaction costs
- Partial observability

RL for trading is active research area; not solved problem

Trading Reward Function Design

Reward with transaction costs:

$$r_t = R_t^{\text{portfolio}} - c \cdot |\Delta w_t|$$

where $R_t^{\text{portfolio}}$ = portfolio return, c = transaction cost rate, Δw_t = position change

Common State Features:

- Price returns (1-day, 5-day, 20-day)
- Technical indicators: RSI, MACD, Bollinger width
- Current position and unrealized P&L

Alternative Rewards:

- Sharpe ratio: $r_t = \frac{\bar{R}_t}{\sigma_{R_t}}$ (risk-adjusted, but non-stationary)
- Log return: $r_t = \log(1 + R_t)$ (additive over time)

Reward design is THE most critical decision in RL for trading

Backtesting RL Trading Strategies

Critical Challenge: RL agents overfit to historical data

Walk-Forward Validation:

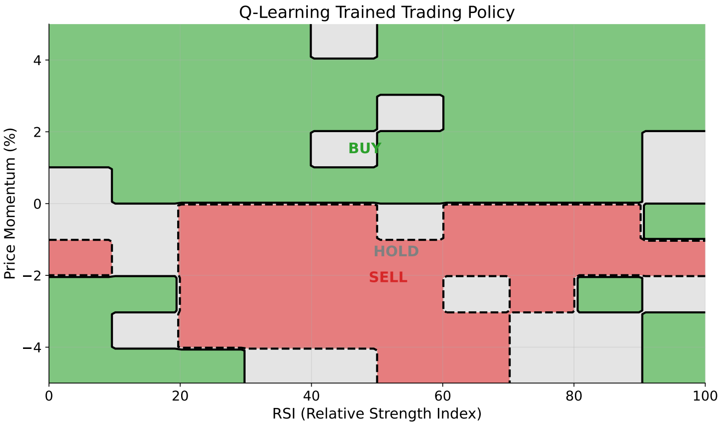
1. Train on period $[t_0, t_1]$, test on $[t_1, t_2]$
2. Roll forward: train on $[t_1, t_2]$, test on $[t_2, t_3]$
3. Report average out-of-sample performance

Honest Evaluation:

- Compare to buy-and-hold benchmark (most RL strategies fail to beat after costs)
- Include realistic transaction costs (0.1–0.5% per trade)
- Test across multiple market regimes (bull, bear, sideways)

If your RL agent beats buy-and-hold after costs, you likely have a bug — verify carefully

Trading Policy



https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L06_Embeddings_RL/06_policy_viz

Q-learning trained policy: agent discovers buy/sell/hold regions from reward signal

Deep Q-Networks (DQN)

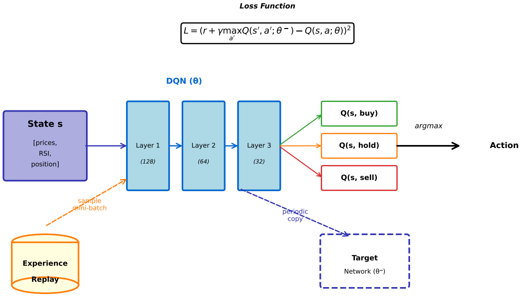
Idea: Neural network approximates Q-function: $Q(s, a; \theta) \approx Q^*(s, a)$

Loss Function:

$$L(\theta) = \mathbb{E}\left[\left(r + \gamma \max_{a'} Q(s', a'; \theta^-) - Q(s, a; \theta)\right)^2\right]$$

Key Innovations:

- **Experience Replay**: Store (s, a, r, s'), sample random mini-batches (breaks temporal correlation)
- **Target Network** θ^-: Separate, slowly-updated copy for stability



Deep Q-Network Architecture for Trading

DQN: Atari-level play from raw pixels (Mnih et al., 2015); loss is mean squared TD error

Policy Gradient Methods

Policy Gradient Theorem (Sutton et al., 2000):

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) \cdot A^{\pi_\theta}(s,a)\right]$$

where $A^\pi(s,a) = Q^\pi(s,a) - V^\pi(s)$ is the **advantage function**

- **REINFORCE**: Uses episode returns G_t as A; high variance
- **Actor-Critic**: Actor (policy π_θ) + Critic (learns V^ϕ); lower variance
- **PPO**: Clips policy ratio to prevent large updates; widely used

Policy gradient handles continuous actions; advantage reduces variance vs raw returns

Statistical Inference for Embeddings & RL

Embedding Uncertainty:

- Bootstrap cosine similarity: resample corpus, retrain, compute CI
- Permutation test: shuffle word-context pairs, check if similarity is significant

RL Uncertainty:

- Q-value confidence: run N independent training runs, report mean ± std
- Off-policy evaluation: importance sampling to estimate policy value from logged data

$$\hat{V}(\pi) = \frac{1}{n} \sum_{i=1}^{n} \prod_{t=0}^{T} \frac{\pi(a_t|s_t)}{\beta(a_t|s_t)} \cdot G_i$$

Always report uncertainty — a single training run is not evidence of a good policy

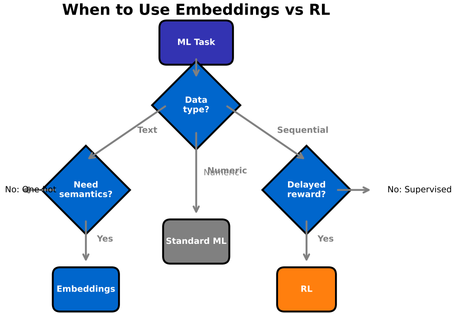
Hands-on Exercise

Open the Colab Notebook

- Exercise 1: Explore word embeddings with Word2Vec
- Exercise 2: Implement basic Q-learning
- Exercise 3: Apply RL to a simple trading environment

Link: `https://colab.research.google.com/github/Digital-AI-Finance/methods-algorithms/blob/master/notebooks/L06_embeddings_rl.ipynb`

Part 4: When to Use What



Embeddings: Text, categorical -> dense vectors (Word2Vec, BERT)

RL: Sequential decisions with delayed rewards (trading, games)

https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L06_Embeddings_RL/07_decision_flowchart

Embeddings for text/categorical; RL for sequential decisions

Comparison Table

Aspect	**Embeddings**	**RL**
Input	Text, categorical	State sequence
Output	Dense vectors	Actions/policy
Learning	Unsupervised/supervised	Trial and error
Signal	Context (words)	Rewards
Key challenge	Semantics	Credit assignment
Finance use	Sentiment	Trading

Both transform complex inputs into learnable representations

Part 5: Implementation

Embeddings in Python:

- `gensim.models.Word2Vec`: Train your own
- `gensim.downloader.load('glove-wiki-gigaword-100')`: Pre-trained
- `transformers.BertModel`: BERT embeddings

RL Libraries:

- `gymnasium`: Environment interface (formerly OpenAI Gym)
- `stable-baselines3`: Pre-implemented algorithms
- `ray[rllib]`: Scalable RL

Start with pre-trained embeddings; use stable-baselines3 for RL

Practical Tips

Embeddings:

- Start with pre-trained, fine-tune if needed
- Check domain match (general vs financial)
- Visualize with t-SNE/UMAP to verify quality

RL:

- Start simple (tabular Q-learning before DQN)
- Reward shaping is crucial (sparse rewards are hard)
- Normalize observations
- Use established environments first (Gym, FinRL)

Both domains: start simple, iterate, validate thoroughly

Summary

Embeddings:

- Dense vector representations of text/categories
- Capture semantic similarity
- Use pre-trained (Word2Vec, GloVe, BERT)

Reinforcement Learning:

- Agent learns from environment interaction
- Q-learning: value-based, tabular or deep (DQN)
- Applications: trading, portfolio optimization

Key Takeaway: Different tools for different problems

Course complete! Apply these methods in your capstone project

References

Embeddings:

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- Pennington et al. (2014). GloVe
- Devlin et al. (2019). BERT
- Levy & Goldberg (2014). Linguistic Regularities in Word Embeddings
- Bolukbasi et al. (2016). Man is to Computer Programmer as Woman is to Homemaker?

Reinforcement Learning:

- Sutton & Barto (2018). RL: An Introduction (free online)
- Mnih et al. (2015). DQN (Atari)
- Schulman et al. (2017). PPO
- Watkins & Dayan (1992). Q-Learning Convergence

Finance Applications:

- Liu et al. (2021). FinRL: Deep RL for Trading
- Araci (2019). FinBERT

Sutton & Barto: the definitive RL textbook (free at incompleteideas.net)