L02: Logistic Regression

Classification with Probability Estimates

Methods and Algorithms

Spring 2026

Outline

Learning Objectives

By the end of this lecture, you will be able to:

1. Explain how logistic regression models binary outcomes
2. Derive the maximum likelihood estimation for logistic regression
3. Interpret classification metrics (precision, recall, AUC)
4. Apply logistic regression for credit scoring decisions

Finance Application: Credit default prediction

These objectives span Bloom's levels: Understand, Apply, Analyze

Why Logistic Regression?

The Business Problem

- Banks must decide: approve or reject loan applications
- Need probability of default, not just yes/no prediction
- Regulatory requirement: interpretable, auditable models

Why Not Linear Regression?

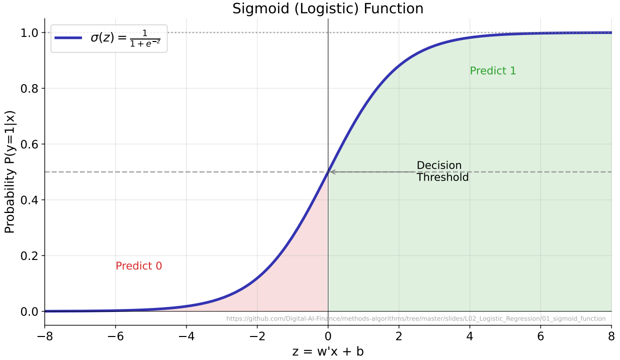
- Linear regression can predict values outside [0,1]
- Binary outcomes need probability-based approach
- Logistic regression outputs calibrated probabilities

Logistic regression: the industry standard for credit scoring since 1980s

The Sigmoid Function

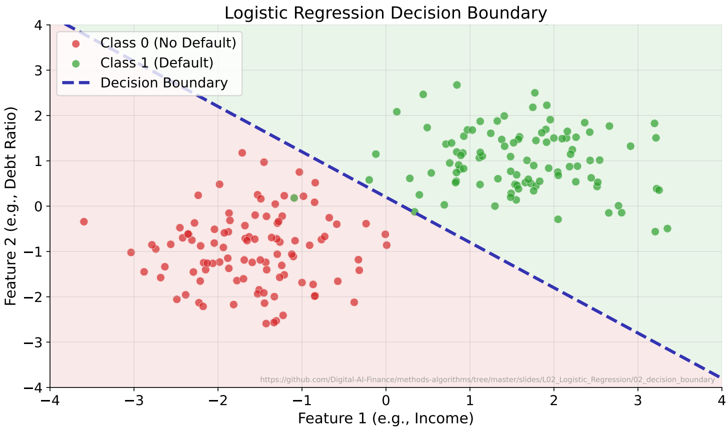
From Linear to Probability

- Maps any real number to (0, 1) range
- Smooth, differentiable, interpretable



$\sigma(z) = 1/(1 + e^{-z})$ **transforms linear combination to probability**

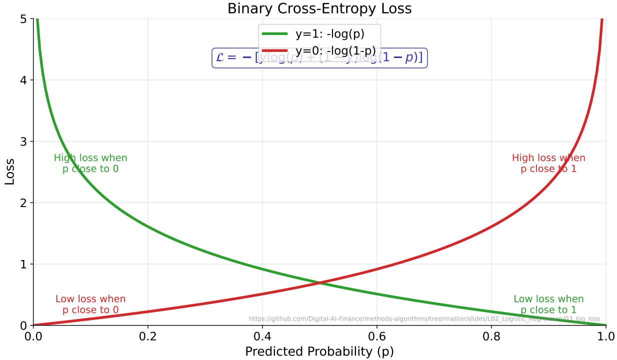
Decision Boundary



The decision boundary is where $P(y = 1|x) = 0.5$**, i.e.,** $w'x + b = 0$

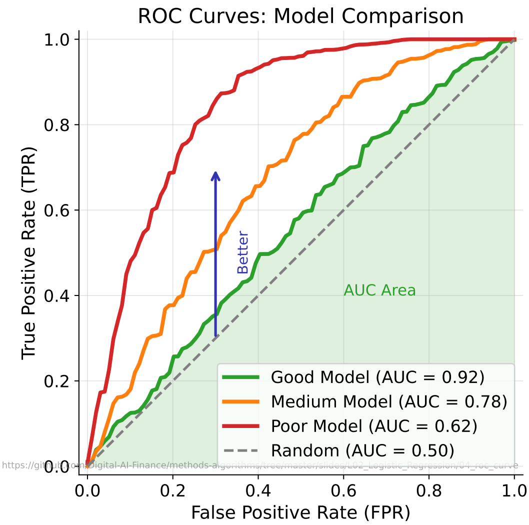
Why Not MSE?

- MSE with sigmoid creates non-convex loss landscape
- Cross-entropy is convex, guarantees global optimum

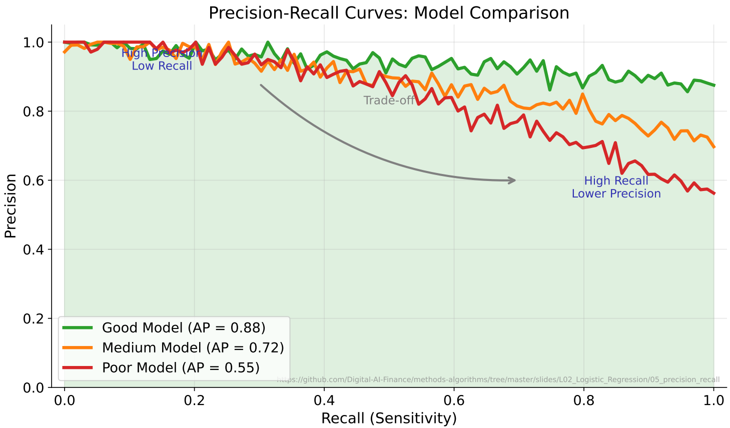


Heavily penalizes confident wrong predictions

ROC Curve and AUC

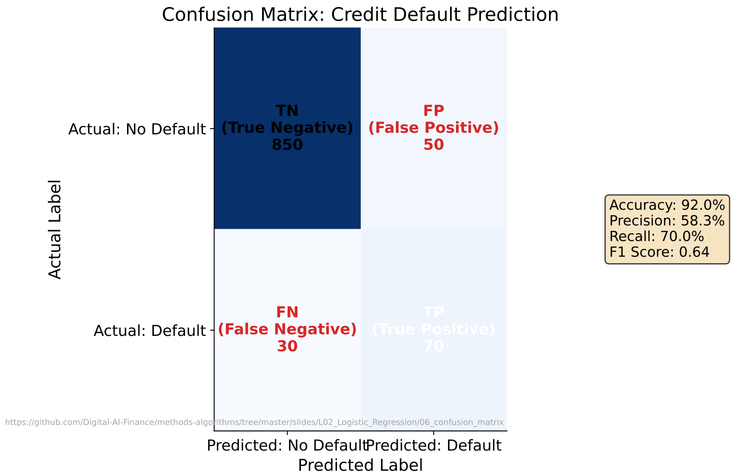


AUC = probability random positive ranks higher than random negative



Use PR curve when classes are imbalanced (common in fraud detection)

Confusion Matrix: Reading the Results



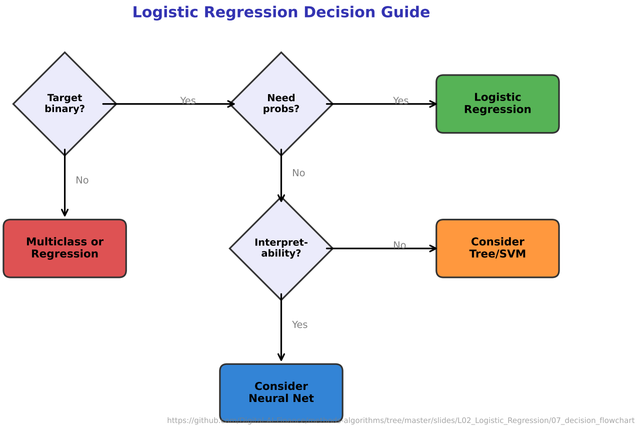
FP = approve bad loans (costly), FN = reject good customers (lost revenue)

Hands-on Exercise

Open the Colab Notebook

- Exercise 1: Implement logistic regression from scratch
- Exercise 2: Train model on credit scoring data
- Exercise 3: Evaluate with ROC curve and confusion matrix

Link: `https://colab.research.google.com/` [TBD]



Key strengths: interpretable coefficients, probability outputs, fast training

References

- James et al. (2021). *Introduction to Statistical Learning*. `https://www.statlearning.com/`
- Hastie et al. (2009). *Elements of Statistical Learning*. `https://hastie.su.domains/ElemStatLearn/`