Introduction & Linear Regression

Overview

Methods and Algorithms

MSc Data Science

Spring 2026

Outline

Learning Objectives

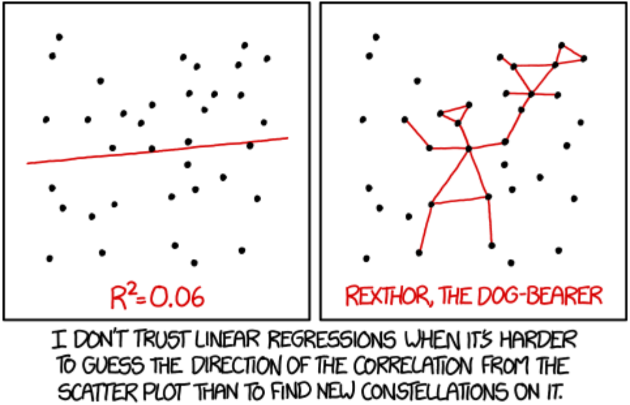
By the end of this session, you will be able to:

1. **Derive** the OLS estimator and prove its optimality under Gauss-Markov assumptions
2. **Analyze** gradient descent convergence and evaluate learning rate selection
3. **Evaluate** regression diagnostics to identify assumption violations
4. **Compare** regularization strategies (Ridge, Lasso, Elastic Net) for different problem structures

Finance Applications: Property valuation, asset pricing (CAPM)

Foundation for all supervised learning methods

The Art of Fitting Lines



XKCD #1725 by Randall Munroe (CC BY-NC 2.5) – Always check if your data supports a linear fit!

The Business Problem

Finance Use Case: Predicting House Prices

- Banks need accurate property valuations for mortgages
- Insurance companies assess property risk
- Investors evaluate real estate portfolios

The Question: Given features (size, location, age), what price?

Why Linear Regression?

- Interpretable coefficients (price per square meter)
- Fast, well-understood method
- Strong baseline for comparison

Linear regression: the “hello world” of machine learning

What is Linear Regression?

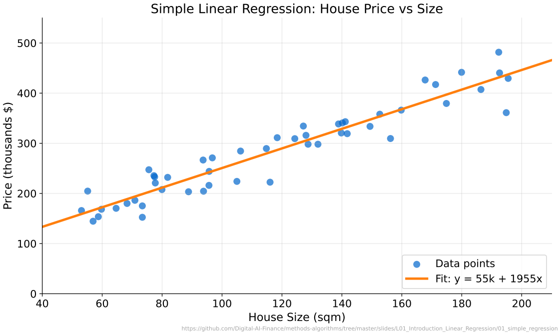
Core Idea: Model the relationship between inputs and a continuous output

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \varepsilon \tag{1}$$

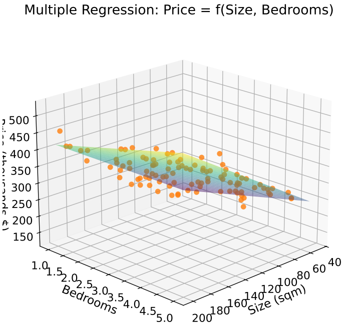
- y: Target variable (house price)
- $x_1, \ldots, x_p$: Features (size, bedrooms, age)
- $\beta_0, \ldots, \beta_p$: Coefficients to learn
- ε: Random error term

Goal: Find coefficients that minimize prediction error

Simple Linear Regression



The best-fit line minimizes the sum of squared distances from points



https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L01_Introduction_Linear_Regression/02_multiple_regression_3d

With multiple features, we fit a hyperplane to minimize squared errors

Objective: Minimize the sum of squared errors (SSE)

$$\min_{\boldsymbol{\beta}} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \min_{\boldsymbol{\beta}} \|\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\|^2 \tag{2}$$

Why Squared Errors?

- Penalizes large errors more than small errors
- Differentiable (enables gradient-based optimization)
- Leads to closed-form solution

This objective function defines "ordinary least squares" (OLS)

Two Solution Approaches

1. Closed-Form (Normal Equation)

$$\hat{\beta} = (\mathbf{X}^\top \mathbf{X})^{-1}\mathbf{X}^\top \mathbf{y} \tag{3}$$

Pros:

- Exact solution
- One computation

Cons:

- Slow for large p
- Memory intensive

2. Gradient Descent

$$\beta_{t+1} = \beta_t - \alpha \nabla L(\beta_t) \tag{4}$$

Pros:

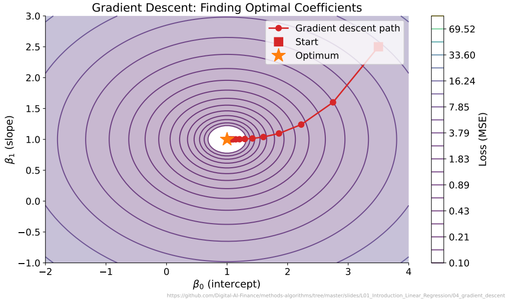
- Scales to big data
- Memory efficient

Cons:

- Requires tuning α
- Iterative process

Both methods yield the same solution for OLS

Gradient Descent in Action



Iteratively update parameters in the direction of steepest descent

Interpreting Coefficients

Example: House price model

$$\text{Price} = 50,000 + 200 \times \text{Size} + 15,000 \times \text{Bedrooms} - 1,000 \times \text{Age} \tag{5}$$

Interpretation:

- $\beta_0 = 50,000$: Base price (all features $= 0$)
- $\beta_1 = 200$: Each extra sqm adds \$200
- $\beta_2 = 15,000$: Each bedroom adds \$15,000
- $\beta_3 = -1,000$: Each year of age subtracts \$1,000

Coefficients show marginal effect, holding others constant

Model Evaluation

Key Metrics:

- R^2 **(Coefficient of Determination):** Variance explained

$$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2} \tag{6}$$

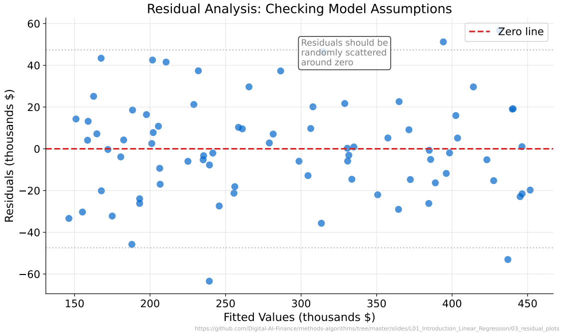
- **RMSE (Root Mean Squared Error):** Prediction accuracy

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum(y_i - \hat{y}_i)^2} \tag{7}$$

In Practice: $R^2 = 0.75$ means model explains 75% of price variance

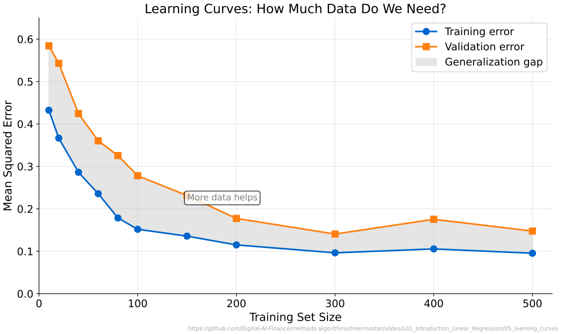
Always evaluate on held-out test data

Residual Diagnostics



Good residuals are random; patterns suggest model problems

Learning Curves



Learning curves help diagnose underfitting vs overfitting

Use When:

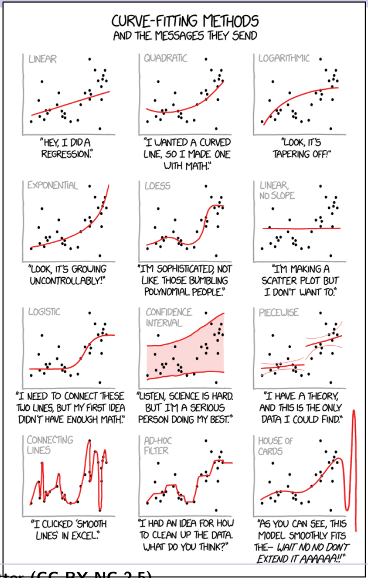
- Target is continuous
- Linear relationships expected
- Interpretability matters
- Fast inference needed

Avoid When:

- Target is categorical
- Strong non-linearities
- High multicollinearity
- Many outliers present

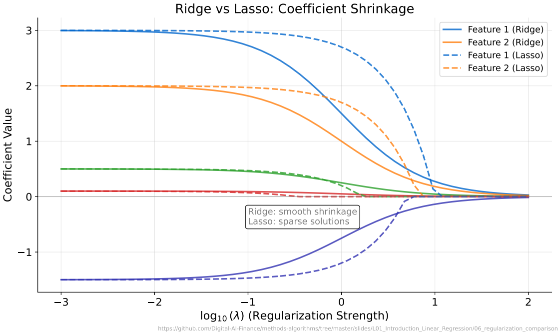
Pros	**Cons**
Interpretable	Assumes linearity
Fast training	Sensitive to outliers
Well-understood	Limited flexibility

The Danger of Overfitting



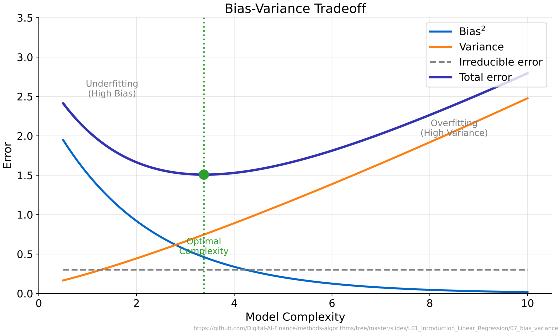
XKCD #2048 – Simpler models often generalize better (CC BY-NC 2.5)

Regularization: Ridge vs Lasso



Ridge shrinks all coefficients; Lasso can set some to exactly zero

Bias-Variance Tradeoff



Model complexity controls the tradeoff between bias and variance

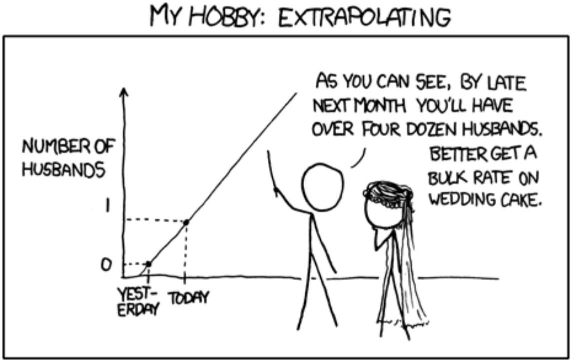
Hands-on Exercise

Open the Colab Notebook

- Exercise 1: Implement OLS from scratch
- Exercise 2: Use scikit-learn LinearRegression
- Exercise 3: Compare with gradient descent

Link: See course materials on GitHub

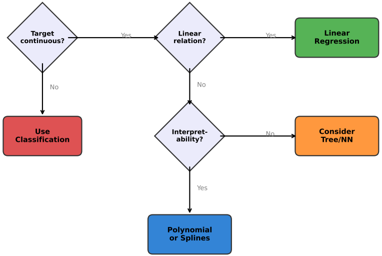
A Word of Caution



XKCD #605 by Randall Munroe (CC BY-NC 2.5) – Extrapolation is dangerous!

Decision Framework

Linear Regression Decision Guide



https://github.com/Digital-AI-Finance/methods-algorithms/tree/master/slides/L01_Introduction_Linear_Regression/08_decision_flowchart

Use this flowchart to decide when linear regression is appropriate

Key Takeaways

1. Linear regression models continuous outcomes as weighted sum of features
2. OLS minimizes sum of squared errors
3. Solve via normal equation (small data) or gradient descent (big data)
4. Coefficients are directly interpretable as marginal effects
5. Evaluate using R^2 (variance explained) and RMSE (prediction error)

Next: Deep dive into mathematics and implementation

References:

- ISLR Chapter 3: Linear Regression
- ESL Chapter 3: Linear Methods for Regression