16. Learning Rate Comparison
Neural Networks - From Brain to Business

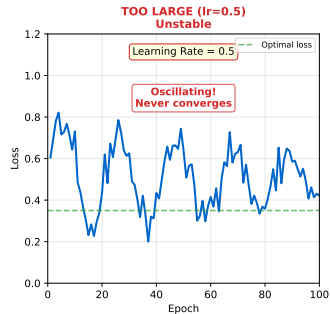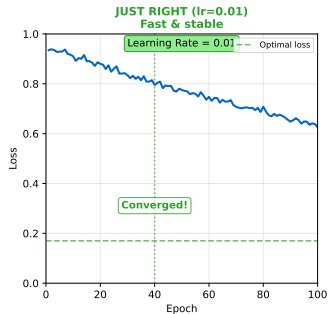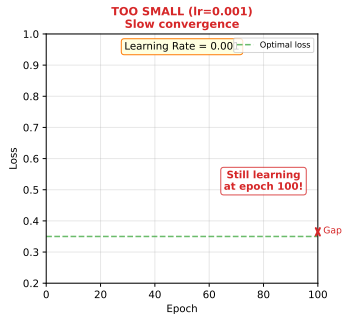Understand how learning rate affects training speed and stability.

The **learning rate** controls how big a step we take during gradient descent. It's one of the most important hyperparameters in neural network training.

Too small (e.g., 0.0001): Training is stable but extremely slow. The network may never reach a good solution in reasonable time.

Too large (e.g., 1.0): Training is unstable. The loss may oscillate wildly or even diverge to infinity. The network overshoots the minimum.

Just right (typically 0.001-0.01): Steady progress toward minimum. Fast enough to converge in reasonable time, stable enough not to diverge.

Finding the optimal learning rate often requires experimentation. Techniques like **learning rate schedules** (reducing rate over time) and **adaptive methods** (Adam, RMSprop) help automate this process.

Key Formula

Weight update:

$$w := w - \eta \cdot \nabla L$$

Effect of learning rate: - Small eta: Δw is small, slow movement - Large eta: Δw is large, fast but potentially unstable movement

Common learning rate schedules:

Step decay:

$$\eta_t = \eta_0 \times \gamma^{\lfloor t/k \rfloor}$$

Exponential decay:

$$\eta_t = \eta_0 \times e^{-\lambda t}$$

Intuitive Explanation

Imagine walking downhill with a blindfold:
- **Too small step**: You inch forward, taking forever to reach the bottom - **Too large step**: You leap so far you might land on the opposite hill - **Just right**: Steady steps that make good progress without losing balance
The terrain (loss landscape) determines what "just right" means. Steeper hills allow smaller steps; flat areas can use larger steps.

Practice Problem 1

Problem 1
You train with learning rate 0.1 and observe: Loss at epoch $1 = 0.8$, epoch $2 = 0.5$, epoch $3 = 0.4$, epoch $4 = 0.38$, epoch $5 = 0.37$. What pattern do you see?

Solution

Pattern: Diminishing returns (normal convergence)
Analysis: - Epoch 1-¿2: Improvement of 0.3 (37.5% reduction) - Epoch 2-¿3: Improvement of 0.1 (20% reduction) - Epoch 3-¿4: Improvement of 0.02 (5% reduction) - Epoch 4-¿5: Improvement of 0.01 (2.6% reduction)
This is **healthy training behavior**: - Rapid early progress (large gradients far from minimum) - Slowing improvement (smaller gradients near minimum) - Convergence to stable value
The learning rate (0.1) appears appropriate for this problem.

Practice Problem 2

Problem 2
You train with learning rate 1.0 and observe: Loss at epoch 1 = 0.8, epoch 2 = 1.5, epoch 3 = 3.2, epoch 4 = 8.7, epoch 5 = NaN. What happened?

Solution

Diagnosis: Learning rate too high - divergence
What happened: - Epoch 1-¿2: Loss increased (0.8 -¿ 1.5) - overshot minimum - Epoch 2-¿3: Continued increasing - oscillating past optimal - Epoch 3-¿4: Accelerating growth - unstable regime - Epoch 5: NaN (Not a Number) - loss exploded to infinity
Why: - Large eta means large weight updates - Updates so large they jump past the minimum to higher loss regions - Positive feedback: higher loss -¿ larger gradient -¿ even larger update - Eventually overflows numerical precision (NaN)
Fix: Reduce learning rate significantly (try 0.01 or 0.001)

Key Takeaways

- Learning rate is critical: too small = slow, too large = unstable
- Look for diminishing returns pattern = healthy training
- Look for increasing loss = learning rate too high
- Learning rate schedules reduce rate over time
- Adaptive optimizers (Adam) adjust rate automatically