December 6, 2025

Learning Goal

Understand the vanishing gradient problem caused by sigmoid saturation.

This slide establishes the learning objective for this topic

Key Concept (1/2)

The sigmoid function **saturates** at extreme values. When z is very large (e.g., z ¿ 3) or very small (z ¡ -3), the sigmoid output is nearly 1 or 0, and barely changes regardless of input changes.

This creates the **vanishing gradient problem**. During backpropagation, gradients flow backward through the network. In saturated regions, gradients become extremely small (near zero), so weights barely update. The network stops learning.

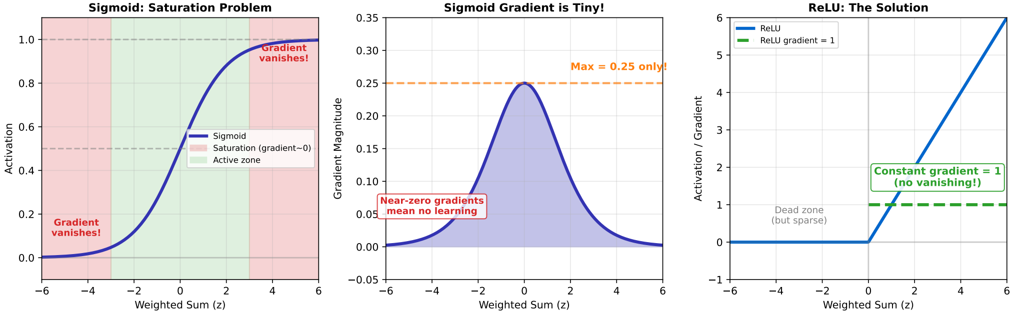
Understanding this concept is crucial for neural network fundamentals

Key Concept (2/2)

The **maximum gradient** of sigmoid is only 0.25 (at $z = 0$). In deep networks, multiplying many small gradients results in effectively zero gradient for early layers - they learn very slowly or not at all.
ReLU solves this by having a constant gradient of 1 for positive inputs. However, ReLU has its own issue: neurons can "die" if z is always negative (gradient = 0).

Understanding this concept is crucial for neural network fundamentals

Visualization



Visual representations help solidify abstract concepts

Key Formula

Sigmoid and its derivative:

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma'(z) = \sigma(z)(1 - \sigma(z))$$

Maximum derivative:

$$\max(\sigma'(z)) = \sigma(0)(1 - \sigma(0)) = 0.5 \times 0.5 = 0.25$$

ReLU and its derivative:

$$\text{ReLU}(z) = \max(0, z)$$

$$\text{ReLU}'(z) = \begin{cases} 1 & z > 0 \\ 0 & z \leq 0 \end{cases}$$

Mathematical formalization provides precision

Intuitive Explanation

Imagine pushing a ball on different surfaces: - **Sigmoid at extremes**: Like pushing a ball on a flat plateau. No matter how hard you push, it barely moves. (Small gradient = little learning) - **Sigmoid at center**: Gentle slope, ball rolls smoothly. (Gradient = 0.25) - **ReLU positive region**: Constant slope, ball rolls consistently. (Gradient = 1) - **ReLU negative region**: Another flat plateau, ball doesn't move.

For efficient learning, we want consistent, non-vanishing gradients throughout.

Intuitive explanations bridge theory and practice

Practice Problem 1

Problem 1
Calculate the sigmoid derivative at $z = 5$. How does it compare to the maximum derivative?

Solution

First, calculate sigmoid:

$$\sigma(5) = \frac{1}{1 + e^{-5}} = \frac{1}{1 + 0.0067} = 0.9933$$

Then, calculate derivative:

$$\sigma'(5) = \sigma(5)(1 - \sigma(5)) = 0.9933 \times 0.0067 = 0.0067$$

Comparison to maximum:

$$\frac{0.0067}{0.25} = 0.027 = 2.7\%$$

At $z = 5$, the gradient is only **2.7% of maximum**. Learning is extremely slow in this saturated region.

Practice problems reinforce understanding

Practice Problem 2

Problem 2
A network has 5 layers, each with sigmoid activation. If each layer's gradient is 0.2, what is the total gradient that reaches the first layer?

Solution

Gradients multiply through layers (chain rule):

$$\text{Total gradient} = 0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2 = 0.2^5$$
$$= 0.00032$$

The first layer receives a gradient of only **0.00032** - practically zero!
With learning rate 0.1:

$$\Delta w = 0.1 \times 0.00032 = 0.000032$$

Weight updates are negligible. The first layer cannot learn effectively.
This is the vanishing gradient problem.

Practice problems reinforce understanding

Key Takeaways

- Sigmoid saturates at extreme values (gradient near 0)
- Maximum sigmoid gradient is only 0.25
- Vanishing gradients prevent learning in deep networks
- ReLU has constant gradient $= 1$ for positive inputs
- Modern networks use ReLU (or variants) for hidden layers

These key points summarize the essential learnings