December 6, 2025

Learning Goal

Observe how increasing the number of neurons improves decision boundaries.

This slide establishes the learning objective for this topic

Key Concept (1/2)

The **expressivity** of a neural network - its ability to represent complex functions - increases with more neurons. This manifests as more flexible decision boundaries.
- **1 neuron**: Can only create a straight line boundary. Fails on XOR-like patterns. - **2-4 neurons**: Can create piecewise linear boundaries with some curvature - **10+ neurons**: Can create smooth, curved boundaries that wrap around complex clusters

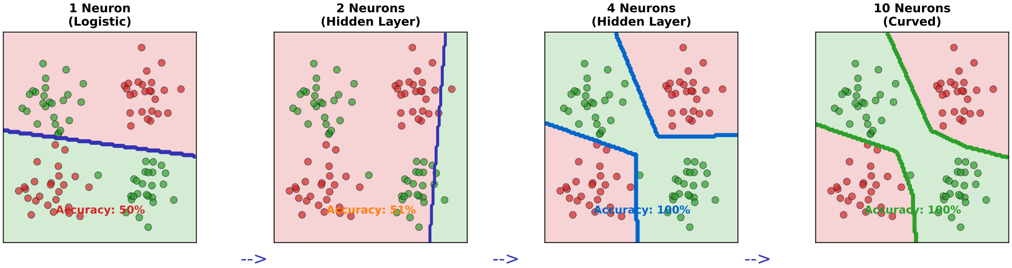
Understanding this concept is crucial for neural network fundamentals

Key Concept (2/2)

With enough neurons, a single hidden layer can theoretically approximate any continuous function (Universal Approximation Theorem). However, practical considerations (training time, overfitting) limit network size.
The key insight: **more neurons = more capacity to represent complex patterns**. But this capacity must be balanced against available training data to avoid overfitting.

Understanding this concept is crucial for neural network fundamentals

Visualization



Visual representations help solidify abstract concepts

Key Formula

Network capacity (approximate):
A single hidden layer with n neurons can represent boundaries with approximately n "bends" or segments.
Universal Approximation Theorem (informal): A feedforward network with one hidden layer containing a finite number of neurons can approximate any continuous function to arbitrary accuracy, given sufficient neurons.

Mathematical formalization provides precision

Intuitive Explanation

Imagine drawing with different tools: - **1 neuron**: A single ruler - can only draw straight lines - **2 neurons**: Two rulers - can make an angle (V-shape) - **4 neurons**: A flexible curve with several bends - **10 neurons**: A smooth, complex curve that can wrap around clusters

Each neuron contributes one "bend" to the decision boundary. The more bends available, the more precisely the boundary can separate complex patterns.

Intuitive explanations bridge theory and practice

Problem 1

A dataset has an XOR pattern (like a checkerboard). What is the minimum number of hidden neurons needed to solve it with a single hidden layer?

Solution

Minimum: 2 hidden neurons

XOR requires separating diagonal corners. With 2 neurons: - Neuron 1: Creates one linear boundary - Neuron 2: Creates another linear boundary - Combined: The intersection creates an XOR-compatible separation

With 1 neuron: Only one straight line = cannot solve XOR

Visualization: ‘‘‘ Neuron 1: / Neuron 2: Combined: X ‘‘‘

The two boundaries intersect, creating four regions that match XOR's four quadrants.

Practice problems reinforce understanding

Practice Problem 2

Problem 2
You have 500 training samples. Would you use 10 neurons or 500 neurons in the hidden layer? Why?

Solution

Use 10 neurons, not 500.
Reasoning:
With 500 neurons: - Parameters (assuming 10 inputs): 10 x 500 + 500 + 500 x 1 + 1 = 6,001 - Parameters exceed training samples (6,001 ¿ 500) - High risk of **overfitting** - network can memorize each training point - Poor generalization to new data
With 10 neurons: - Parameters: 10 x 10 + 10 + 10 x 1 + 1 = 121 - Parameters much less than training samples (121 ¡¡ 500) - Forces network to find **generalizable patterns** - Better expected test performance
Rule of thumb: Keep parameters well below training samples.

Practice problems reinforce understanding

Key Takeaways

- More neurons = more flexible decision boundaries
- 1 neuron creates linear boundary; many neurons create curved boundaries
- Universal Approximation: enough neurons can fit any pattern
- But more neurons also increases overfitting risk
- Balance capacity with available data

These key points summarize the essential learnings