The Simplest Decision Maker

What is a Perceptron?
The simplest possible neural network:

- One artificial neuron
- Multiple inputs, one output
- Binary decision: Yes or No

Think of it as:

- A filter for data
- A simple classifier
- A linear decision maker

Finance Application:
Stock screener that outputs "Buy" or "Don't Buy" based on financial metrics.

Real-World Examples

Email Spam Filter:

- Inputs: word frequencies
- Output: spam or not spam

Loan Approval:

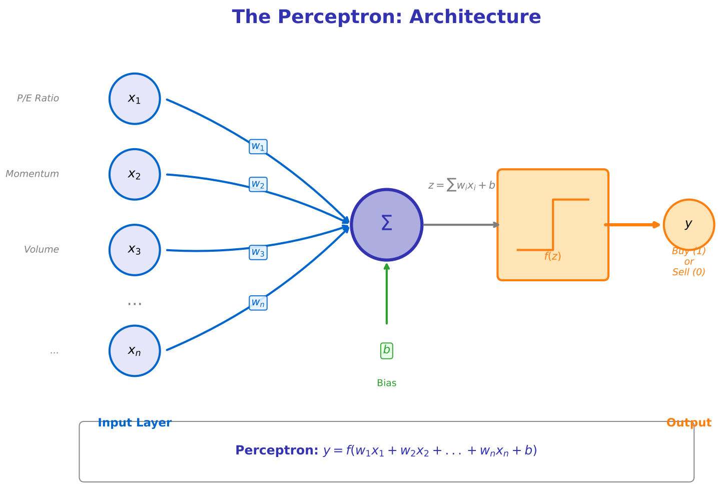
- Inputs: income, credit score, debt
- Output: approve or reject

Stock Screening:

- Inputs: P/E, momentum, volume
- Output: buy or pass

All these are binary classification problems that a perceptron can solve (if the data is linearly separable).

A single perceptron is a stock screening filter



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Inputs, weights, sum, activation, output

Finance Scenario: Buy or Sell?

Problem Setup
You want to build a simple stock screener:

- **Goal**: Decide Buy or Pass
- **Data**: Historical financial metrics
- **Method**: Perceptron classifier

Available Features:

1. P/E Ratio (valuation)
2. 6-month momentum (%)
3. Average daily volume
4. Debt-to-Equity ratio
5. Earnings surprise (%)

The Question
Given these features for a new stock, should we add it to our portfolio?

Example Stock:

- P/E $= 18$
- Momentum $= +12\%$
- Volume $=$ 2M shares
- D/E $= 0.8$
- Surprise $= +5\%$

Traditional Approach:
Analyst manually weighs factors and decides.

Perceptron Approach:
Learn the weights from historical winners/losers.

Given financial indicators, should we buy this stock?

Inputs: The Raw Data

What Are Inputs?

Each input x_i is a numerical feature:

- A measurement
- A statistic
- A signal

In Finance:

- Price-based: returns, volatility
- Fundamental: P/E, ROE, debt ratios
- Technical: RSI, moving averages
- Sentiment: news scores, analyst ratings

Key Requirement:

All inputs must be **numerical**. Categorical data needs encoding.

Notation

For a stock with n features:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

Example (n=3):

$$\mathbf{x} = \begin{pmatrix} 18 \\ 0.12 \\ 0.8 \end{pmatrix} = \begin{pmatrix} \text{P/E} \\ \text{Momentum} \\ \text{D/E} \end{pmatrix}$$

Note: Features often need **normalization** (covered in Module 3).

What data feeds into our decision?

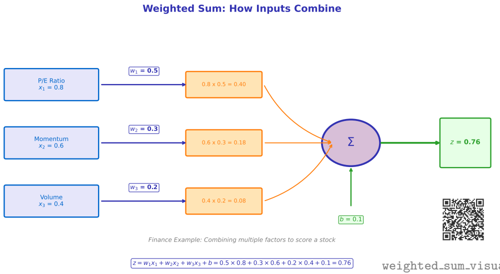
What Are Weights?

Each weight w_i represents:

- Importance of input x_i
- Direction of influence
- Learned from data

Interpretation:

- $w_i > 0$: Higher x_i pushes toward "Buy"
- $w_i < 0$: Higher x_i pushes toward "Sell"
- $|w_i|$ large: Strong influence
- $|w_i|$ small: Weak influence



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"Not all data is equally important" - weights encode importance

Discussion Question

"If you could only look at 3 metrics for a stock, which would you choose and why? How would you weight them?"

Consider:

Value Investor Might Choose:

- P/E ratio ($w = 0.5$)
- Book value ($w = 0.3$)
- Dividend yield ($w = 0.2$)

Growth Investor Might Choose:

- Revenue growth ($w = 0.5$)
- Momentum ($w = 0.3$)
- Market share ($w = 0.2$)

Key Insight: Different investors would assign different weights. The perceptron *learns* these weights from historical performance.

Think-Pair-Share: 3 minutes

The Weighted Sum: Adding Up Evidence

Computing the Weighted Sum

$$z = \sum_{i=1}^{n} w_i x_i + b = w_1 x_1 + w_2 x_2 + \cdots + w_n x_n + b$$

What This Means:

- Multiply each input by its weight
- Sum all the products
- Add the bias term b
- Result: a single "score"

The Bias b:

- Shifts the decision threshold
- Like a "base rate" or prior
- Can be thought of as $w_0 \cdot x_0$ where $x_0 = 1$

Worked Example

Inputs:

- $x_1 = 0.8$ (normalized P/E)
- $x_2 = 0.6$ (normalized momentum)

Weights:

- $w_1 = 0.5$
- $w_2 = 0.7$
- $b = -0.3$

Calculation:

$$\begin{aligned} z &= w_1 x_1 + w_2 x_2 + b \\ &= (0.5)(0.8) + (0.7)(0.6) + (-0.3) \\ &= 0.4 + 0.42 - 0.3 \\ &= \mathbf{0.52} \end{aligned}$$

Combine all weighted inputs into a single score

Analogy: The Voting Committee

The Perceptron as a Committee

Member	Vote	Weight	Contribution
P/E analyst	+1	0.5	+0.5
Momentum	+1	0.7	+0.7
Bias (skeptic)	-1	0.3	-0.3
Total			**+0.9**

If Total $>$ 0: Committee recommends **Buy**

Key Insight:
The perceptron is just a weighted voting system where the weights are learned from data.

Why This Works

Traditional Committee:

- Human experts set weights
- Based on experience/intuition
- May have biases
- Hard to scale

Perceptron Committee:

- Weights learned from data
- Based on historical performance
- Consistent application
- Scales to any volume

Trade-off: Data-driven weights may not capture regime changes or rare events.

Some votes count more than others

The Activation Function
After computing z, we need a final decision.

Step Function:

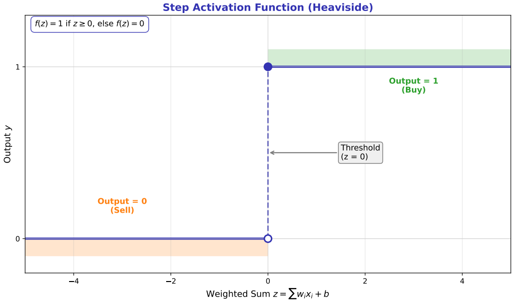
$$f(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

Interpretation:

- $z \geq 0$: Evidence favors "Buy" $\rightarrow$ output 1
- $z < 0$: Evidence favors "Sell" $\rightarrow$ output 0

Why Step Function?

- Binary classification needs binary output
- Mimics neuron firing (all-or-nothing)
- Simple to implement



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Above threshold = Buy, Below threshold = Sell

The Complete Perceptron Flow

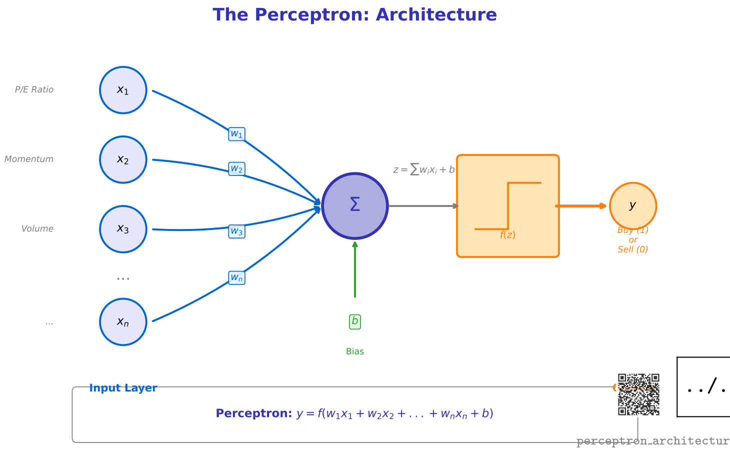
The Pipeline

1. **Input**: Receive features $\mathbf{x}$
2. **Weight**: Multiply by $\mathbf{w}$
3. **Sum**: Add all products + bias
4. **Activate**: Apply step function
5. **Output**: Return prediction

Compact Notation:

$$y = f(\mathbf{w}^T\mathbf{x} + b)$$

where $\mathbf{w}^T\mathbf{x} = \sum_i w_i x_i$



Inputs -¿ Weights -¿ Sum -¿ Threshold -¿ Decision

Now Let’s Formalize

What You Already Know

From the intuition section:

- Inputs are weighted
- Weights encode importance
- Sum is compared to threshold
- Output is binary

What’s Next

- Precise mathematical notation
- Geometric interpretation
- Foundation for learning algorithm

Why Math Matters

Without Math:

- “The network kind of learns”
- “Adjust weights somehow”
- “It works, probably”

With Math:

- Precise learning rules
- Convergence guarantees
- Understanding of limitations

The next 8 slides formalize what you already understand intuitively.

You understand the intuition. Let’s write it precisely.

The Perceptron Equation

Scalar Form

$$y = f\left(\sum_{i=1}^{n} w_i x_i + b\right)$$

where f is the step function:

$$f(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Vector Form

$$y = f(\mathbf{w}^T \mathbf{x} + b)$$

where:

- $\mathbf{w} = (w_1, \ldots, w_n)^T$
- $\mathbf{x} = (x_1, \ldots, x_n)^T$

Alternative Notation
We can absorb the bias into weights:

$$\tilde{\mathbf{w}} = \begin{pmatrix} b \\ w_1 \\ \vdots \\ w_n \end{pmatrix}, \quad \tilde{\mathbf{x}} = \begin{pmatrix} 1 \\ x_1 \\ \vdots \\ x_n \end{pmatrix}$$

Then:

$$y = f(\tilde{\mathbf{w}}^T \tilde{\mathbf{x}})$$

Note: This "bias trick" simplifies notation but they are equivalent.

The complete mathematical model

Term by Term

Symbol	Meaning
x_i	Input feature i
w_i	Weight for feature i
b	Bias (threshold shift)
z	Weighted sum (pre-activation)
f	Activation function
y	Output prediction
n	Number of features

Dimensions:

- $\mathbf{x} \in \mathbb{R}^n$
- $\mathbf{w} \in \mathbb{R}^n$
- $b, z, y \in \mathbb{R}$

What Gets Learned?

Learned (trainable):

- Weights $w_1, \ldots, w_n$
- Bias b

Fixed (architecture):

- Number of inputs n
- Activation function f

Given (data):

- Input values $x_1, \ldots, x_n$
- Target labels (for training)

Total Parameters: $n + 1$
(For a 3-feature perceptron: 4 parameters)

Each symbol has a meaning

The Bias Term

What Does Bias Do?

Without bias ($b = 0$):

$$z = \mathbf{w}^T \mathbf{x}$$

The decision boundary passes through origin.

With bias ($b \neq 0$):

$$z = \mathbf{w}^T \mathbf{x} + b$$

The decision boundary can be anywhere.

Interpretation:

- $b > 0$: Default toward "Buy"
- $b < 0$: Default toward "Sell"
- Like a prior belief

Finance Analogy

Without Bias:
"I have no opinion until I see data"

With Positive Bias:
"I'm generally bullish; you need to convince me to sell"

With Negative Bias:
"I'm skeptical by default; you need strong evidence to buy"

Key Point: Bias shifts the "bar" that evidence must clear. It's learned from data just like weights.

Bias shifts the decision threshold

Formal Definition

The Heaviside step function:

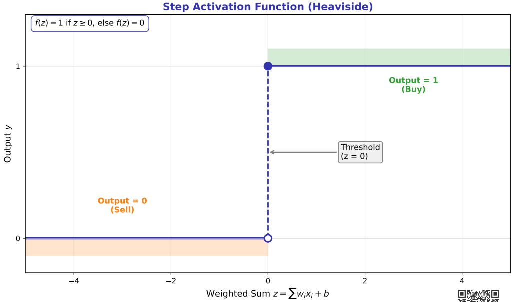
$$f(z) = \mathbf{1}_{z \geq 0} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

Properties:

- Output $\in \{0, 1\}$
- Discontinuous at $z = 0$
- Not differentiable (problem for gradient-based learning!)

Variants:

- Sign function: outputs $\{-1, +1\}$
- Same idea, different labels



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Preview: The non-differentiability of the step function is why we'll need smoother activations (sigmoid, ReLU) in later modules.

Binary output: yes or no

The Perceptron as a Hyperplane

The equation $\mathbf{w}^T\mathbf{x} + b = 0$ defines a hyperplane:

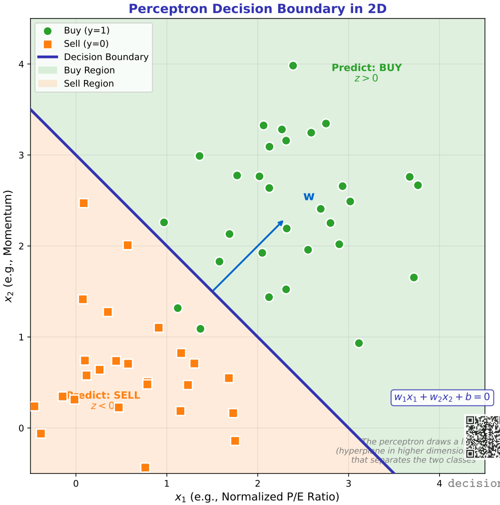
- In 2D: a line
- In 3D: a plane
- In nD: a hyperplane

Regions:

- $\mathbf{w}^T\mathbf{x} + b > 0$: Class 1 (Buy)
- $\mathbf{w}^T\mathbf{x} + b < 0$: Class 0 (Sell)
- $\mathbf{w}^T\mathbf{x} + b = 0$: Decision boundary

Weight Vector Direction:

$\mathbf{w}$ is perpendicular to the decision boundary, pointing toward the positive class.



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The perceptron draws a line between classes

Two-Feature Stock Screener

Features:

- x_1: P/E ratio (normalized)
- x_2: 6-month momentum (%)

Classes:

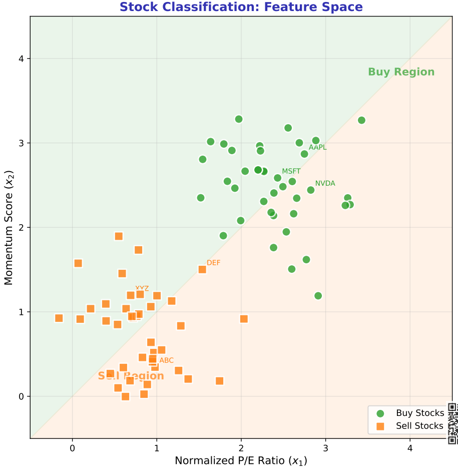
- Green: Outperformed (Buy)
- Red: Underperformed (Sell)

Goal:

Find w_1, w_2, b such that:

$$w_1 \cdot \text{P/E} + w_2 \cdot \text{Momentum} + b = 0$$

separates the classes.



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Challenge: Can we draw a single line to separate Buy from Sell?

Separating "good" stocks from "bad" stocks

In 2D: The Line Equation
From $w_1 x_1 + w_2 x_2 + b = 0$:

$$x_2 = -\frac{w_1}{w_2} x_1 - \frac{b}{w_2}$$

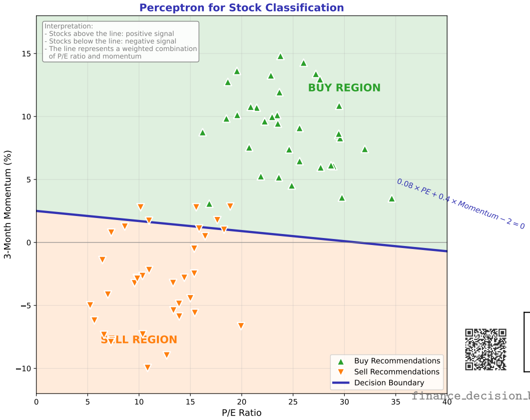
This is a line with:

- Slope: $-\frac{w_1}{w_2}$
- Intercept: $-\frac{b}{w_2}$

Example:
If $w_1 = 2, w_2 = 1, b = -3$:

$$x_2 = -2x_1 + 3$$

Stocks above this line: Buy
Stocks below this line: Sell



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The line that separates buy from sell

How Does the Perceptron Learn?

The Learning Problem
Given:

- Training data: $\{(\mathbf{x}^{(i)}, y^{(i)})\}_{i=1}^{m}$
- Each $\mathbf{x}^{(i)}$: feature vector
- Each $y^{(i)} \in \{0, 1\}$: true label

Find:

- Weights $\mathbf{w}$
- Bias b
- Such that predictions match labels

The Approach:
Start with random weights, then iteratively adjust based on mistakes.

The Core Idea

If prediction is correct:
Do nothing. Weights are fine.

If prediction is wrong:
Adjust weights to make this example more likely to be correct next time.

Repeat:
Keep cycling through training data until no mistakes (or convergence).

Key Insight: Learning = adjusting weights based on errors.

Learning = adjusting weights based on mistakes

Learning from Mistakes

Two Types of Errors

False Negative ($\hat{y} = 0,\ y = 1$):

- Predicted Sell, should be Buy
- The score z was too low
- Need to *increase* score for this $\mathbf{x}$
- Solution: Add $\mathbf{x}$ to $\mathbf{w}$

False Positive ($\hat{y} = 1,\ y = 0$):

- Predicted Buy, should be Sell
- The score z was too high
- Need to *decrease* score for this $\mathbf{x}$
- Solution: Subtract $\mathbf{x}$ from $\mathbf{w}$

Visual Intuition

Before update:
Point is on wrong side of boundary.

After update:
Boundary moves to include the point on the correct side.

The Update Rule:

$$\mathbf{w}_{\text{new}} = \mathbf{w}_{\text{old}} + (y - \hat{y}) \cdot \mathbf{x}$$

Check:

- If $y = 1, \hat{y} = 0$: add $\mathbf{x}$
- If $y = 0, \hat{y} = 1$: subtract $\mathbf{x}$
- If $y = \hat{y}$: no change

Each mistake is a learning opportunity

Why Adding x Works

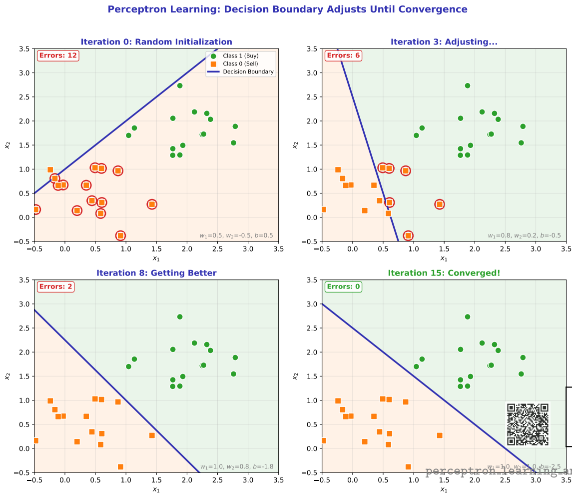
For a false negative (missed Buy):

- Current: $\mathbf{w}^T\mathbf{x} + b < 0$
- After adding $\mathbf{x}$ to $\mathbf{w}$:
- New score: $(\mathbf{w} + \mathbf{x})^T\mathbf{x} + b$
- $= \mathbf{w}^T\mathbf{x} + \mathbf{x}^T\mathbf{x} + b$
- $= \mathbf{w}^T\mathbf{x} + \|\mathbf{x}\|^2 + b$

Since $\|\mathbf{x}\|^2 > 0$, the new score is higher!

Geometrically:

Adding $\mathbf{x}$ rotates the decision boundary toward classifying $\mathbf{x}$ correctly.



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If wrong, move the boundary

The Perceptron Learning Rule

The Update Equations

For each training example $(\mathbf{x}, y)$:

Weight update:

$$\mathbf{w} \leftarrow \mathbf{w} + \eta(y - \hat{y})\mathbf{x}$$

Bias update:

$$b \leftarrow b + \eta(y - \hat{y})$$

where:

- $\eta > 0$ is the learning rate
- $\hat{y} = f(\mathbf{w}^T\mathbf{x} + b)$ is prediction
- y is true label

The Complete Algorithm

1. Initialize $\mathbf{w} = \mathbf{0}$, $b = 0$
2. **repeat**:
   a. For each $(\mathbf{x}^{(i)}, y^{(i)})$ in training set:
   b. Compute $\hat{y}^{(i)} = f(\mathbf{w}^T\mathbf{x}^{(i)} + b)$
   c. If $\hat{y}^{(i)} \neq y^{(i)}$:
      $\mathbf{w} \leftarrow \mathbf{w} + \eta(y^{(i)} - \hat{y}^{(i)})\mathbf{x}^{(i)}$
      $b \leftarrow b + \eta(y^{(i)} - \hat{y}^{(i)})$
3. **until** no errors (or max iterations)

The mathematical update rule

The Learning Rate

What is η?
The learning rate controls step size:

- How much weights change per update
- Typical values: 0.01 to 1.0
- For perceptron: often $\eta = 1$

Effects:
η **too small:**

- Very slow learning
- Many iterations needed
- But stable

η **too large:**

- May overshoot
- Oscillate around solution
- But faster initially

For the Perceptron

Good news:
For linearly separable data, the perceptron converges regardless of $\eta > 0$.

Why?
The convergence theorem (next slides) guarantees finding a solution if one exists.

In Practice:
$\eta = 1$ is common for perceptron. Learning rate matters more for:

- Gradient descent (Module 3)
- Non-separable data
- Multi-layer networks

Step size matters: too big or too small both cause problems

Worked Example: Stock Classification

Setup

Two stocks, two features:

- $\mathbf{x}^{(1)} = (0.5, 0.8)$, $y^{(1)} = 1$ (Buy)
- $\mathbf{x}^{(2)} = (0.2, 0.3)$, $y^{(2)} = 0$ (Sell)

Initialize: $\mathbf{w} = (0, 0)$, $b = 0$, $\eta = 1$

Iteration 1: Example 1

- $z = 0 \cdot 0.5 + 0 \cdot 0.8 + 0 = 0$
- $\hat{y} = f(0) = 1$ (threshold at 0)
- $y = 1$, correct! No update.

Iteration 1: Example 2

- $z = 0$, $\hat{y} = 1$
- $y = 0$, wrong!
- $\mathbf{w} \leftarrow (0, 0) + 1(0 - 1)(0.2, 0.3) = (-0.2, -0.3)$
- $b \leftarrow 0 + 1(0 - 1) = -1$

Iteration 2: Example 1

- $z = -0.2(0.5) - 0.3(0.8) - 1 = -1.34$
- $\hat{y} = 0$
- $y = 1$, wrong!
- $\mathbf{w} \leftarrow (-0.2, -0.3) + (0.5, 0.8) = (0.3, 0.5)$
- $b \leftarrow -1 + 1 = 0$

Iteration 2: Example 2

- $z = 0.3(0.2) + 0.5(0.3) + 0 = 0.21$
- $\hat{y} = 1$, $y = 0$, wrong!
- $\mathbf{w} \leftarrow (0.3, 0.5) - (0.2, 0.3) = (0.1, 0.2)$
- $b \leftarrow 0 - 1 = -1$

Continue until convergence...

Following the math with real numbers

Convergence: Does It Always Work?

The Perceptron Convergence Theorem

Theorem (Rosenblatt, 1962):

If the training data is **linearly separable**, the perceptron learning algorithm will find a separating hyperplane in a **finite** number of updates.

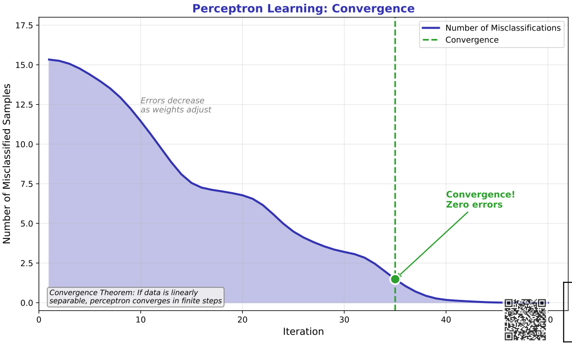
Key Conditions:

- Data must be linearly separable
- Learning rate $\eta > 0$
- Cycling through all examples

Bound on Updates:

$$\text{mistakes} \leq \frac{R^2}{\gamma^2}$$

where $R = \text{max norm}$, $\gamma = \text{margin}$



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The perceptron convergence theorem guarantees finding a solution IF one exists

Discussion Question

"What happens when data isn't linearly separable in financial markets? Can you think of examples?"

Consider:

Examples of Non-Separable Data:

- High P/E growth stocks AND low P/E value stocks both outperform
- Medium-risk investments underperform both conservative and aggressive
- "Buy the rumor, sell the news" patterns

What Happens to the Perceptron?

- Never converges
- Oscillates forever
- Best we can do: minimize errors
- Need something more powerful...

Foreshadowing: This is exactly why we need **multi-layer** networks (Module 2).

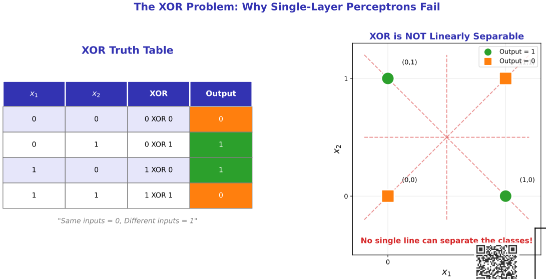
Think-Pair-Share: 3 minutes

The Exclusive OR Function

x_1	x_2	XOR
0	0	0
0	1	1
1	0	1
1	1	0

In Words:
Output is 1 if inputs are *different*, 0 if inputs are *same*.

The Challenge:
Try to draw a single line that separates the 1s from the 0s...



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Some patterns cannot be separated by a single line

Geometric Impossibility

Perceptron decision boundary:
$w_1x_1 + w_2x_2 + b = 0$
This is always a **straight line**.

XOR requires:
A boundary that curves or has multiple segments.

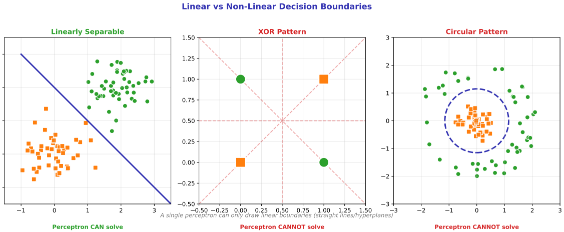
Linear vs Non-Linear

Linearly Separable:

- AND, OR, NAND, NOR
- One line can separate

Not Linearly Separable:

- XOR, XNOR
- No single line works



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No single hyperplane can separate XOR

1969: The Critique That Changed Everything

Minsky and Papert's Book
"Perceptrons: An Introduction to Computational Geometry" (1969)

Key Arguments:

1. Single-layer perceptrons cannot compute XOR
2. Many important functions are non-linear
3. No known training algorithm for multi-layer networks
4. Scaling limitations

The Impact:
The book was rigorous and influential. It convinced funding agencies that neural networks were a dead end.

The Controversy

Valid Points:

- Single layers *are* limited
- XOR problem is real
- No training algorithm existed (then)

Overstated Points:

- "Neural networks can't work"
- Implied multi-layer networks wouldn't help
- Discouraged research for 15+ years

Lesson: Valid criticism of current methods shouldn't stop research into future improvements.

Marvin Minsky and Seymour Papert: "Perceptrons" book

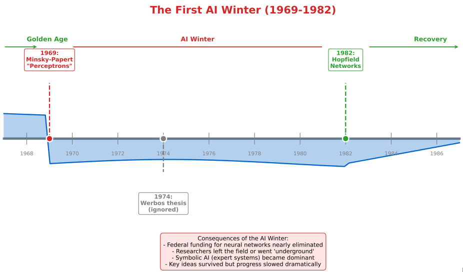
The Collapse
After 1969:

- Funding dried up
- Researchers left the field
- "Neural networks don't work"
- Symbolic AI took over

Duration: 1969 to ∼1982

What Survived:

- A few dedicated researchers
- Theoretical work continued quietly
- Hopfield networks (1982)
- Backpropagation (1986)



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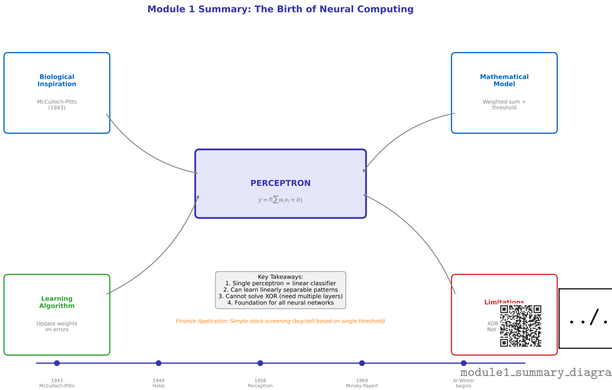
1969-1982: The dark ages of neural network research

Module 1: Key Takeaways

What We Learned

1. **Historical Foundation**
   - McCulloch-Pitts (1943): neurons compute
   - Hebb (1949): learning strengthens connections
   - Rosenblatt (1958): perceptron learns
2. **The Perceptron Model**
   - Weighted sum + threshold
   - Linear decision boundary
   - Learns from mistakes
3. **Limitations**
   - Only linearly separable problems
   - XOR is impossible
   - Led to AI Winter



From biological inspiration to mathematical limitation

Preview: Module 2

"What if we stack multiple perceptrons?"

The Problem We Face
Single perceptrons can only solve linearly separable problems. Real financial data is rarely that simple.

The Solution Preview:

- Add "hidden" layers
- Non-linear activation functions
- Multi-Layer Perceptrons (MLPs)

Coming in Module 2:

- How XOR gets solved
- MLP architecture
- Activation functions (sigmoid, ReLU)
- Universal Approximation Theorem
- Loss functions

Spoiler: Adding just one hidden layer changes everything.

Mathematical details for this module: See Appendix A (Perceptron Convergence Proof)

Next: Solving XOR with Multi-Layer Perceptrons