Gradient Descent and Backpropagation

Neural Networks for Finance

Neural Networks for Finance

BSc Lecture Series

November 30, 2025

The Central Question

"We have the architecture. How does it LEARN?"

What We Know:

- MLP architecture (Module 2)
- Forward pass computation
- Loss functions measure error
- Good weights exist (universal approximation)

This module bridges the gap from architecture to learning.

What We Don't Know:

- How to find good weights
- How errors guide updates
- Why training sometimes fails
- How to avoid overfitting

The fundamental challenge of neural network training

Finance Parallel: The Trading Desk

How Traders Improve
A trader's learning process:

1. Make a trade (forward pass)
2. Wait for P&L (loss function)
3. Analyze what went wrong (gradient)
4. Adjust strategy (weight update)
5. Repeat thousands of times (epochs)

Key Insight:
Mistakes are information. Each error tells you how to adjust.

Neural Network Training

Trading	Neural Net
Trade execution	Forward pass
P&L calculation	Loss function
Post-trade analysis	Backpropagation
Strategy adjustment	Weight update
Experience	Training epochs

Both learn by **iteratively correcting mistakes**.

How does a trader improve? By analyzing what went wrong.

Module 3 Roadmap

Today's Journey

1. **Loss Functions (Review)**
   - Measuring prediction error
   - MSE intuition
2. **Gradient Descent**
   - Finding the minimum
   - Learning rate tuning
3. **Backpropagation**
   - Credit assignment
   - Chain rule in action
4. **Training Dynamics**
   - Batch vs. stochastic
   - Epochs and convergence
5. **Overfitting**
   - The enemy of generalization
   - The backtest trap

Learning Objectives:

- Understand gradient descent intuitively
- Grasp backpropagation as "blame assignment"
- Recognize and prevent overfitting

From measuring error to updating weights

The Learning Problem

The Challenge
Given:

- Training data: $\{(\mathbf{x}^{(i)}, y^{(i)})\}_{i=1}^{m}$
- Network architecture
- Loss function $\mathcal{L}$

Find:

- Weights **W** and biases **b**
- That minimize $\mathcal{L}$
- And generalize to new data

Scale of the Problem:
A 4-10-5-1 network: 111 parameters
A ResNet-50: 25 million parameters

Why Is This Hard?

Dimensionality:

- Millions of weights to tune
- Exponentially many combinations
- Can't try them all

Non-Convexity:

- Many local minima
- Saddle points
- Flat regions

The Solution:
Gradient-based optimization
"Move downhill in weight space"

Thousands of weights to tune - how do we find the right values?

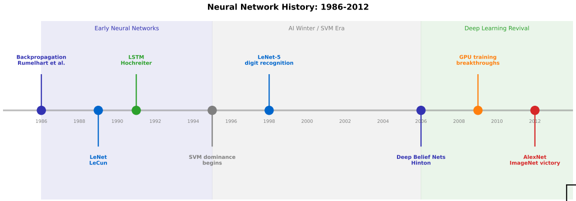
1989: LeNet and Practical Success

Yann LeCun at Bell Labs
First commercially deployed neural network:

- Handwritten digit recognition
- Used by US Postal Service
- Read millions of checks
- Proved neural nets could work

Key Innovations:

- Convolutional architecture
- Shared weights
- Backprop through convolutions



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timeline_1986_201

Yann LeCun: First commercially deployed neural network

1991: The Vanishing Gradient Problem

The Discovery
Sepp Hochreiter (1991) identified why deep networks fail:

The Problem:

- Gradients multiply through layers
- Sigmoid derivative: max 0.25
- Through 10 layers: $0.25^{10} \approx 10^{-6}$
- Early layers learn nothing

Symptoms:

- Later layers learn quickly
- Early layers stuck at random
- Network never converges

Why Sigmoid Causes Problems

For sigmoid: $\sigma'(z) = \sigma(z)(1 - \sigma(z))$
Maximum value: $\sigma'(0) = 0.25$

Layers	Max Gradient
1	0.25
5	10^{-3}
10	10^{-6}
20	10^{-12}

Implication: Deep networks seemed impossible until ReLU (2010).

Deep networks couldn't learn - gradients disappeared

1997: LSTM Networks

Long Short-Term Memory
Hochreiter & Schmidhuber solution:

- Designed for sequences
- Explicit “memory” cells
- Gating mechanisms
- Gradients can flow unchanged

Key Innovation:
The “constant error carousel” – a path where gradients don’t decay.

Applications:

- Speech recognition
- Machine translation
- Time series prediction

Finance Relevance

LSTMs became popular for:

- Stock price prediction
- Volatility forecasting
- Sentiment analysis
- Algorithmic trading

Why LSTM for Finance?

- Financial data is sequential
- Long-term dependencies matter
- Regime changes persist

Note: Now largely replaced by Transformers (2017).

Hochreiter and Schmidhuber: Long Short-Term Memory

2012: The ImageNet Moment

AlexNet Wins ImageNet
Alex Krizhevsky, Ilya Sutskever, Geoffrey Hinton:

- 15.3% error rate
- Second place: 26.2%
- **40% relative improvement**
- Used GPUs for training

What Made It Work:

1. ReLU activation (not sigmoid)
2. Dropout regularization
3. GPU training (60x faster)
4. Large dataset (1.2M images)
5. Data augmentation

Why This Was Different

Previous Attempts:

- Shallow networks
- Hand-crafted features
- Small datasets
- CPU training

AlexNet:

- 8 layers deep
- Learned features
- Massive data
- GPU parallelism

The Result: Deep learning became the dominant paradigm. Every major AI company pivoted.

AlexNet: Deep learning proves its superiority

What Changed Between 1990 and 2012?

The Ingredients for Success

1. **Big Data**
   - ImageNet: 1.2M labeled images
   - Internet made data collection possible
   - 1990: thousands of samples
2. **Compute Power**
   - GPUs: 100x speedup
   - Moore's law compounding
   - Training in days, not years
3. **Algorithmic Improvements**
   - ReLU: no vanishing gradients
   - Dropout: better generalization
   - Batch normalization (2015)
4. **Open Research Culture**
   - arXiv preprints
   - Open-source frameworks
   - Reproducibility

Key Insight: The core ideas from 1986 worked – they just needed scale and engineering.

Big data + GPUs + ReLU + dropout = breakthrough

What Does “Wrong” Mean?

Quantifying Prediction Error
We need a function that:

- Takes predictions and labels
- Returns a single number
- Higher = worse predictions
- Differentiable (for gradients)

The Loss Function:

$$\mathcal{L}(\hat{y}, y)$$

Properties We Want:

- $\mathcal{L} \geq 0$ (non-negative)
- $\mathcal{L} = 0$ iff perfect prediction
- Smooth (for optimization)

Different Tasks, Different Losses

Task	**Loss**
Regression	MSE
Binary classification	Cross-entropy
Multi-class	Categorical CE
Ranking	Hinge loss

Finance Examples:

- Return prediction: MSE
- Buy/sell: Binary CE
- Sector classification: Categorical CE

We need a way to measure how wrong our predictions are

Finance Analogy: Profit and Loss

P&L as a Loss Function
For traders:

- P&L = realized gain/loss
- Negative P&L = bad trades
- Goal: maximize P&L

Connection to ML Loss:

- ML loss = prediction error
- Higher loss = worse model
- Goal: minimize loss

Key Difference:
P&L is a *performance* metric.
ML loss is an *optimization* target.
They may not align perfectly!

When P&L $\neq$ Loss

A model might have:

- Low MSE (accurate predictions)
- But low P&L (wrong on big moves)

Or:

- High MSE (noisy predictions)
- But high P&L (right when it matters)

Implication:
Consider using custom loss functions that better align with trading goals.

Module 4 explores this tension.

P&L is the loss function of trading

Loss Function: Error Measurement

Total Loss Over Dataset
For m training examples:

$$\mathcal{L}(\mathbf{W}) = \frac{1}{m}\sum_{i=1}^{m} \ell(\hat{y}^{(i)}, y^{(i)})$$

where:

- ℓ: loss per example
- $\hat{y}^{(i)} = f(\mathbf{x}^{(i)}; \mathbf{W})$: prediction
- $y^{(i)}$: true label
- $\mathbf{W}$: all network weights

Goal:

$$\mathbf{W}^* = \arg\min_{\mathbf{W}} \mathcal{L}(\mathbf{W})$$

Why Average?

Sum vs Average:

- Sum: scales with dataset size
- Average: comparable across datasets
- Gradient magnitude consistent

The Optimization Landscape:
$\mathcal{L}(\mathbf{W})$ defines a surface over weight space.

- High regions: bad weights
- Low regions: good weights
- We seek the lowest point

The loss function quantifies prediction error

Mean Squared Error: Intuition

The Formula

$$\mathcal{L}_{MSE} = \frac{1}{m}\sum_{i=1}^{m}(y^{(i)} - \hat{y}^{(i)})^2$$

In Words:

1. Compute error: $y - \hat{y}$
2. Square it: $(y - \hat{y})^2$
3. Average over all samples

Why Squaring?

- Makes all errors positive
- Penalizes large errors heavily
- Mathematically convenient

Example

y	$\hat{y}$	$(y - \hat{y})^2$
5%	3%	4
-2%	1%	9
8%	7%	1
MSE		**4.67**

Units: (percentage points)^2

RMSE: $\sqrt{MSE} = 2.16\%$
"On average, we're off by about 2%"

"How far off were we, on average?"

MSE: Visual Interpretation

Squared Errors as Areas

Each error $(y - \hat{y})^2$ is the area of a square with side length $|y - \hat{y}|$.

MSE = Average Square Area

Why This Matters:

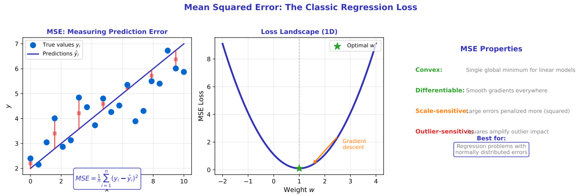
- Error of 4 is 16x worse than error of 1
- Large errors dominate
- Outliers have huge impact

Alternative: MAE

Mean Absolute Error:

$$\mathcal{L}_{MAE} = \frac{1}{m} \sum |y - \hat{y}|$$

More robust to outliers.



Squaring emphasizes large errors

Finance Application: Return Prediction

Worked Example

Predictions for 5 Stocks:

Stock	$\hat{y}$	y	Error^2
AAPL	$+5\%$	$+2\%$	9
MSFT	$+3\%$	$+4\%$	1
GOOG	-1%	$+2\%$	9
AMZN	$+4\%$	$+4\%$	0
META	$+2\%$	-3%	25
MSE			**8.8**

RMSE $= 2.97\%$

Interpretation

"On average, our return predictions are off by about 3 percentage points."

Is This Good?

Depends on context:

- Market daily vol: $\sim 1\%$
- 3% RMSE $=$ 3 std devs
- Not very predictive

Reality Check:

Even small predictability (RMSE slightly $<$ volatility) can be valuable in trading.

Worked example with stock returns

Discussion Question

"Why might we want to penalize large errors more than small ones in stock prediction?"

Consider:

Arguments For (Use MSE):

- Big errors are costlier
- Crashes matter more than small moves
- Position sizing affected
- Risk management

Arguments Against (Use MAE):

- Markets have fat tails
- Outliers can dominate MSE
- May optimize for rare events
- Robustness to noise

Reality: Many practitioners use MAE or Huber loss (combines both) for financial applications.

Think-Pair-Share: 3 minutes

The Loss Landscape

Loss as a Function of Weights

$$\mathcal{L}(\mathbf{W})$$

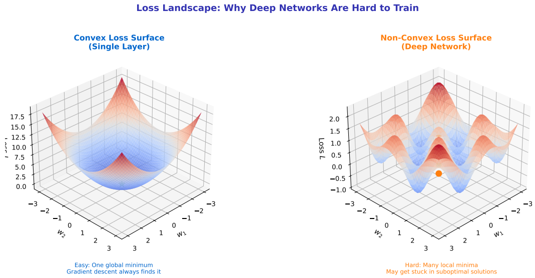
For every choice of weights, there's a loss value.

In 2D (two weights):
A surface we can visualize.

In High Dimensions:
A hypersurface we navigate blindly.

Features:

- Global minimum (best)
- Local minima (traps)
- Saddle points
- Flat regions (plateaus)



Finding the minimum of a high-dimensional function

The Optimization Problem

The Challenge
Find:

$$\mathbf{W}^* = \arg\min_{\mathbf{W}} \mathcal{L}(\mathbf{W})$$

Difficulties:

- Millions of dimensions
- Non-convex landscape
- No closed-form solution
- Can't try all possibilities

We Need:
An *iterative* algorithm that gradually improves weights.

Possible Approaches

Random Search:

- Try random weights
- Keep best so far
- Hopelessly slow

Grid Search:

- Try all combinations
- 10^{100} possibilities
- Impossible

Gradient-Based:

- Use local slope information
- Move toward improvement
- Tractable!

How do we find the weights that minimize loss?

The Blind Hiker Analogy

The Scenario

Imagine you're:

- Blindfolded
- On a mountainside
- Trying to reach the valley
- Can only feel the local slope

What Would You Do?

1. Feel the ground around you
2. Determine which way is downhill
3. Take a step in that direction
4. Repeat until you reach a valley

Neural Network Translation

Hiker	**Network**
Position	Weights $\mathbf{W}$
Altitude	Loss $\mathcal{L}$
Slope	Gradient $\nabla\mathcal{L}$
Step	Weight update
Valley	Minimum loss

Key Insight:

We don't need to see the whole landscape. Local slope is enough!

"You're blindfolded on a mountain. How do you find the valley?"

Answer: Feel the Slope

The Strategy

1. Compute the slope (gradient)
2. Move opposite to the slope
3. Repeat until convergence

Why Opposite?

- Gradient points uphill
- We want to go downhill
- Move in negative gradient direction

The Update Rule:

$$\mathbf{W} \leftarrow \mathbf{W} - \eta \nabla_{\mathbf{W}} \mathcal{L}$$

Gradient Descent Algorithm

```
1. Initialize $\mathbf{W}$ randomly
2. repeat:
    a. Compute loss $\mathcal{L}(\mathbf{W})$
    b. Compute gradient $\nabla \mathcal{L}$
    c. Update: $\mathbf{W} \leftarrow \mathbf{W} - \eta \nabla \mathcal{L}$
3. until convergence
```

$\eta =$ learning rate (step size)

Move in the direction that goes down

The Gradient: Direction of Steepest Ascent

What Is the Gradient?
The gradient $\nabla\mathcal{L}$ is a vector of partial derivatives:

$$\nabla_{\mathbf{w}}\mathcal{L} = \begin{pmatrix} \frac{\partial\mathcal{L}}{\partial w_1} \\ \frac{\partial\mathcal{L}}{\partial w_2} \\ \vdots \\ \frac{\partial\mathcal{L}}{\partial w_n} \end{pmatrix}$$

Each Component:
$\frac{\partial\mathcal{L}}{\partial w_i}$ = How much does loss change if we change w_i slightly?

Properties

Direction:

- Points toward steepest increase
- $-\nabla\mathcal{L}$ points toward steepest decrease

Magnitude:

- $\|\nabla\mathcal{L}\|$ = slope steepness
- Near minimum: gradient ≈ 0

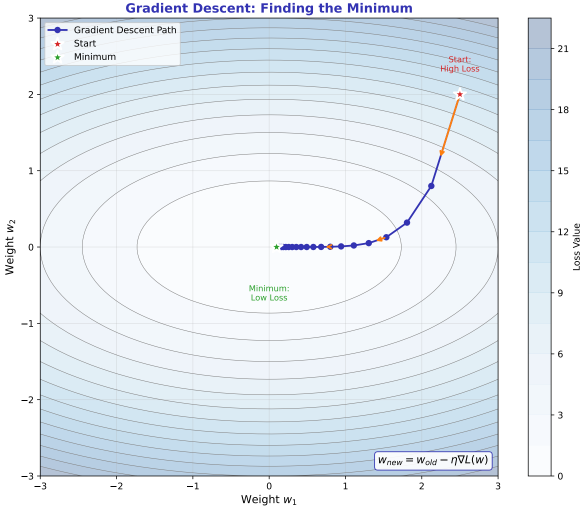
At a Minimum:

$$\nabla\mathcal{L} = \mathbf{0}$$

No direction goes further down.

The gradient tells us which way is “up”

Gradient Descent: Move Downhill



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Step in the negative gradient direction

Finance Parallel: Portfolio Optimization

Portfolio Adjustment
Similar iterative process:

1. Evaluate current portfolio
2. Estimate sensitivities ("greeks")
3. Adjust positions to reduce risk
4. Repeat periodically

Delta Hedging:

- Measure option delta
- Adjust stock position
- Move toward neutral

Comparison

GD	Portfolio
Loss	Risk/Variance
Weights	Positions
Gradient	Sensitivities
Learning rate	Trading aggressiveness
Convergence	Optimal allocation

Key Difference:
Markets change continuously. Portfolios must adapt.
Neural networks train once (mostly).

Similar to iterative portfolio rebalancing

The Learning Rate: Step Size

The Hyperparameter η

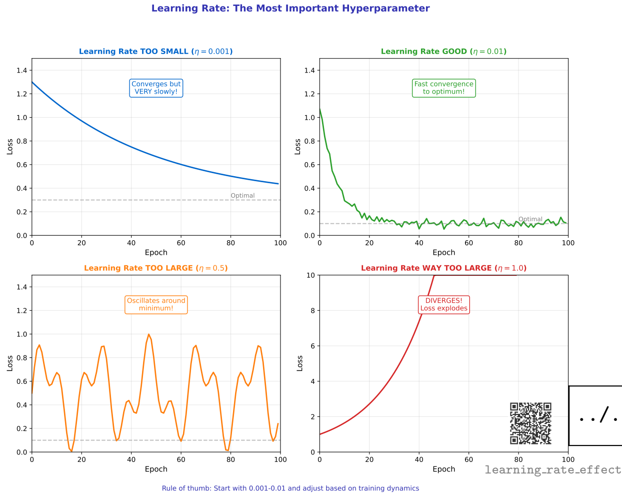
$$\mathbf{W} \leftarrow \mathbf{W} - \eta \nabla \mathcal{L}$$

η Controls:

- Size of each weight update
- Speed of convergence
- Stability of training

Typical Values:

- 10^{-4} to 10^{-1}
- Often starts at 0.01 or 0.001
- May decrease during training



Learning rate controls how far we move each step

Learning Rate Too High

The Problem

When η is too large:

- Steps overshoot the minimum
- May jump to worse regions
- Loss oscillates or explodes
- Training diverges

Symptoms:

- Loss goes up, not down
- Loss becomes NaN
- Weights grow very large
- Erratic training curves

Finance Analogy

Overtrading:

- Adjusting positions too aggressively
- Chasing every signal
- Transaction costs accumulate
- Portfolio becomes unstable

Solution:

Reduce learning rate until stable.

Rule of Thumb: If loss explodes, halve η.

Too big = overshoot the minimum

Learning Rate Too Low

The Problem
When η is too small:

- Steps are tiny
- Progress is slow
- May get stuck in flat regions
- Training takes forever

Symptoms:

- Loss decreases very slowly
- Many epochs with little improvement
- May stop before reaching minimum
- Wasted computation

Finance Analogy

Underreacting:

- Ignoring market signals
- Missing opportunities
- Portfolio drifts from target
- Slow adaptation to regime changes

Solution:
Increase learning rate or use adaptive methods.

Modern Practice: Adaptive optimizers (Adam, RMSprop) adjust η automatically.

Too small = converge too slowly

Discussion Question

"In trading, what's analogous to learning rate? What happens if you adjust positions too aggressively or too conservatively?"

Consider:

Position Sizing:

- How much to trade per signal
- Kelly criterion vs. fractional Kelly
- Risk management constraints

Rebalancing Frequency:

- How often to adjust
- Transaction cost vs. tracking error
- Market impact considerations

Key Insight: Both trading and ML require balancing responsiveness against stability.

Think-Pair-Share: 3 minutes

The Attribution Problem

The Challenge
We know:

- The output was wrong
- We need to update weights
- There are thousands of weights

The Question:

Which weights caused the error?

Credit Assignment:
Attributing output error to individual weights deep in the network.

Why Is This Hard?

Direct Attribution:

- Output layer weights: clear influence
- Hidden layer weights: indirect
- Early layers: very indirect

The Chain of Influence:
$w_1 \to h_1 \to h_2 \to \cdots \to \hat{y} \to \mathcal{L}$
Each weight affects the loss through many intermediate steps.

The output was wrong. Which weights caused it?

Finance Analogy: Post-Trade Analysis

Attribution in Trading
A portfolio lost money. Why?

1. Macro call wrong?
2. Sector allocation off?
3. Stock selection bad?
4. Timing poor?
5. Execution costly?

Performance Attribution:

- Decompose returns by factor
- Trace P&L to decisions
- Learn which calls were wrong

Neural Network Attribution

Trading	Neural Net
Macro view	Early layers
Sector allocation	Hidden layers
Stock picks	Later layers
Final trades	Output
P&L	Loss

Backpropagation is the neural network's performance attribution algorithm.

"Which decisions led to this P&L?"

Backpropagation: Blame Assignment

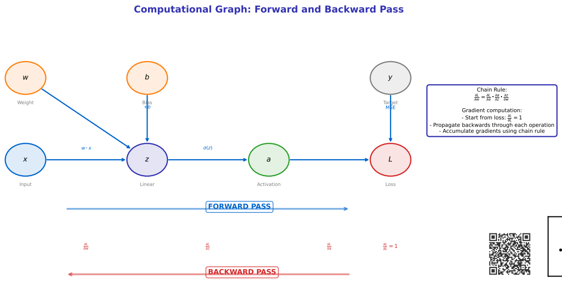
The Algorithm
Backpropagation computes $\frac{\partial \mathcal{L}}{\partial w}$ for every weight w in the network.

Key Idea:
Work backward from output to input, propagating error attribution.

Two Passes:

1. **Forward Pass:** Compute outputs
2. **Backward Pass:** Compute gradients

Efficiency:
Computes ALL gradients in time proportional to one forward pass.



Computational Graph: Forward and Backward Pass
w
Weight
b
Bias
$+b$
y
Target
MSE
x
Input
$w \cdot x$
z
Linear
$\sigma(z)$
a
Activation
L
Loss
Chain Rule:
$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial a} \cdot \frac{\partial a}{\partial z} \cdot \frac{\partial z}{\partial w}$
Gradient computation:
- Start from loss: $\frac{\partial L}{\partial L} = 1$
- Propagate backwards through each operation
- Accumulate gradients using chain rule
FORWARD PASS
$\frac{\partial L}{\partial w}$
$\frac{\partial L}{\partial z}$
$\frac{\partial L}{\partial a}$
$\frac{\partial L}{\partial L} = 1$
BACKWARD PASS

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backprop_computational_grap

Propagating error backward through the network

The Chain Rule: Intuition

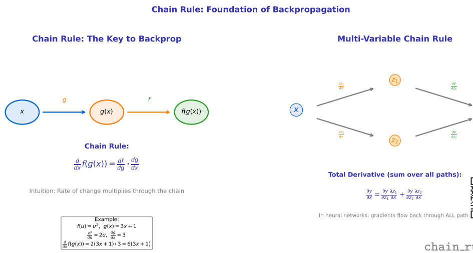
The Core Mathematical Tool
If A affects B and B affects C:

$$\frac{\partial C}{\partial A} = \frac{\partial C}{\partial B} \cdot \frac{\partial B}{\partial A}$$

Example:
Temperature $\rightarrow$ Ice cream sales $\rightarrow$ Profit

How does temperature affect profit?

$$\frac{\partial \text{Profit}}{\partial \text{Temp}} = \frac{\partial \text{Profit}}{\partial \text{Sales}} \cdot \frac{\partial \text{Sales}}{\partial \text{Temp}}$$


Chain Rule: Foundation of Backpropagation
Chain Rule: The Key to Backprop
x
g
$g(x)$
f
$f(g(x))$
Chain Rule:
$\frac{d}{dx}f(g(x)) = \frac{df}{dg} \cdot \frac{dg}{dx}$
Intuition: Rate of change multiplies through the chain
Example:
$f(u) = u^2, \ g(x) = 3x + 1$
$\frac{df}{du} = 2u, \ \frac{dg}{dx} = 3$
$\frac{d}{dx}f(g(x)) = 2(3x+1) \cdot 3 = 6(3x+1)$
Multi-Variable Chain Rule
x
$\frac{\partial z_1}{\partial x}$
$\frac{\partial z_2}{\partial x}$
z_1
z_2
$\frac{\partial y}{\partial z_1}$
$\frac{\partial y}{\partial z_2}$
y
Total Derivative (sum over all paths):
$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial z_1}\frac{\partial z_1}{\partial x} + \frac{\partial y}{\partial z_2}\frac{\partial z_2}{\partial x}$
In neural networks: gradients flow back through ALL paths

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chain_rule_visualizatio

“If A affects B and B affects C, how does A affect C?”

Finance Chain Example

Chain of Effects

Fed Rate → Mortgages → Housing → Banks → Portfolio

How does Fed rate affect your portfolio?

$$\frac{\partial \text{Portfolio}}{\partial \text{Fed}} = \frac{\partial P}{\partial B} \cdot \frac{\partial B}{\partial H} \cdot \frac{\partial H}{\partial M} \cdot \frac{\partial M}{\partial F}$$

Each Link:

- Fed → Mortgages: rate sensitivity
- Mortgages → Housing: demand elasticity
- Housing → Banks: credit exposure
- Banks → Portfolio: position size

Neural Network Parallel

Finance	**Neural Net**
Fed rate	Input x
Mortgages	Hidden layer 1
Housing	Hidden layer 2
Banks	Hidden layer 3
Portfolio	Output

Backprop does this automatically:
Chains together all the local sensitivities to get the total effect of each input/weight on the loss.

Effects propagate through chains of influence

Backprop: Output Layer

At the Output
For output weight $w^{(L)}$:

$$\frac{\partial \mathcal{L}}{\partial w^{(L)}} = \frac{\partial \mathcal{L}}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z^{(L)}} \cdot \frac{\partial z^{(L)}}{\partial w^{(L)}}$$

Each Term:

- $\frac{\partial \mathcal{L}}{\partial \hat{y}}$: How loss changes with output
- $\frac{\partial \hat{y}}{\partial z^{(L)}}$: Activation derivative
- $\frac{\partial z^{(L)}}{\partial w^{(L)}}$: Input from previous layer

For MSE + Sigmoid:

$$\frac{\partial \mathcal{L}}{\partial w^{(L)}} = (\hat{y} - y) \cdot \hat{y}(1 - \hat{y}) \cdot a^{(L-1)}$$

Output Error ($\delta^{(L)}$)

Define the “error signal”:

$$\delta^{(L)} = \frac{\partial \mathcal{L}}{\partial z^{(L)}}$$

For MSE loss + sigmoid:

$$\delta^{(L)} = (\hat{y} - y) \cdot \sigma'(z^{(L)})$$

Then:

$$\frac{\partial \mathcal{L}}{\partial w^{(L)}} = \delta^{(L)} \cdot a^{(L-1)}$$

This is just error $\times$ input!

At the output, error attribution is straightforward

Backprop: Hidden Layers

The Key Insight
Hidden layer error comes from downstream:

$$\delta^{(l)} = ((W^{(l+1)})^T \delta^{(l+1)}) \odot \sigma'(z^{(l)})$$

In Words:

1. Take error from next layer ($\delta^{(l+1)}$)
2. Multiply by weights connecting to next layer
3. Scale by local activation derivative

Error Flows Backward:
Output $\rightarrow$ Last hidden $\rightarrow$ ... $\rightarrow$ First hidden

Why This Works

Chain rule connects layers:

$$\frac{\partial \mathcal{L}}{\partial z^{(l)}} = \sum_j \frac{\partial \mathcal{L}}{\partial z_j^{(l+1)}} \cdot \frac{\partial z_j^{(l+1)}}{\partial z^{(l)}}$$

Gradient for Hidden Weight:

$$\frac{\partial \mathcal{L}}{\partial w^{(l)}} = \delta^{(l)} \cdot a^{(l-1)}$$

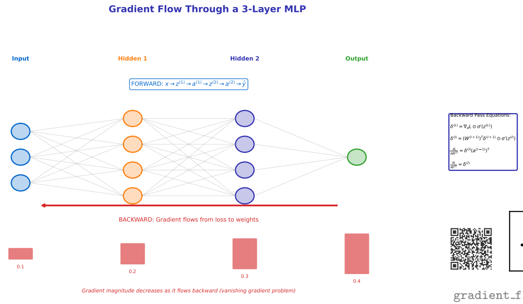
Same formula as output layer!

Hidden layer gradients require the chain rule

The Complete Training Loop

One Training Step

1. **Forward Pass**
   - Compute all activations
   - Get prediction $\hat{y}$
2. **Compute Loss**
   - $\mathcal{L} = \ell(\hat{y}, y)$
3. **Backward Pass**
   - Compute $\delta^{(L)}$ at output
   - Propagate backward to get all $\delta^{(l)}$
   - Compute all weight gradients
4. **Update Weights**
   - $\mathbf{W} \leftarrow \mathbf{W} - \eta \nabla \mathcal{L}$


Gradient Flow Through a 3-Layer MLP
Input
Hidden 1
Hidden 2
Output
FORWARD: $x \to z^{(1)} \to a^{(1)} \to z^{(2)} \to a^{(2)} \to \hat{y}$
Backward Pass Equations:
$\delta^{(L)} = \nabla_a L \odot \sigma'(z^{(L)})$
$\delta^{(l)} = (W^{(l+1)})^T \delta^{(l+1)} \odot \sigma'(z^{(l)})$
$\frac{\partial L}{\partial W^{(l)}} = \delta^{(l)} (a^{(l-1)})^T$
$\frac{\partial L}{\partial b^{(l)}} = \delta^{(l)}$
BACKWARD: Gradient flows from loss to weights
0.1
0.2
0.3
0.4
Gradient magnitude decreases as it flows backward (vanishing gradient problem)

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gradient_flow_ml

Forward pass, compute loss, backward pass, update weights

Why “Backpropagation”?

The Name
“Back-propagation of errors”

Information Flow:
Forward:

- Data flows input $\rightarrow$ output
- Activations computed layer by layer

Backward:

- Errors flow output $\rightarrow$ input
- Gradients computed layer by layer

Symmetry:
Each layer: one forward operation, one backward operation.

Historical Note

The Algorithm:

- Werbos (1974): first derivation
- Rumelhart et al. (1986): popularized
- Now standard in all deep learning

Modern Perspective:
Backprop is just automatic differentiation applied to neural networks.

Frameworks (PyTorch, TensorFlow):
Compute gradients automatically – you just specify the forward pass!

Error information flows from output to input

Discussion Question

"Why do deeper networks make training harder? What happens to gradients as they flow backward through many layers?"

Consider:

Vanishing Gradients:

- Sigmoid: max derivative 0.25
- Through 10 layers: 0.25^{10}
- Early layers get tiny gradients
- Learn extremely slowly

Exploding Gradients:

- If derivatives > 1
- Gradients grow exponentially
- Weights become huge
- Training diverges

Solutions: ReLU, batch normalization, residual connections, careful initialization.

Think-Pair-Share: 3 minutes