

Divisive Synthesis

1st Edition

Digital Down

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Abstract

This paper introduces and explores Divisive Synthesis, an approach in sound synthesis that utilizes the last unexplored common arithmetic operation in audio synthesis. While additive, subtractive and multiplicative synthesis have long been staples in the field, divisive synthesis has remained conspicuously absent.

1 Introduction

Despite the field's long history of innovation, Divisive Synthesis emerges as an unexplored "low-hanging fruit," offering a rich palette of timbral possibilities with relatively simple implementation. This research introduces a new synthesis technique but also serves as a reminder that even in mature fields, there remains simple concepts that have not been fully realized.

1.1 Historical Context

1.1.1 Additive Synthesis

Additive Synthesis, one of the oldest methods of sound synthesis, is rooted in Fourier's theorem, which states that any periodic waveform can be constructed from a sum of sinusoidal waves of various frequencies, amplitudes, and phases [1].

The fundamental principle of Additive Synthesis can be expressed mathematically as:

$$y(t) = \sum_{n=1}^N A_n \sin(2\pi f_n t + \phi_n)$$

where $y(t)$ is the resulting waveform, A_n is the amplitude of the n th partial, f_n is its frequency, and ϕ_n is its phase.

1.1.2 Subtractive Synthesis

Subtractive Synthesis, popularized in the 1960s and 1970s with the advent of analog synthesizers, operates on the principle of filtering harmonically rich waveforms [2].

The process can be described as:

$$y(t) = H(f) * x(t)$$

where $y(t)$ is the output signal, $x(t)$ is the input signal, and $H(f)$ is the transfer function of the filter.

1.1.3 Multiplicative Synthesis (Ring Modulation)

Multiplicative Synthesis, often referred to as Ring Modulation, involves the multiplication of two input signals [3]. The basic form is expressed as:

$$y(t) = x_1(t) * x_2(t)$$

where $y(t)$ is the output signal, and $x_1(t)$ and $x_2(t)$ are the input signals.

1.1.4 Divisive Synthesis

Divisive Synthesis represents a novel approach to sound generation, operating on the principle of dividing one waveform by another. The fundamental equation for Divisive Synthesis can be expressed as:

$$y(t) = \frac{x_1(t)}{x_2(t)}$$

where $y(t)$ is the output signal, $x_1(t)$ is the numerator waveform, and $x_2(t)$ is the denominator waveform.

2 Methodology

2.1 Waveform Generation

Two distinct waveforms are generated for the Divisive Synthesis process:

1. Active Waveform (W_a): This is the primary waveform that will be divided.
2. Divisive Waveform (W_d): This waveform's samples will serve as the divisors in the synthesis process.

2.2 Divisive Power

1. The divisive waveform's samples are normalized to a range determined by a parameter called "divisive power" (P_d). This process can be expressed mathematically as:

$$W_{d_{norm}}(t) = \frac{W_d(t)}{2^{15}} \cdot 2^{P_d-1}$$

where $W_d(t)$ is the original 16-bit sample value at time t , and P_d is the divisive power.

Table 1 illustrates the relationship between divisive power and the corresponding acceptable ranges for the divisive waveform.

Divisive Power	Acceptable Range
1	-1 to 1
2	-2 to 2
3	-4 to 4
4	-8 to 8
5	-16 to 16
6	-32 to 32
7	-64 to 64
8	-128 to 128
9	-256 to 256
10	-512 to 512
11	-1024 to 1024
12	-2048 to 2048
13	-4096 to 4096
14	-8192 to 8192
15	-16384 to 16384
16	-32768 to 32768

Table 1: Acceptable Ranges for Each Divisive Power

As shown in Table 1, the acceptable range for the divisive waveform's samples expand exponentially as the divisive power increases. This relationship can be expressed mathematically as:

$$\text{Range} = [-2^{(P_d-1)}, 2^{(P_d-1)}]$$

where P_d is the divisive power.

2. Rounding: The values are then rounded to the nearest integer:

$$W_{rounded}[n] = \text{round}(W_{result}[n])$$

2.3 Division Operation

The core of Divisive Synthesis lies in the sample-by-sample division of the input signals. This operation is performed for each sample in the digital domain:

$$y[n] = \frac{x_1[n]}{x_2[n]}$$

where n is the sample index.

2.3.1 Zero Conditional

One of the primary challenges in implementing Divisive Synthesis is handling division by zero. To address this, a conditional statement is used to skip division when the denominator is zero. This strategy can be expressed mathematically as:

$$y(t) = \begin{cases} \frac{x_1(t)}{x_2(t)} & x_2(t) \neq 0 \\ x_1(t) & x_2(t) = 0 \end{cases}$$

where $x_1(t)$ is the sample of the active waveform at time t , and $x_2(t)$ is the sample of the divisive waveform at time t .

2.4 Post Processing

After the division operation, the resulting waveform undergoes two final processing steps:

1. Rounding: Each sample's division result is rounded to the nearest integer to maintain compatibility when converting the raw audio values into output:

$$W_{rounded}[n] = \text{round}(result[n])$$

2. Normalization: All samples are normalized to ensure it fits within the desired bit depth range (typically 16-bit). This can be expressed as:

$$W_{final}[n] = \frac{W_{rounded}[n]}{\max(|W_{rounded}|)} \cdot (2^{15} - 1)$$

where $W[n]$ represents the n th sample in the sequence of samples.

2.5 Output

The division operation in the time domain leads to complex spectral interactions. Consider two sinusoidal inputs:

$$x_1(t) = A_1 \sin(2\pi f_1 t)$$

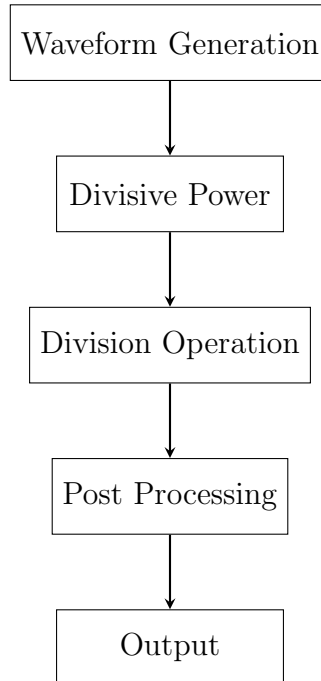
$$x_2(t) = A_2 \sin(2\pi f_2 t)$$

The resulting output can be expressed as:

$$y(t) = \frac{A_1 \sin(2\pi f_1 t)}{A_2 \sin(2\pi f_2 t)}$$

This operation leads to a series expansion involving Chebyshev polynomials of the second kind, resulting in a rich harmonic spectrum [4].

3 Conclusion



Divisive Synthesis stands as a testament to the ongoing potential for innovation in audio synthesis. It reminds us that even in mature fields, there remain fundamental concepts waiting to be explored, inviting further exploration and creative applications in sound design.

References

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