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Bernoulli versus Bernoulli

Sibling Rivalry of the Highest Order

The Bernoullis were an astonishing Swiss family, from which came eight noted mathematicians over a span of three generations.

The two main characters in our story are the brothers Jakob and Johann. Jakob, born in 1654, was the fifth of ten children. Their father, a successful spice merchant, wanted Jakob to go into the ministry. He even studied for it for a while. But Jakob's real interest was in mathematics, which he studied on his own. By 1676, at age 22, he was tutoring other students in the subject, and by 1687 he had become a professor at the University of Basel. At about the same time, and very shortly after Leibniz had published his first papers on his calculus (1684 and 1686), Jakob was already delving into it. In 1690, Leibniz would say of him, "The devices of this [Leibniz's] calculus are yet known to few people, and I do not know anybody who has understood my meaning better than this famous man."[1]

Johann, the tenth child in the Bernoulli family, was born in 1667. He was twelve and a half years younger than Jakob and, to his father's distress, proved unsuited to a business career. In 1685, he began the study of medicine and even got a degree in it, but, like Jakob, his heart was in mathematics. He began to study it privately with Jakob, probably in 1687. After perhaps two years, he was already a match for his older brother. These two were the first mathematicians to recognize the calculus's importance, to put it to use, and to spread the word about its significance.

By 1691, Johann was teaching the new mathematics to Guillaume François L'Hospital–an experienced and gifted mathematician. Using Johann's lesson plans, L'Hospital went on to write the first systematic text in the calculus (*Analyse des infiniment petits*, 1696). Johann also taught mathematics to Leonhard Euler, who himself went on to become a giant in 18th-century mathematics. In fact, nearly all of the half dozen or so major mathematicians of the time had been pupils of one or the other of these two Bernoulli brothers. By an interesting coincidence, Johann also had as a student J. C. Fatio de Duillier, whose brother Nicolas, as we already saw in the last chapter, was to play a major part in the Newton-Leibniz feud.

More important to our story, Johann taught mathematics to his own two sons, Daniel and Nicholas, both of whom went on to become quite respectable mathematicians in their own right. In fact, the tradition continued, and a third son of Johann's also became a professor of mathematics, and then *his* two sons became active in the fields of science and mathematics. Today there are half a dozen mathematical equations, theorems, or functions named after a Bernoulli.

It's easy to imagine the Bernoullis as just one big, happy family and the two brothers as especially pleased with their accomplishments and teaching careers.

Well, that's not quite the way it went. For although Jakob and Johann were both successful and busy, and kept up an almost continuous exchange of ideas with Leibniz, with other mathematicians, and with each other, they also challenged, argued with, and sniped at each other at every opportunity. This was sibling rivalry on a grand scale, for Jakob could never accept the fact that his much younger brother actually became his equal and, in some ways, even surpassed him. And Johann–well, we'll see shortly how Johann responded to his "kid brother" position.

Some Background

By the early 1690s, Jakob had done more to formalize the integral calculus than Leibniz himself had, for Leibniz had treated individual problems and had not set down general rules on the subject. Along the way, Johann became more proficient, and then, with their strange way of goading each other, both brothers used the new mathematics as a tool to solve problems that had bedeviled mathematicians for years, or even centuries.

For example, in 1659 Christiaan Huygens (1629–1695), a Dutch mathematician and physicist, had sought the curve along which an object descending under the influence of gravity would take the same amount of time to reach the bottom, from whatever point on the curve the descent began. He showed geometrically that the curve was a cycloid. Huygens then used this concept for his design of a pendulum clock that would keep accurate time. This design is sometimes referred to as an isochrone or a tautochrone. Galileo had earlier come up with the idea of using a pendulum for a clock, and Leibniz had done some preliminary work on the mathematics.

In May 1690, Jakob published his analysis of the equal-time problem, based on the calculus, in the *Acta Eruditorum*. He did it by setting up the differential equation for this curve of constant descent. The curve, he showed, is that of a cycloid. In essence, he proved Huygens's result analytically. The paper is important for another reason: the term *integral* appears for the first time as a calculus term.

Proud of his success with the isochrone problem, Jakob then proposed in the same paper an allied problem: determine the shape of a flexible but inelastic cord hung between two fixed points at the same height. The problem had been worked on at least as far back as the 15th century by Leonardo da Vinci; Galileo had considered the problem and had speculated that the curve was a parabolic arc.

The Kid Brother

Thirteen months after Jakob's paper, several solutions to the problem appeared in the June 1691 *Acta Eruditorum.* They were for a curve called a catenary, and they were by Leibniz, Huygens–and Johann! There was an entry from Jakob, which he called an "Additamentum ad Problema Funicularium." In it, he stated that after the solution was given by his brother, he continued the research further into some variations of the problem, such as cases where the rope is of different thickness or weight, for which he gave solutions.[2] As we'll see, Jakob's entry was not strictly an answer to the original problem and could be construed in different ways.

Johann made much of the fact that he was able to solve the catenary problem while, he maintained, his own brother, his teacher, could not. That was in 1691. The state of their later relations is shown by a letter Johann sent in 1718, some 27 years later, to his colleague and friend Pierre Rémond de Montmort. In it, he still writes in deprecating fashion of his brother, who had died 13 years earlier, and we still hear the strains of their never-ending competition. Msr. Montmort had apparently been under the impression that Jakob had been able to solve the catenary problem; Johann would have none of that. He wrote:

> The efforts of my brother were without success; for my part, I was more fortunate, for I found the skill (I say it without boasting, why should I conceal the truth?) to solve it in full and to reduce it to the rectification of the parabola. It is true that it cost me study that robbed me of rest for an entire night. . . . [B]ut the next morning, filled with joy, I ran to my brother, who was still struggling miserably with this Gordian knot without getting anywhere, always thinking like Galileo that the catenary was a parabola. Stop! Stop! I say to him, don't torture yourself any more to try to prove the identity of the catenary with the parabola, since it is entirely false. . . . [T]he two curves are so different that one is algebraic, the other is transcendental. . . . But then you [Montmort] astonish me by concluding that my brother found a method of solving this problem. . . . I ask you, do you

really think, if my brother had solved the problem in question, he would have been so obliging to me as not to appear among the solvers, just so as to cede me the glory of appearing alone on the stage in the quality of the first solver, along with Messrs. Huygens and Leibniz?[3]

Johann had in fact seen the important difference in the solutions. For the parabola, the simplest form of the standard equation in Cartesian coordinates is

$$y^2 = 4ax$$

which is clearly an algebraic equation.

For the catenary, Johann showed that the equation is transcendental:

$$y = (a/2)(e^{x/a} + e^{-x/a}).$$

Shortly thereafter, Johann solved the differential equation for the velaria.* Not one to exult in private, Johann went around boasting of his achievements.

Already, though, we are beginning to face the difficulties involved in teasing out the realities of the two brothers' respective claims and counterclaims. Although Johann's claims to have solved the catenary and the velaria are backed up by several writers,[4] others dispute both claims. For example, W. W. Rouse Ball, a respected historian of mathematics, argues that credit for both of these developments belongs to Jakob. In fact, he claims that Jakob's solution to the problem of the velaria, as well as his proof that the construction given by Leibniz of the catenary had been correct, are among his greatest discoveries.[5] In addition, several sources say that it was Jakob's solution that later proved useful in the design of such structures as suspension bridges and high-tension towers.[6] As I noted earlier, Jakob's contribution in the 1691 article does contain a treatment of variants on the catenary problem, as well as further generalizations, which suggests that he did have some understanding of the original problem.

*Curve for a rectangular sail filled with wind.

A possible explanation for the confusion is given by Florian Cajori, who maintained that Jakob tended to publish answers without explanations, while Johann gave in addition their theory.[7]

Well, no matter. It's quite likely that by this time Johann was trying to break out of the kid brother role and was boasting of, and perhaps exaggerating, his accomplishments. Who could stop him from maintaining that he had the solution before Jakob, and a better one at that?

Their Young Years

Rüdiger Thiele, a professor of mathematics at the University of Leipzig, argues that the negative attitude of the two mathematicians' father had an unfortunate effect on both of their personalities. He feels that Johann, however, as the kid brother, suffered the most. As time went on, Johann compensated by developing an enormous sense of self-importance, and he tried in every way he could to achieve fame. Yet he always found himself under the shadow of Jakob. As a result, he attempted to exaggerate his own importance.

Thiele even argues that Johann's emotional problems actually made it difficult for him to properly evaluate his *own* mathematical achievements.[8] As for who deserves credit for the brothers' various discoveries, Thiele points out that in the early days of their relationship they worked closely together, so it is sometimes hard to distinguish their respective contributions.[9] This could explain the confusion about the solutions to the catenary and the velaria I spoke of earlier. Similarly, while the *Encyclopædia Britannica* states that Johann exceeded his brother in the number of contributions he made to mathematics,[10] Thiele feels that Johann perpetrated so many untruths (*Unwahrheiten*) that the fame he did accrue is unwarranted.[11]

But he reveled in it. In 1701, Johann wrote to his father, "That I never got a letter from my father indicates that he preferred my brothers and had no affection for me. Am I not worthy of as much consideration as my siblings? . . . I would be grateful if you could tell me how they have earned this trust and affection from you, which you are depriving me of. I have placed myself under God's guidance because my father will not allow me to lead the kind of life I would wish to lead. So don't come to Basel and take my fame and say that you had anything to do with it."[12]

Jakob felt just as strongly about their father and worked under his own motto, which might be translated as, "I went against my father's will, and yet I am up there among the stars."[13]

In any case, by the beginning of the 1690s Johann was already a bona fide mathematician and had developed to the point where his claims and his actual accomplishments made him a real threat to Jakob's own need for celebrity. The seeds were being sown for a serious battle. Thiele and others feel that as the feud escalated, Johann was usually the instigator, but the feeling is by no means universal. The mathematics historian J. E. Hofmann states that Jakob was "self-willed, obstinate, aggressive, vindictive, beset by feelings of inferiority, and yet firmly convinced of his own abilities. With these characteristics, he necessarily had to collide with his similarly disposed brother."[14]

It was during this period (early 1690s) that Johann had his dealings with L'Hospital. Johann was in Paris in 1691, where he was already earning respect as a practitioner of the new mathematics. L'Hospital, quickly seeing its true importance and value, hired Johann as his private tutor. L'Hospital was well off, and he paid Johann well. Even after Johann returned to Basel, he continued the lessons by correspondence. The written lessons provided the substance for the first textbook in differential calculus–L'Hospital's *Analyse des infiniment petits* (1696)–and is the basis for his respected name in the field.

Although Jakob was smarting from Johann's boasting, both brothers went on carving out their own mathematical careers, while at the same time interacting between themselves and with other major mathematicians of their time. They also continued working with the new mathematics, applying it to a variety of problems. In 1694, for example, Jakob came up with the analysis for a fascinating eight-shaped curve that is often seen in mathematics classes today. It is commonly referred to as the lemniscate of Bernoulli.

The Relationship Changes

In 1695, Johann was offered a professorship at Halle and at the same time the chair in mathematics at Groningen, in the Netherlands. He accepted the latter, though not without some resentment toward Jakob–he would have preferred Jakob's position at Basel but knew that was not possible as long as Jakob held it. Furthermore, Jakob had begun doing some damage control; in retaliation for Johann's boasting, he went about terming Johann his pupil, who could only repeat what he has learned from his teacher.

Yet Johann now ranked as high as Jakob, and in June of 1696 Johann posed, first in *Acta Eruditorum* and then via a leaflet, the problem of the brachistochrone: determine the curve linking any two points, not in the same vertical line, along which a body would most quickly descend from a higher to a lower point under its own gravity. Intuitively, one might expect the answer to be a straight line, that is, the shortest distance between the two points. Galileo had already realized, however, that that was not so, but his speculation that the path would be the arc of a circle was also not correct. As I noted in the previous chapter, the solutions offered by several mathematicians were published together in the May 1697 *Acta*. The honored group consisted of the two Bernoullis, Leibniz, Newton, and L'Hospital. It's worth noting that L'Hospital needed Johann's help.

The methods used by the two brothers to solve the problem are particularly interesting, for they illustrate well the difference in their characters and abilities. In essence, Johann performed a kind of trick. His ingenious mind saw a connection between the path of quickest descent, that is, the mechanical problem at hand, and Fermat's principle of least time and its application in an optical problem. From Snell and Descartes, he knew what happened when a ray of light passes from one optical medium to another. He figured he could combine the refractive sine law (chapter 2) with the equation for the velocity of a body under gravity:

$$v = \surd(2gy).$$

Johann then divided the vertical plane of the problem into a series of very thin horizontal strips whose material densities varied slightly from one to the next. Although the particle would travel in a straight line through each strip, its path would bend slightly as it traveled from one to the next, as in a series of optical media with slightly varying refractive indexes. The particle's path of least time was therefore equivalent to the curved path of a light ray whose direction changed infinitesimally as it passed from one layer to the next.

Then, as Johann wrote, "But now we see immediately that the brachistochrone is the curve that a light ray would follow on its way through a medium whose density is inversely proportional to the velocity that a heavy body acquires during its fall. Indeed, whether the increase of the velocity depends on the constitution of a more or less resisting medium, or whether we forget about the medium and suppose that the acceleration is generated by another cause according to the same law as that of gravity, in both cases the curve is traversed in the shortest time. Who prohibits us from replacing one by the other?"[15] By letting the number of layers go to infinity, he was able to derive the curve for the brachistochrone.

Jakob, on the other hand, worked out a method that was more geometric and that seemed at first more cumbersome–he constructed a curve and used it as a basis for the analysis. As he put it, "The problem can therefore be reduced to the purely geometric one of determining the curve of which the line elements are directly proportional to the elements of the abscissa and indirectly proportional to the square roots of the ordinates."[16] The resulting curve was the one being sought. Both men had shown, each in his own way, that the correct form of the curve was a cycloid! The advantage of Jakob's method was that it was both more direct and more general. That is, it provided some general rules for solving several other problems of the same kind.

Calculus of Variations

The mathematics historian E. T. Bell argues, "James[17] [that is, Jakob] Bernoulli's signal merit was his recognition that the problem of selecting from an infinity of curves one having a given maximum or minimum property was of a novel genus, not amenable to the differential calculus and demanding the invention of new methods. This was the mathematical origin of the calculus of variations."[18]

The calculus of variations is a sort of generalization of the calculus. It seeks to find the path, the curve, or the surface for which a given function has a stationary value. In physical problems, this is usually a maximum or a minimum.

Though Bell credits Jakob for his pioneering input on this form of the calculus, there is a range of thought on this question–yet another example of how controversy seems to swirl around everything connected with the Bernoullis. Morris Kline, a well-known scholar, agrees with Bell, saying that while both solutions turned out to be early steps forward, Jakob's solution was even stronger in this regard.[19]

Yet Johann's solution, with its dependence on Fermat's principle of least action, surely points in this direction. Others feel even more strongly that the credit should go to Johann. David Eugene Smith, the editor of an important sourcebook in mathematics, writes, "The calculus of variations is generally regarded as originating with the papers of Jean [that is, Johann] Bernoulli on the problem of the brachistochrone." Smith's argument centers about the fact that Johann "attained a fairly complete if not precise idea of the simpler problems of the calculus of variations *in general*."[20]

The mathematics historian Stuart Hollingdale feels even more strongly: "It was Jean Bernoulli who started Euler on his researches into the calculus of variations."[21]

One of the problems here is that the question apparently hinges on which of Johann's solutions is being considered–and once again, the situation is cloaked in fog. J. J. O'Connor and E. F. Robertson, who did a series of articles on the Bernoullis for an online mathematics history forum, argue that Johann later built an elegant solution, published in 1718, that used a work of Brook Taylor's.[22] Smith maintains that this is not so, that "such a direct solution is mentioned in several of the letters which passed between Leibniz and Johann in 1696 as well as in the remarks which the former made on the subject of the brachistochrone problem in the *Acta Eruditorum* for May, 1697."

Smith admits that "this solution was not published until 1718 when both Jacques and Leibniz were dead." But he argues: "This fact is apparently regarded by those who believe Jean plagiarized from his brother Jacques as invalidating the former's claim of having secured a second solution. Jean for his part asserted that he delayed the publication of his second method in deference to counsel given by Leibniz in 1696."[23]

In rebuttal, Hollingdale argues, "However, until Euler took up the subject, no general methods were available."[24] In other words, the Bernoullis had used the method to solve specific problems, such as the brachistochrone. Euler, who began his work in this area around 1732, was more interested in a general theory. But the form in which the work is seen today was the work of yet another great mathematician, Joseph-Louis Lagrange.

What was Lagrange's take on the origins of the calculus of variations? Smith argues that he [Lagrange] "emphasizes the part of Jean no less than that of Jacques in pioneering work on a general method in the calculus of variations."[25]

And so we are pretty much back where we started. Well, let's go on.

Hollingdale adds an interesting point: "The development of the calculus of variations received a strong boost from physics, from the adoption by the eighteenth-century scientists of the 'principle of least action' as a guiding principle in nature."[26]

Ironically, the principle also had strong theological support. Euler stated, "For since the fabric of the Universe is most perfect and the work of a most wise Creator, nothing at all takes place in the Universe in which some rule of maximum or minimum does not appear."[27] Once again, Johann's use of Fermat's principle of least action comes to mind.

In any case, when the Bernoullis found that the cycloid was also the solution of the brachistochrone problem, they were amazed and delighted. Johann wrote in his article, "With justice we admire Huygens because he first discovered that a heavy particle falls down along a *common cycloid* in the same time no matter from what point on the *cycloid* it begins its motion. But you will be petrified with astonishment when I say that precisely this *cycloid*, the *tautochrone* of Huygens, is our required *brachistochrone*." Later, he picked up the notion again:

> Before I conclude, I cannot refrain from again expressing the amazement which I experienced over the unexpected identity of Huygens's *tautochrone* and our *brachistochrone*. . . . For, as nature is accustomed to proceed always in the simplest fashion, so here she accomplishes two different services through one and the same curve, while under every other hypothesis two curves would be necessary, the one for oscillations of equal duration, the other for quickest descent. If, for example, the velocity of a falling body varied not as the square root but as the cube root of the height [fallen through], then the *brachistochrone* would be algebraic, the *tautochrone* on the other hand [would be] transcendental.[28]

At the end of Jakob's paper on the brachistochrone, he laid out three other kinds of problems that can be solved by his method, the third of which was, "To find isoperimetric figures of different kinds." The origin of this problem may date back to pre-Greek times. In essence, it seeks to find which closed plane curve with a given perimeter will have the largest area. Within this group, Jakob crafted a complicated example and challenged Johann by name. He even offered Johann a prize of 50 ducats if he could solve it by the end of the year, or six months from then.

Now the fur really began to fly.

Battle Lines

Johann came up with a solution in 1697 and claimed the award. He failed, however, to perceive the isoperimetric problem's variational character and thereby offered an incomplete solution, one in which the resulting differential equation was one order too low. Jakob, delighted, criticized his brother unmercifully.

E. A. Fellman and J. O. Fleckenstein, writing in the *Dictionary of Scientific Biography*, argue, "This was the beginning of alienation and open discord between the brothers–and also the birth of the calculus of variations."[29] (Yet another variation on the origin of the calculus of variations.)

Johann's analysis of Jakob's isoperimetric problem was presented via the French mathematician Pierre Varignon to the Paris Academy of Sciences in February 1701. Jakob presented his solution in the *Acta Eruditorum* in May 1701. A later comparison of it with Johann's solution clearly shows Jakob's to be superior. Unfortunately, Jakob could not revel in this particular triumph. For reasons unknown, Johann's solution was put into a sealed envelope and not opened until April 1706, almost a year after Jakob's death.

Was this because Johann realized the truth even then? He never admitted it. Much later, though–after his brother's death, and after having been able to digest work by Brook Taylor (*Methodus Incrementorum*, 1715)–he produced an elegant solution of the isoperimetric problem. The concepts in this 1718 paper contain elements of the modern concept of the calculus of variations, which Euler and Lagrange would carry forward. The solution was, however, strangely reminiscent of Jakob's solution and style.[30]

Does Johann deserve the credit here? It would seem more appropriate to say that both men contributed–in their wondrously contentious fashion–possibly starting with the original solution of Jakob's.

Even in Death

Jakob's death in 1705 produced yet another instance of the strange relationship between the brothers. Among Jakob's many interests was the topic of probability, which he had pursued fairly intensely during the years 1684 to 1690. And while most of the mathematical contributions of the brothers were to be found in journals, especially in the *Acta Eruditorum*, Jakob spent the last two years of his life working on a manuscript for a text on probability. This was the *Ars Conjectandi*, or *The Art of Conjecture.*

It would contain a general theory of combination and permutation: the so-called weak law of large numbers–also known as Bernoulli's theorem and today used as a main tool in the theory of probability–and much else. It was his most important single piece of work. It was also the first substantial work on probability, and it still has application wherever statistical methods are used today, as in insurance, weather prediction, and population sampling.

The book is set up in four sections, the second of which, on permutations and combinations, he used for a proof of the binomial theorem for positive integral exponents. Contained in this section is a formula and a table for the sum of the r^{th} powers of the first n integers. Using a table of his so-called numbers of Bernoulli, he calculated the sum of the 10th powers of the first 1,000 integers. Then, showing his sweet nature, Jakob wrote:

> With the help of this table it took me less than half of a quarter of an hour to find that the tenth powers of the first 1000 numbers being added together will yield the sum
>
> 91,409,924,241,424,243,424,241,924,242,500
>
> From this it will become clear how useless was the work of Ismael Bullialdus [which he] spent on the compilation of his voluminous *Arithmetica infinitorum* in which he did nothing more than compute with immense labor the sums of the first six powers, which is only a part of what we have accomplished in a single page.[31]

When Jakob died, the manuscript was nearly completed, but even in death, the animosity between the brothers played a role. It would seem logical for the work to have been published under Johann's supervision; but Jakob's widow was categorically against the idea, fearing that the vengeful brother might use the opportunity to damage or even sink the project. Nicholas, Johann's eldest son, had read the manuscript when he was studying with Jakob and, in true family spirit, had used it for his own thesis after Jakob died, and for other purposes as well. When it was finally published in 1713, it included a short preface by Nicholas. After admitting that he had been too young and inexperienced to do much with it, he says he advised the printers to give it to the public as the author had left it. It went on to become the centerpiece of Jakob's considerable reputation.

Jakob seems to have foreseen, or at least feared, an early death.

In the course of his studies, he worked with the curious equiangular spiral. This is a curve that can be seen in sea shells and the spider's web. It has some similarity with the circle, but there is one major difference. A circle crosses its radii at right angles; the equiangular spiral also crosses its radii at a constant angle, but not at 90 degrees. Jakob, who had some mystical leanings, was fascinated by the fact that the curve reproduces itself under various mathematical transformations. He asked that the curve be engraved on his tombstone, along with the inscription "*Eadem mutata resurgo*" (Though changed, I arise again the same). He died at the still young age of 51 in 1705, having held the chair in mathematics at Basel until his death. His chair at Basel was now available; it was offered to Johann, who was happy to accept it.

Johann Carries On

With Jakob out of the way, it was almost as if the argumentative Johann needed to find others to battle with. A healthy, vigorous man, he had another forty-three years in which to do so. L'Hospital's calculus text, for example, had been published in 1696. Initially, Johann seemed quite pleased with the way things had worked out. Upon receiving a copy from L'Hospital, he wrote back and expressed his thanks for being mentioned. He even promised to return the compliment when and if he, Johann, published such a work. L'Hospital actually suggested a follow-up, namely and reasonably, a text on integral calculus, since Leibniz didn't seem to be doing anything along these lines. Bernoulli, however, replied that he was, unhappily, preoccupied with domestic problems—which we'll get to in a moment.

That was Bernoulli's initial reaction, but the book had provided entry to a new and exciting world for Continental mathematicians and was received eagerly by them. As it became increasingly successful, Johann began to show an equivalent increase in jealousy and annoyance. In the years following Jakob's death, he complained bitterly about the situation. He attacked both the work and its author, virtually accusing L'Hospital of plagiarism. L'Hospital had acknowledged Johann's part in the work in the preface. He wrote, "And then

I am obliged to the gentlemen Bernoulli for their many bright ideas; particularly to the younger Mr. Bernoulli, who is now a professor in Groningen."[32] Johann now felt this was not sufficient credit, however, and he tried with all his might to tell the world who the real author was. But his cries seemed to fall on deaf ears.

For example, section 9 presents "Solution of some problems . . . ," involving what we would now refer to as indeterminate forms. Although the presentation is mainly geometrical, the result is what came to be called L'Hospital's rule–a mathematical method for evaluating indeterminate forms. This was particularly annoying to Johann, who felt that L'Hospital should have made it clear in the text that this was Bernoulli's work, not his. To be fair, though, L'Hospital never actually claimed this to be his invention. It was only by a quirk of fate, namely, the book's widespread use, that the result was so named–not by him but by others.

L'Hospital, it might be worth noting, was no longer around to defend himself. He had died in 1704. Nevertheless, and this would aggravate Johann even further, the *Analyse* remained the standard text for higher mathematics throughout Johann's long life–he died in 1748, at age 81–and even well beyond. A later comparison between Johann's lecture notes and L'Hospital's work showed them to be virtually identical, though a number of the mistakes in the notes do not appear in the book. So L'Hospital–or someone–did do some useful and knowledgeable editing.

According to one scholar, Gerard Sierksma, Johann's financial agreement with L'Hospital meant that Johann had sold his discoveries to L'Hospital and therefore could not publish his own work, at least for a while.[33] This could also explain Johann's unhappiness.

What were the domestic problems Johann referred to in his reply to L'Hospital? There were several that he might have had in mind. In 1697, he had lost a beloved daughter. Not long after, he suffered a serious illness.

More likely, it had to do with the years he was spending at Groningen, which ran from 1695 to 1705. Thanks to animosity between the city councilors and the provincial legislators because of religious differences, it was a trying time for everyone at the university. Johann had been brought up as a strict Calvinist and had remained a fervent member of the church, yet at least partly because of his work, he was accused of grievous heresies. Recall that Johann had also had medical training, and he had been expected to perform some medical duties as well. It seems that in the course of these duties, he made some comment about the continuous metabolism of the human body. He was attacked bitterly by both a student and a well-known theologian, Paulus Hulsius, and accused of denying the resurrection of the body.[34]

As he often did, he defended himself by going on the attack: "I would not have minded so much if he [the student] had not been one of the worst students, an utter ignoramus, not known, respected, or believed by any man of learning, and he is certainly not in a position to blacken an honest man's name and honour, let alone a professor known throughout the learned world, and distract the young from their fine studies."[35]

He was also attacked for espousing the use of experiments to learn about nature. He came through all of this, but there were some worrisome times.

Still More Controversy

Another man who can be counted among Johann's numerous adversaries was Brook Taylor (1685–1731). Taylor, in his *Methodus Incrementorum* of 1715, worked through many of the problems that Johann and others had dealt with, but the only credit Taylor gave was to Newton–not surprisingly, an Englishman. Bernoulli did not like being ignored and published an anonymous essay that accused Taylor of plagiarism. Taylor figured out who the author was and published an answer in defense–also anonymously. He also made fun of a mathematical error that Bernoulli had made years earlier. Johann's colleague, Pierre Rémond de Montmort, tried to mediate the dispute but got nowhere. The battle raged on until 1719, when Taylor published another insulting diatribe and then decided that enough was enough; Johann, characteristically, would have continued the battle.

Even years later, when Taylor died in 1731, Johann commented, "Taylor is dead. It is a kind of fate that my antagonists died before me, all younger than I. He is the sixth one of them to die in the last fifteen years. . . . All these men attacked and harrassed me . . . though I did them no wrong. It seems that heaven would avenge the wrong they have done me."[36] Note his reading of the turns of fate's wheel. The prior deaths of his adversaries proved, at least to him, that he was in the right!

As it turned out, neither Johann nor Taylor was aware that their battle was rather pointless. The general feeling today is that Taylor was not guilty of plagiarism, but rather of not keeping up with new developments on the Continent. Furthermore, the young Scotsman James Gregory had come up with the "Taylor series," which is basic in the method, some 40 years earlier.

And Yet Again

With the next controversy, we circle back to the family. Johann's second son was Daniel, who was born in Groningen in 1700. Johann, repeating his father's strange behavior, tried his best to prevent Daniel from pursuing mathematics. According to the mathematics historian James R. Newman, Johann went so far as to attempt to destroy the child's self-confidence through cruel treatment.[37] He tried, as his father had tried with him, to push Daniel into the business world, but something in the Bernoulli genes objected. Of course, in those days a young man was not likely to say, "Sorry, Pop, that's not for me. I want to study mathematics."

So, Daniel was first sent off as a commercial apprentice. When that didn't work, Johann sent him to study medicine, and he eventually earned his doctorate in 1721. His heart, though, as with so many other Bernoullis, was in mathematics. Historians differ over whether Johann taught Daniel any mathematics. If he did, it wasn't a great deal, and Daniel learned most of what he was taught from his older brother, Nicholas. As was the case with Johann, however, it was as a sideline. Nevertheless, he went on to become by far the ablest mathematician of the younger group of Bernoullis. By 1724, he had earned a solid reputation in the field with his *Exercitationes Mathematicae*, which covered several different mathematical subjects.

This led to a position in mathematics at the St. Petersburg Academy. He was, however, a scientific polymath, and he worked not only in mathematics but also in such fields as medicine, biology, physiology, physics, mechanics, astronomy, and oceanology. The following year he won a prize awarded by the Paris Academy, the first of 10 he would earn in these various fields.

Nicholas had also gotten an appointment at St. Petersburg. But within eight months of their appointments, Nicholas died, leaving Daniel feeling both lonely and not very happy with the harsh climate. Johann once again entered the scene, and we see an excellent example of his complicated personality. Argumentative, irascible, and jealous, he nevertheless arranged for one of his best pupils, no less a one than Leonhard Euler, to move to St. Petersburg to work with Daniel in 1727. The following few years were among Daniel's most productive. One of his main topics of interest was vibrating systems; by 1728, Daniel and Euler were doing important work on the mechanics of flexible and elastic bodies.

Ironically, all of Johann's three sons gained some reputation as mathematicians and scientists, but Daniel was to become the most famous of all. When Johann began feeling the hot breath of competition from this youngster, he reacted badly to it.

The battle between the optics and the dynamics of Newton and the once-dominant Cartesian description of the world was still being fought, including at the Paris Academy of Sciences. Johann, arguing against Newton's ideas, had won the Paris Academy's prestigious prize competition twice, in 1727 and 1730. He won again in 1734, but this time he had to share the prize with Daniel, who was arguing in support of Newton!

Daniel had been wanting to come back to Basel for years. He had finally obtained the chair of anatomy and botany at Basel in 1734. He had come home. Unfortunately, Johann was increasingly unhappy with having to share credit with his own son. The occasion of the shared prize led to a break between them; Johann even barred Daniel from the family home.

Then things went from bad to worse. Both men continued to work in mathematics and its application to physical problems. Daniel had been working on a text titled *Hydrodynamica*, which covered the properties important in fluid flow, including pressure, density, and velocity. It included the key relationship, now called Bernoulli's principle, which states that pressure in a fluid decreases as its velocity increases. He had the manuscript ready by about 1734, but for various reasons it was not published until 1738. This was to be Daniel's most important work, and the work that would make him famous.

His father, not to be outdone, published a competing book, which he titled *Hydraulica*, at about the same time. Again, history is somewhat ambiguous. At the very least, Johann tried to predate his book to 1732, to make it seem like Daniel took his material from Johann.[38] At worst, he actually stole material from Daniel's book and tried to pass it off as his own, and then attempted to make it look as if Daniel had stolen from him![39] Later, in a 1743 letter to Euler, Daniel referred to "my complete *Hydrodynamic*, for which I do not have to thank my father," and complained, "I was robbed of and lost the fruit of my ten years of labor."[40]

This hardly sounds like the behavior of a proud and loving father, yet this was the same man who had thoughtfully sent Euler up to St. Petersburg to help Daniel through a difficult time. In any case, fate would serve up another unpleasant dish to Johann, for he lived another 10 years, long enough to see Daniel's work become a classic text in the field.

Why end on a sour note, though? Daniel was a first-class scientist, as well as a competent mathematician, and he has been described as one of the founding fathers of mathematical physics. If Johann couldn't find pleasure in his son's success, that was his problem. He had two other sons, who also became competent mathematicians, and several grandsons, with the same outcome. The only real blight I can see is that the eldest of his three sons, Nicholas, died at Petrograd, where he was a professor, at the young age of 31. Yet Galton describes Nicholas as "a great mathematical genius" and "one of the principal ornaments of the then young academy."[41] Who can say what Johann's feelings really were?

In any case, Johann, like his brother Jakob, and like Daniel, had plenty to feel good about. We've seen that Johann and Jakob were the very first to see the importance of the new calculus, and that both of them performed some very useful mathematics during their time on earth. After the death of Jakob in 1705, Leibniz in 1716, and Newton in 1727, Johann reigned as perhaps the foremost mathematician of his time. It was largely due to his efforts–both through his teaching and through his many demonstrations of the calculus's wonderful powers–that the differential notation of Leibniz, rather than Newton's fluxional notation, was generally adopted on the Continent. He was surely one of the great teachers of all time. He was also, it turns out, one of the great communicators. His scientific correspondence adds up to some 2,500 letters, and he exchanged letters with no fewer than 110 scholars.