# Real-time autofocus of holograms based on image sharpness

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A comprehensive method for autofocus of optically-acquired holograms based on classical image features assessment is reported. The focus criterion relies on the image intensity, and the recorded optical phase is used for image rendering at a given distance. Focus seeking over a wide range of reconstruction distances is ensured by the continuity of the two holographic rendering algorithms used, which makes it suitable for both microscopic and macroscopic imaging. Calculations are performed on graphics processing units to enable versatile and robust real-time autofocus operation in experimental conditions.

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# I. INTRODUCTION

Various imaging and sensing applications benefit from the unconventional nature of holograms acquired by interferometry, enabling powerful features such as phase imaging of optical wavefronts, shot-noise sensitivity with traditional sensor arrays in low-light conditions, aperture synthesis, compressed sensing, and heterodyne detection. Some applications of this imaging scheme include microscopy [1–5], non-contact vibrometry [6–9], fluid dynamics metrology [10–14]. A powerful feature of optically-acquired holograms is to enable post-measurement refocus. Computational image rendering of complex-valued optical fields from holographic measurements can be performed by wave propagation algorithms. Classical methods used for numerical reconstruction rely on free-space propagation of the diffracted optical field in the Fresnel approximation. Nevertheless, manual focus procedures for image plane retrieval are still prevalent.

Classical image sharpness assessment methods are mostly based on the estimation of the variance, the gradient or the Laplacian of the spatial distribution of intensity [15-19]. Holographic interferometry can benefit from state-of-the-art focus metrics based on image sharpness criteria [20]. Optically-acquired interferograms need to be processed numerically to yield a hologram image, because of the specific nature of complex-valued diffracted optical field recordings. In contrast with direct imaging schemes, this digital treatment is performed after data acquisition in holography. Numerical image rendering is referred to as hologram refocusing. It was shown that at the focus distance, the amplitude of the hologram is minimum for a pure amplitude object and maximum for a pure phase object. Hence the amplitude of the reconstructed hologram can be used as a focus metric [4, 21–23]. The complex amplitude [24], the spectral content [25], sparsity metrics [26-28], the speckle pattern [29], axial

variations [30] or the discrete cosine transform [31] of the hologram can also be used as sharpness metrics for the reconstruction of holograms. Furthermore, resampling of the holographic complex field has been proposed for fast focus assessment [32]. Focus criteria were also tested onto downsampled holograms to increase autofocus speed [33] while maintaining a stable global maximum in the sharpness curve. Focus metrics for color holograms were also proposed [4, 34]. The contrast of the rendered images affected by speckle was found to be a good focus metric, providing a single focus-value without ambiguity [35]. An active method, requiring the the use of structured illumination with a spatial light modulator was introduced for the microscopy of phase and amplitude objects [36].

In this paper, we report on a numerical autofocus method for optically-acquired digital holograms implemented on graphics processing units for real-time processing. After a definition of the discrete operators used for focus estimate computations, we will describe the two numerical optical field propagation methods used for hologram rendering, based on one and two fast Fourier transform (FFT) algorithms respectively. We will define their domains of validity and verify their continuity, in order to ensure that the autofocus method is valid over a wide range of reconstruction distances, and thus usable for both microscopic and macroscopic imaging. We will then define the focus criteria on the squared amplitude of the optical field we chose to combine for the proposed focus retrieval algorithm. The accuracy, robustness and versatility of this approach will be illustrated with experimental holograms.

# II. DISCRETE OPERATORS USED FOR IMAGE RENDERING AND FOCUS RETRIEVAL

In digital holography, optical interferograms are recorded on a discrete sensor array. Therefore, the operators used in this article will be defined with their actual discrete formulation. The proposed autofocus method is based on the estimation of the spatial features of images reconstructed by standard propagation algorithms. Most of the criteria used in this article to assess the best focus distance are based on bi-dimensional spatial convolution products calculations. According to the convolution theorem, the bi-dimensional spatial convolution of functions of discrete variable sequences  $\boldsymbol{x}$  and  $\boldsymbol{K}$  can the be written

$$x * K = \mathcal{F}^{-1} \{ \mathcal{F} \{ x \} \times \mathcal{F} \{ K \} \}$$
 (1)

The 2D fast Fourier transform of an array x is noted  $\mathcal{F}\{x\}$ , and the 2D inverse fast Fourier transform of x is  $\mathcal{F}^{-1}\{x\}$ . The kernel K is zero-padded to the size of the array x, to ensure size matching of both arrays, in practice. The operator  $\times$  denotes the multiplication of matching array elements. One can finally define the averaging operator  $\langle . \rangle$  by  $\langle x \rangle = (MN)^{-1} \sum_{m=1}^{M} \sum_{n=1}^{N} x_{m,n}$ 

#### III. HOLOGRAM RECONSTRUCTION

#### A. Introduction

Typical hologram acquisition schemes involve interferometric techniques [37–43]. Under spatial and/or temporal modulation of the recorded interference pattern, one can retrieve the spatial distribution in the sensor plane of the complex optical field E diffracted by the object (Fig. 1). The magnitude and the phase of the digitized object wave, assessed at each pixel of a sensor array of  $M \times N$  pixels of pitch  $d_x = d_y = d$ , is indexed by the integers (m,n). The complex-valued discrete scalar field  $E(x,y,z) = \{E_{m,n}(z)\} \equiv E(z)$  in the plane z is the field's dimensionless magnitude. The intensity of the optical field in the plane z is computed as

$$I(z) = |E(z)|^2 \tag{2}$$

Holograms can be rendered numerically via the use of scalar diffraction theories. Several methods can be considered for hologram reconstruction [44–52]. In this article, we make use of the Fourier approach (one-FFT algorithm) and the angular spectrum propagation approach (two-FFT algorithm) for the computation of the propagation integral of the optical field.

#### B. Propagation integrals of the optical field

Provided only small angles are involved during the propagation process, under paraxial conditions, the optical field E at point (x, y, z) is given with good accuracy by the Fresnel diffraction integral [53, 54]

$$E(x, y, z) \approx \frac{e^{ikz}}{i\lambda z} \iint E(x', y', 0)$$

$$\times \exp\left[\frac{ik}{2z} \left( (x - x')^2 + (y - y')^2 \right) \right] dx' dy' \qquad (3)$$

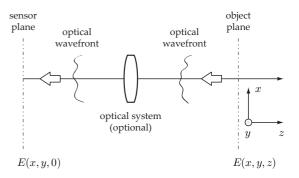


FIG. 1. Optical configuration. The optical radiation field E propagates along the optical axis z from the object plane to the sensor plane in free space; an optical system can be present. The diffracted field is recorded in the plane z=0.

where  $\lambda$  is the wavelength and  $k = 2\pi/\lambda$  is the wavenumber of the optical wave. For large diffraction angles, the propagation of the angular spectrum  $\hat{E}(k_x, k_y, z) = \hat{E}(k_x, k_y, 0) \exp(ik_z z)$  of the field's distribution E(x, y, z), yields to another diffraction integral [53, 54]

$$E(x, y, z) \approx \iint \hat{E}(k_x, k_y, 0)$$

$$\times \exp(ik_z z) \exp(ik_x x + ik_y y) dk_x dk_y$$
(4)

where  $\hat{E}(k_x, k_y, 0)$  is the two-dimensional Fourier transform of the field E(x, y, 0), and  $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$  is the wavevector projection along z.

#### C. One-FFT rendering algorithm

Under the paraxial approximation, the discrete form of the propagation integral of the optical field can be written as [53]

$$E(z) = \mathcal{F}\{E(z=0) \times h(z)\}\tag{5}$$

where  $h(z) = \{h_{m,n}(z)\}$  is free-space propagation impulse response

$$h_{m,n}(z) = \frac{1}{i\lambda z} \exp\left[\frac{2i\pi z}{\lambda} + \frac{i\pi}{\lambda z}(m^2d^2 + n^2d^2)\right]$$
 (6)

Here, z=0 is associated with the sensor plane, and z corresponds to the object plane. For optimal reconstruction, the distance parameter z has to be accurately determined. As discussed further, the one-FFT rendering method will be suited for reconstruction of holograms of extended objects located far from the sensor (i.e. macroscopic imaging). The pitch  $\Delta x_1$  of the reconstructed hologram (Fig. 2) can be derived geometrically from the ratio of the angular acceptance  $\sim \lambda/d$  of one pixel behaving as a heterodyne antenna [55], the number of pixels

of the receiver in the considered direction N, and the reconstruction distance z.

$$\Delta x_1(z) = \frac{\lambda z}{Nd} \tag{7}$$

Hence the discrete lateral coordinates of a hologram rendered with the one-FFT algorithm are  $(x, y) = (m\Delta x, n\Delta y)$ . The lateral field of view scales up linearly with z (Fig. 2).

#### D. Two-FFT rendering algorithm

The propagation integral of the optical field can also be viewed as the propagation of its angular spectrum, and can be expressed as a spatial convolution product [53]

$$E(z) = \mathcal{F}^{-1} \{ \mathcal{F} \{ E(z=0) \} \times H(z) \}$$
 (8)

where  $H(z) = \{H_{m,n}(z)\}$  is the angular spectrum propagation kernel defined as

$$H_{m,n}(z) = \exp \left[ \frac{2i\pi z}{\lambda} \sqrt{1 - \left(\frac{m\lambda}{Md}\right)^2 - \left(\frac{n\lambda}{Nd}\right)^2} \right]$$
(9)

The function H(z) is the Fourier transform of the free-space impulse response h (Eq. 6). The angular spectrum propagation will have to be used as a reconstruction method when dealing with small objects located near the sensor (i.e. microscopic imaging). With two-FFT rendering, the pitch  $\Delta x_2$  of the reconstructed hologram does not depend on the reconstruction distance (Fig. 2). It is equal to the pitch in the sensor plane

$$\Delta x_2 = d \tag{10}$$

In order to make the proposed autofocus procedure versatile and applicable for both macroscopic and microscopic imaging, the switching conditions from one formalism to the other are now investigated.

# E. Hologram rendering method versus reconstruction distance

To ensure an effective autofocus procedure for both macroscopic and microscopic holographic imaging, the influence of the parameter z on spatial sampling of the kernels defined in Eq. 6 and Eq. 9 was investigated. For accurate sampling, the argument of the exponent in the right member of equation Eq. 6 has to be assessed at reconstruction distances z satisfying [56–58]

$$z \ge \frac{Nd^2}{\lambda} = z_b \tag{11}$$

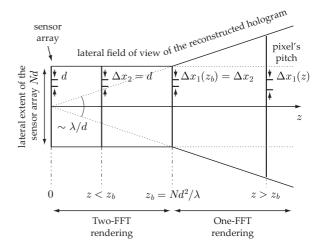


FIG. 2. Pixel's pitch  $\Delta x$  and lateral field of view  $M\Delta x$  versus reconstruction distance z, in the case of free-space propagation.

The boundary value  $z_b$  of the reconstruction distance parameter is determined by equaling the highest spatial frequency of the quadratic phase in the exponent of Eq. 6 with the Nyquist frequency  $(Nd)^{-1}$ . In the same manner, according to the Nyquist-Shannon sampling theorem, the argument of the exponent in the right member of equation Eq. 9 has to be calculated at reconstruction distances z satisfying

$$z \le z_b \tag{12}$$

For the value  $z_b$  of the parameter z, image pitches of both reconstruction methods are equal

$$\Delta x_1(z_b) = \Delta x_2 \tag{13}$$

Hence, a piecewise continuous rendering method valid for all values of the reconstruction parameter  $z \in \mathbb{R}$  has been defined. The reconstruction distance  $z_b$  is the boundary value of the parameter z at which image rendering is switched between one-FFT (Eq. 5) and two-FFT algorithms (Eq. 8).

## IV. SELECTED FOCUS CRITERIA

A focus function processes an image and condenses its in-focus estimate into a scalar value. The basic assumption on which are based most of these functions is that well-focused images contain more information and details than unfocused images [59], consequently they contain more high spatial frequency information, are more contrasted and their edges are sharper than for defocused images. A good immunity to noise is also a requirement for a focus function as it permits the use of the algorithm under real conditions. In addition to its immunity to noise, the ideal focus function response should

be unique, accurate and sharp. The reconstruction distance given by the focus function maximum and the actual object-sensor distance should be as close as possible. The presence of an irrelevant maximum can trap the autofocus algorithm, thus the number of local maxima should be as low as possible. Finally, the sharper the response is, the more precisely the focus distance can be determined. The proposed focus metric uses three classical focus criteria based on the spatial features of the reconstructed holograms: the global variance of the intensity, the average local variance of the intensity, and the average magnitude of the intensity gradient.

#### A. Global variance of the intensity

The global variance evaluates the non-normalized contrast of the image and characterizes its focusing as it measures the variations among the pixels of the image. Bright and dark pixels have the same influence. The global spatial variance of the intensity is defined as

$$V\{I\} = \langle (I - \langle I \rangle)^2 \rangle \tag{14}$$

where  $\langle . \rangle$  is the spatial averaging operator.

#### B. Average local variance of the intensity

The local variance characterizes the local sharpness of image features. The local variance at pixel (m,n) in the neighborhood delimited by the extent of the kernel  $K_E$ , used for its computation in reciprocal space, is noted  $V_L\{I\}_{m,n}$ . Its average value over the reconstructed image I is

$$\langle V_L\{I\}\rangle = \langle (I - I * K_E)^2 \rangle \tag{15}$$

where the kernel  $K_E$  is a made of a  $3 \times 3$  matrix of ones

$$K_E = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$
 (16)

# C. Average magnitude of the intensity gradient computed by the Sobel operator

The Sobel operator is a discrete differentiation operator that computes an approximation of the image gradient [60, 61]. For each pixel, it outputs a bidimensional vector that contains the value of the spatial derivatives in the horizontal and vertical directions. A large magnitude reveals a sharp edge in the direction pointed by the vector. The average value of the Sobel gradient's magnitude on the intensity image I is defined as

$$\langle ||S\{I\}|| \rangle = \left\langle \left[ (I * K_S)^2 + \left( I * K_S^T \right)^2 \right]^{1/2} \right\rangle$$
 (17)

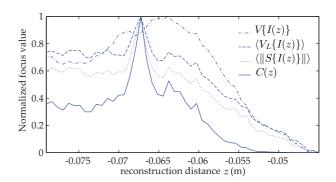


FIG. 3. Focus criteria versus reconstruction distance parameter z. The refocused hologram is shown in Fig. 7.

where the convolution products  $I * K_S$  and  $I * K_S^T$  are the directional gradients. The matrix  $K_S^T$  is the transpose of the matrix  $K_S$ . The Sobel kernel  $K_S$  is defined as

$$K_S = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \tag{18}$$

## V. COMPOUND FOCUS METRIC

Our choice of focus metric was guided by the need to comply with a a broad range of experimental conditions. To avoid a case-specific approach, we selected three classical focus criteria of intensity images calculated by Eq. 2. Four objects were studied in various recording conditions, as reported in Fig. 5, Fig. 4, Fig.6, and Fig. 7). The chosen focus metric combines the assessment of the global variance of the intensity (Eq. 14), the average local variance of the intensity (Eq. 15), and the average magnitude of the intensity gradient (Eq. 17). The focus metric  $C\left(z\right)$  is the product of these three criteria

$$C(z) = V\{I(z)\} \times \langle V_L\{I(z)\}\rangle \times \langle ||S\{I(z)\}||\rangle$$
 (19)

To demonstrate the versatility and robustness, we applied the proposed autofocus method to the reconstruction of various experimental holograms acquired in macroscopic and microscopic imaging conditions.

#### A. Autofocus of macroscopic images

Holographic interferometry is widely used in vibration imaging [9, 63–69]. For accurate vibration pattern mapping, the assessment of the reconstruction distance is important. Our approach has two steps. First, a coarse logarithmic sweep is performed on a wide range of reconstruction distances, from z=-5 m to z=5 m, leading to an estimation of the refocusing distance. Then, a fine linear sweep is realized around the distance z for which a maximum of C(z) was found during the coarse sweep.

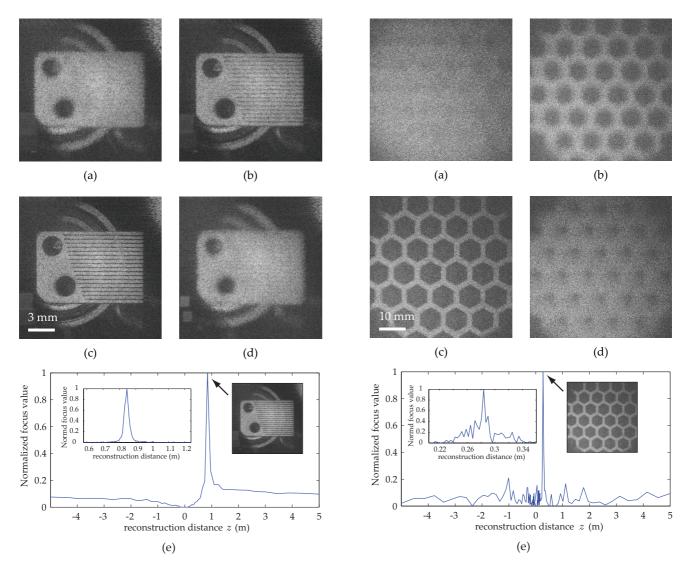


FIG. 4. Image rendering of the lamellophone of a musical box [62]. Intensity holograms reconstructed for distance parameters  $z=0.75\,\mathrm{m}$  (a),  $z=0.81\,\mathrm{m}$  (b),  $z=0.845\,\mathrm{m}$  (c),  $z=1\,\mathrm{m}$  (d). Image (c) was calculated for the maximum of C(z). (Media 1)

FIG. 5. Image rendering of a thin metallic plate with hexagonal holes [70]. Intensity holograms reconstructed for distance parameters  $z=0.22\,\mathrm{m}$  (a),  $z=0.26\,\mathrm{m}$  (b),  $z=0.28\,\mathrm{m}$  (c),  $z=0.35\,\mathrm{m}$  (d). Image (c) was calculated for the maximum of C(z). (Media 2)

It should be noted that the ability of the proposed algorithm to operate at any reconstruction distance z is ensured by the switching condition at  $z=z_b$  from Eq. 11 to Eq. 12. Results obtained for the image of the metallic lamellophone of a musical box [62] are reported in Fig. 4. Here the values of the focus metric C(z) are plotted against z for both coarse and fine (inset) sweeps of the reconstruction distance. Reconstructed images, around the maximum of the fine sweep curve, corresponding to out-of-focus (Fig. 4(a), (b), and (d)) and in-focus holograms (Fig. 4(c)) attest of the sensitivity of the proposed algorithm. The same focus procedure was also applied to the hologram of a thin metallic plate carved with periodic hexagonal holes, used for vibrometry purposes [70]. Performing a coarse logarithmic sweep followed by a fine

linear sweep makes it possible to bring the acquired hologram back to focus accurately, as reported in Fig. 5.

## B. Autofocus of laser Doppler blood flow images

Holographic detection enables characterization of biological samples in-vivo [1, 2, 72, 73]. In particular, it provides a convenient way to perform wide-field optical heterodyne detection with a sensor array at video frame rate. Time-averaged holography with a frequency-shifted reference beam permits narrow band detection and imaging of local Doppler broadenings at low radiofrequencies. With this approach, blood flow contrasts can be imaged in the cerebral cortex [71] and in the eye fundus [74] of ro-

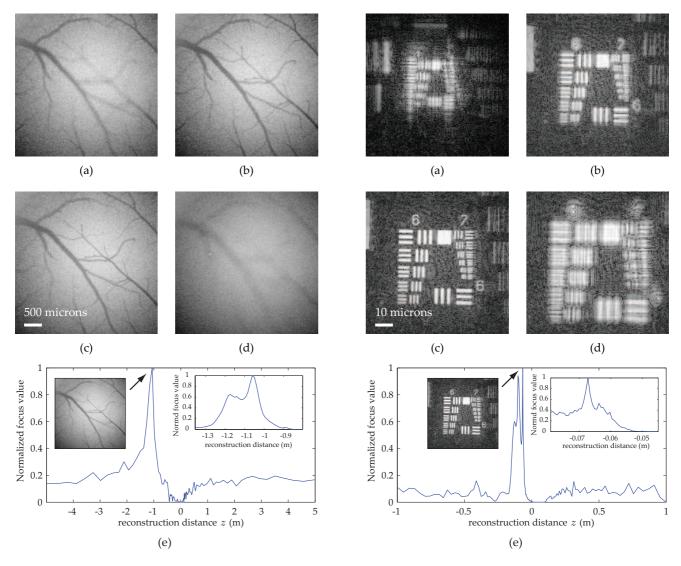


FIG. 6. Image rendering of the cerebral cortex of a mouse, acquired by laser Doppler holography [71]. Intensity holograms reconstructed for distance parameters  $z=-1.5\,\mathrm{m}$  (a),  $z=-1.2\,\mathrm{m}$  (b),  $z=-1.055\,\mathrm{m}$  (c),  $z=-0.7\,\mathrm{m}$  (d). Image (c) was calculated for the maximum of C(z). (Media 3)

FIG. 7. Image rendering of a negative resolution target observed by holographic microscopy [50]. Intensity holograms reconstructed for distance parameters  $z=-0.08\,\mathrm{m}$  (a),  $z=-0.07\,\mathrm{m}$  (b),  $z=-0.0677\,\mathrm{m}$  (c),  $z=-0.06\,\mathrm{m}$  (d). Image (c) was calculated for the maximum of C(z). (Media 4)

# dents with a standard camera. The autofocus procedure was performed on the recording of a rodent's cerebral cortex, in vivo. The two-step sweep procedure was used to assess the maximum of the focus function C(z) on the Doppler hologram of the living tissue to estimate the refocus distance (Fig. 6). A coarse logarithmic sweep from z=-5 m to z=5 m, leads to a first estimate of the refocusing distance of $z\simeq -1.1$ m. Then, a fine linear sweep is realized around this value. The image of the vessels at the surface of the cerebral cortex is accurately refocused at the maximum of the focus function C(z).

#### C. Autofocus of microscopic images

Microscopic imaging benefits from the threedimensional content of digital holography [75–78]. However precise refocusing of the investigated objects remains a tedious task. The proposed autofocus procedure was tested in microscopic imaging conditions. Image acquisition of a resolution target was performed in heterodyne holographic microscopy configuration [5, 79, 80]. As for macroscopic imaging, a two-step sweep of the focus distance is performed to estimate the best reconstruction distance (Fig. 7(e)). Values of the focus metric C(z) are plotted against the reconstruction distance z for both coarse and fine (inset) sweep attest for the applicability of the presented

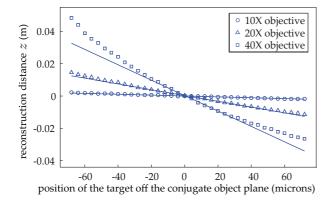


FIG. 8. Reconstruction distance versus position from the conjugate object plane, for three microscope objectives :  $10 \times (NA.25)$ ,  $20 \times (NA.40)$ ,  $40 \times (NA.63)$ .

scheme to holographic microscopy with a coherent light source. These results are confirmed by the focus images, depicted Fig. 7(a-d). The accuracy of focus distance retrieval is reported in Fig. 8. Actual positions of the target with respect to the focal plane are plotted against the reconstruction distance. For three microscope objectives with the following magnification factors and numerical apertures (NA):  $10 \times$  (NA .25),  $20 \times$  (NA .40), 40× (NA .63), holograms for object positions ranging from -80  $\mu$ m to 80  $\mu$ m from the conjugate object plane are acquired. For each hologram, the proposed autofocus procedure is applied, leading to the evaluation of the estimated distance against the actual object position with respect to the focal plane reported in Fig. 8. The experimental results obtained with  $10 \times (dots)$ ,  $20 \times$ (circles), and  $40\times$  (crosses) objectives are superimposed with linear fits. The nominal magnification of microscope objectives is only achievable when the object is positioned in the object focal plane of the objective, and when the imaging sensor is positioned at the end of the microscope objective tube (at the image focal plane of the tube lens for infinity corrected microscope objectives). Within these conditions, the microscope objective works in optimal imaging conditions in terms of aberration correction. When these conditions are not fulfilled, magnification will both depend on the object-to-focal plane distance, and on the sensor-to-best correction plane of the microscope objective. In that case, the relationship between the actual object position and retrieved object position will not be linear. This effect is clearly noticeable in Fig. 8 for the  $40\times$  objective. Both the  $10\times$ , and  $20\times$  objective will exhibit the same behavior, however due to their smaller numerical aperture that affects their depth of field, this effect will appear at higher reconstruction distances.

#### VI. CONCLUSION

In conclusion, we have demonstrated a real-time autofocus procedure for holographic image rendering. Our approach is based on the analysis of spatial features of reconstructed intensity holograms. The retained focus method is the compound of three selected contrast criteria chosen for their versatility, allowing several imaging applications, and their compliance with long-distance and short-distance image rendering methods, with oneand two-FFT rendering algorithms, respectively. continuity at both rendering methods' boundaries was investigated and used to enable focus retrieval at any reconstruction distance. Focus criteria were expressed in terms of discrete spatial convolutions and computed via discrete Fourier transforms. The proposed focus criterion was shown to enable robust and unique renderings when applied to various recording conditions. The accuracy of the method was validated over several detection configurations and a wide range of reconstruction distances. The proposed focus metric for holographic imaging of intensity images is versatile and robust. However, though focus planes are accurately found in the case of macroscopic and microscopic imaging, the retrieved reconstruction distance does not always match the real object position. This can usually happen because (i) a lens in the object channel is present, and (ii) the reference beam's wavefront is not perfectly flat in the sensor plane : the effective curvature of the measured field results from the difference between the object and the reference wavefronts. Therefore, the relation between the reconstruction distance and the physical object position might be calibrated for quantitative focus distance measurements.

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