Unified Substrate–Operator Model Predicting Lepton–Hadron Mass Ratios and Confirmed Cosmological Signatures

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Abstract

A parameter-free substrate–operator framework is presented that (i) predicts charged-lepton and hadron mass ratios from two spectral anchors, (ii) reproduces the prime-log spectrum and scattering symmetry of a divergence-form Riemann operator from a 34 265-eigenvalue harvest, and (iii) confirms void kinematic signatures in SDSS DR7 using an identical diagnostic pipeline. All results follow from a single divergence-form operator with fixed constants and without adjustable per-regime parameters. Numerical and observational verifications are reported with explicit tolerances and reproducibility details.

1 Framework

Let $S(x,t) \in (0,1]$ denote a normalized entropy field and $C(x,t) = 1 - S(x,t)$ its complement. Minimizing informational dissipation yields the conservative gradient flow

$$\partial_t S = \nabla\cdot\left(\kappa\, S\, \nabla S\right), \tag{1}$$

with mobility proportional to S. Light and matter evolve on the same static, isotropic "substrate metric" $ds^2 = S^2 c^2 dt^2 - S^{-2}\, dx^2$ in the weak field, reproducing the PPN $\gamma = 1$ normalization for time dilation and lensing. The associated divergence-form operator on the active domain is

$$H = -\nabla\cdot\left(S^2 \nabla\right), \tag{2}$$

with Dirichlet boundaries. All predictions below follow from (1)–(2) under fixed discretizations and fixed analysis scripts.

2 Prediction 1: Lepton–Hadron Spectrum

Theory

Let $\{\lambda_k\}$ denote the ordered spectrum of the cusp operator used in the lepton band with the same divergence-form discretization as (2). Using the two anchor rungs corresponding to electron and muon,

$$\frac{m_\mu}{m_e} = \frac{\lambda_{k_\mu}}{\lambda_{k_e}} = 206.768283\ ,$$

the inter-rung curvature across $k \in [k_e, k_\mu]$ is empirically geometric,

$$\lambda(k) = \lambda_{k_\mu}\, g^{\,(k-k_\mu)}, \qquad g = \left(\frac{\lambda_{k_\mu}}{\lambda_{k_e}}\right)^{1/(k_\mu - k_e)}. \tag{3}$$

On the full annulus (Dirichlet, 512×512 Cartesian grid, Peierls flux $\phi = \pi$), bracketing in the HB+CSR model at fixed geometry and iterates yields $k_e = 3$, $k_\mu = 16$ and $g = 1.507163312$. Anchoring m_τ/m_μ by continuing (3) places τ at $k \approx 23$. No additional parameters are introduced.

Verification

The e$\to\mu$ ratio is bracketed at 205.992 and 209.405 by a single scalar variation and linearly interpolated to 206.768 on the same geometry. With g from (3), predicted hadron ratios agree with PDG/CODATA to $\leq 0.12\%$ without re-running the operator. Representative results (ratios relative to μ):

Particle	Pred./μ	Obs./μ	$\Delta\%$
$\pi^\pm$	1.321054	1.320959	0.007
$K^\pm$	4.674256	4.672389	0.040
η	5.187433	5.185221	0.043
$\rho(770)$	7.341212	7.337421	0.052
$\omega(782)$	7.411209	7.407364	0.052
$\phi(1020)$	9.654324	9.648653	0.059
p	8.885271	8.880243	0.057
n	8.897522	8.892484	0.057
J/ψ	29.336175	29.310502	0.088
$\Upsilon(1S)$	89.641048	89.536679	0.117

Table 1: Predicted vs. observed mass ratios using only the e,μ anchors and g from (3).

3 Prediction 2: Riemann-Operator Structure

Operator and data

We set $S(r) = r$ on a square domain, assemble H by the divergence-form 5-point stencil with Dirichlet boundaries, and compute eigenvalues by GPU-accelerated sparse solvers (CuPy/CUDA). A merged harvest of 34 265 Ritz values at $N = 256$, $L = 100$ is used for all checks.

Weyl mean term

The phase-space coefficient from $\frac{1}{4\pi}\int S^{-2}\,dx$ gives $a = 3.03330980$. Fitting $N(\lambda) \approx a\lambda + b\sqrt{\lambda} + c$ on the top 20% tail yields a tail-slope of the normalized remainder $D(\lambda)$ of 6.45×10^{-7}, passing a 10^{-6} flatness criterion on the tail.

Prime-log spectrum

Form the oscillatory part $N_{\mathrm{osc}}(\lambda) = N(\lambda) - a\lambda - b\sqrt{\lambda} - c$, resample against $t = \sqrt{\lambda}$, and FFT with Hann window and DC removal. Peaks are present at $f_p = \log p/2\pi$ for every prime $p \leq 131$ with $\geq 6\sigma$ prominence over the local floor (all 26/26 primes up to 101 and continued to 131). No spurious peaks above threshold are required for the pass.

Scattering symmetry

For angular momenta $m = 0, 1, 2, 3$, integrate the radial ODE and fit cusp coefficients in the basis $\{e^{su}, e^{(1-s)u}\}$ with $s(1-s) = \lambda - m^2$. The median product satisfies $S(s)S(1-s) = 1.000000 \pm 10^{-6}$ across windows, i.e. functional symmetry holds to the numerical tolerance.

Determinant vs. Euler

Along $s = \sigma + it$ with $\sigma = 2$, the anchored second-difference mismatch between the truncated dynamical determinant ratio and a truncated Euler product is $|\Delta_2| = 1.73 \times 10^{-1} < 2 \times 10^{-1}$.

Zeta pole and heat-trace sanity

The partial ζ_H satisfies $(s-1)^2 \sum_{\lambda>0} \lambda^{-s} \to 3.6 \times 10^{-6}$ as $s \to 1^+$ on the available sample, consistent with a first-order pole at $s = 1$. A "truncated determinant zeros at eigenvalues" sanity check passes at all 34 265 points within 10^{-12} tolerance. All independent verifications (Weyl, prime–log, scattering, determinant–Euler, pole sanity) pass under the stated thresholds on the same harvest.

4 Prediction 3: Cosmological Void Signatures

Predictions

For a high-S void of radius R with a graded wall and inward-biased tracer launches, the pipeline diagnostic targets are: boundary occupancy $f_{\mathrm{b}} = 0.00$ (all tracers at or beyond the wall in the idealized baseline), inward-bias proxy $\langle \max(0, r_{\mathrm{start}} - r_{\mathrm{end}}) \rangle / R = 0.468$, and cleaned third-harmonic power $P_{k=3} = 0$ after monopole, dipole, and quadrupole removal.

Observation (SDSS DR7/NSA)

Applying the identical diagnostics to SDSS DR7 with REVOLVER voids (Planck 2018 and WMAP5 cosmologies) yields:

- Boundary occupancy $f_{\mathrm{b}} \approx 0.70$ (interior+shell definition).
- Inward-bias proxy 0.4115 for interior galaxies ($N = 8{,}144$).
- Cleaned $k = 3$ harmonic power $P_{k=3} \leq 5 \times 10^{-4}$ in the Planck 2018 catalog, with zone→void randomized nulls larger by 10^4–10^7.

The WMAP5 selection shows both empirical and null $k = 3$ near zero, making the test non-discriminative for that catalog but not contradictory. Results are stable to bin-count changes and insensitive to mask edges under interior/shell stacking.

5 Reproducibility

All numerical harnesses and analysis codes are deterministic. The lepton/hadron spectrum used fixed geometry with bracketing and interpolation on a single scalar, with no domain or boundary changes between brackets. The Riemann-operator harvest used GPU sparse eigensolvers on a 256×256 grid; 34 265 unique eigenvalues are merged and archived, and all five verification steps execute on the same dataset and pass under the stated thresholds. The SDSS DR7 pipeline uses REVOLVER void catalogs (Planck 2018 and WMAP5), fixed zone→void mappings, fixed RNG seeds for null remaps, and bootstrap resampling for error control. No per-block tuning is introduced after constants are locked.

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Data and Reproducibility

All simulation and analysis code is openly available at:
`https://github.com/DigitalMasterworks/Entropic-Substrate-Theory`.

A full replication package, including scripts and archived results, has been submitted to *ReScience C* for independent verification:
(`https://github.com/ReScience/submissions/issues/94`).

All numerical results reported in this Letter can be reproduced directly from the provided repository without modification.