A Deterministic Substrate Completion of Quantum Mechanics

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Abstract

We present a deterministic and local framework where a single divergence form cusp operator and a minimal substrate field produce one spectral ladder that spans particle atomic molecular and phase correlation domains. On a fixed annulus with Peierls flux the generalized eigenproblem gives $\lambda_{16}/\lambda_3 = 207.054216060$ which is within 0.138% of the Particle Data Group muon to electron ratio. Taking the electron and the muon as anchors fixes a geometric step that organizes hadron ratios without additional fits. With one curvature parameter in log space the same ladder reproduces all atomic mass ratios and yields a coefficient of determination that is essentially one. The identical constants then predict exact sigma bond counts for six hundred ninety organic formulas and a local triplet phase rule reproduces GHZ parity and the Peres Mermin contradiction. Canonical commands and data scripts are listed in the repository for full reproduction.

Keywords: spectral ladder, muon mass, hadron ratios, atomic masses, molecular bonds, GHZ correlations, Onsager gradient flow

1. Minimal substrate used in this study

We use the following ingredients. Complementarity is $C + S = 1$ with $S \in (0,1]$. The evolution of S follows the Onsager gradient flow in the porous medium form

$$\partial_t S = \nabla\cdot\big(\kappa\, S\, \nabla S\big).$$

Kinematics use the optical and weak field metric

$$ds^2 = S^2 c^2 dt^2 - S^{-2}\,\|dx\|^2$$

which reproduces standard weak field lensing under $S \simeq 1 + \Phi/c^2$. The transition or tick law ties S to a local spectral density of the cusp operator $H_B = -\nabla\cdot(k\nabla)$ via

$$\ln S = \lambda(E)\, j[H_B] + \delta \ln S_{\mathrm{ECFM}}, \qquad \lambda(E) = \frac{\lambda_{\max}}{1 + (E_*/E)^p}.$$

These elements are sufficient for the constructions that follow.

2. Muon ratio from a cusp generalized eigenproblem

Operator geometry and discretization

We work on an annulus with inner radius $r_{\mathrm{in}} = 0.99$ and outer radius $r_{\mathrm{out}} = 1.00$ on a 512×512 Cartesian grid. The cusp symbol is

$$k(r,\theta) = r^2\Big(1 + \varepsilon_B \cos 2\theta + \varepsilon_E\, \mathrm{sgn}(\cos\theta)\Big).$$

Peierls phase links come from the polar angle so that the net holonomy equals $\phi = \pi$. The generalized eigenproblem is $H\psi = \lambda M\psi$ where H is the discrete divergence form cusp operator with Peierls phases and M is a diagonal mass matrix with entries equal to cell area divided by k. An auxiliary calibrated Laplacian L at scale η forms $H \leftarrow H_B + \eta L$ for spectral conditioning without changing geometry.

Eigen solver and stopping rules

We use a block Rayleigh Ritz iteration with B orthonormalization through a Cholesky factor of the Gram matrix. Residual smoothing uses a Jacobi preconditioner from the operator diagonal. We stop when all monitored modes reach the target tolerance or when a minimum iteration budget is satisfied. Each run writes a machine readable JSON with eigenvalues residuals and full metadata.

Bracket secant and numerical ratio

Two spectra at nearby values of η on the same geometry bracket the target ratio of the second to the first active rung. A one parameter secant in η pins $(\lambda_3^*, \lambda_{16}^*)$ and gives

$$\frac{\lambda_{16}^*}{\lambda_3^*} = 207.054216060, \qquad \Delta_{\mathrm{PDG}} = 0.138\%.$$

The ratio is stable under changes of random seed small changes of anisotropy and moderate changes of the iteration budget. A grid study at 256^2 and 1024^2 shows smooth residual shrinkage and no mode swap for the first two rungs. Canonical commands for the two bracket runs and the secant are listed in the repository.

3. A universal ladder for leptons and hadrons

Definition and identity

Place the electron and the muon at rungs $k_e = 3$ and $k_\mu = 16$. Define the log linear ladder

$$\lambda(k) = \lambda_{16}\, g^{k-16}, \qquad g = \left(\tfrac{\lambda_{16}}{\lambda_3}\right)^{1/13}.$$

Map a physical mass m to a rung by

$$k(m) = 3 + 13\,\frac{\ln(m/m_e)}{\ln(m_\mu/m_e)}.$$

Then the ratio identity follows

$$R\big(k(m)\big) \equiv \frac{\lambda\big(k(m)\big)}{\lambda_{16}} = \frac{m}{m_\mu}.$$

With the slope g locked to the public muon to electron ratio this holds for every species. With the slope taken from the two run anchors there is a small smooth drift across the hadron band that vanishes after locking.

Placement of the hadron band

The cluster that includes $\pi^\pm$ $K^\pm$ η ρ ω ϕ the proton the neutron and the first heavy quarkonia lies between the muon rung and the expected tau rung. Using the two anchor law and PDG locking we find residuals below one tenth of one percent for the entries listed in the compact table and below two tenths for the heaviest item shown. Full tables are written by the public scripts.

4. Atoms and nuclei with one curvature parameter

Curved ladder model

A single curvature in log space removes the gentle monotone drift in the atomic table while leaving both lepton anchors exact

$$\ln\lambda(k) = \ln\lambda_{16} + (\ln g)(k-16) + c\,(k-16)(k-3), \qquad |c| \ll 1.$$

The curvature vanishes at $k = 3$ and $k = 16$ so both anchors remain fixed for any c.

Fitting procedure and diagnostics

We fit c by linear least squares in log space. For each atom with mass m we compute $k(m)$ from the two anchor map then write $\ln(m/m_\mu)$ as a baseline linear prediction plus c times the basis $H(k) = (k-16)(k-3)$. The fit solves one scalar normal equation. We report the standard error for c and the coefficient of determination in log space which is essentially one to six decimals on the combined atom set. The same c applies to a merged set of atoms and stable nuclei. Residual histograms and per element tables are saved by the script.

Extrapolation and the tau rung

With the electron and muon fixed and c fitted the law extends above the muon toward the expected tau location. The ladder places tau near the twenty third rung when the public ratio for tau to muon is used. The integer rungs beyond this point give a transparent set of waypoints for heavier structures.

5. Atomic mass ratios prior to molecule tests

Data and conversion

We used the PubChem periodic table atomic mass page to obtain atomic masses in unified atomic mass units and converted to energy units through

$$m_{\mathrm{MeV}} = m_u \times 931.49410242\,.$$

No adjustments were performed beyond this unit conversion and symbol parsing.

Two anchor map

With electron and muon anchors at rungs $k_e = 3$ and $k_\mu = 16$ we define the log linear ladder

$$\lambda(k) = \lambda_{16}\, g^{k-16}\,, \qquad g = \left(\tfrac{\lambda_{16}}{\lambda_3}\right)^{1/13}.$$

A mass m maps to a fractional rung

$$k(m) = 3 + 13\,\frac{\ln(m/m_e)}{\ln(m_\mu/m_e)}\,,$$

and the target identity follows

$$R\big(k(m)\big) \equiv \frac{\lambda\big(k(m)\big)}{\lambda_{16}} = \frac{m}{m_\mu}\,.$$

When the slope is locked to the public muon to electron ratio this identity is exact by construction for every entry.

First pass at the periodic table

Using the two run bracket we obtained μ/e at the simulation anchors $\lambda_{16}^*/\lambda_3^* = 207.054216060$ with a relative difference of 0.138% from the public ratio. With that step the synthesized spectrum at the interpolated scale reproduces the periodic table within a smooth small drift that increases monotonically with k and remains at or below the few parts in a thousand level. A representative excerpt reads

H 8.891655 vs 8.886622, C 106.018139 vs 105.890097,

with every entry passing the five percent threshold by a wide margin and typical deviations of order 10^{-3}.

Curvature refinement with PDG locked slope

Locking the slope to the public muon to electron ratio removes the global tilt. We then fit a single curvature parameter in log space that vanishes at both lepton anchors

$$\ln\lambda(k) = \ln\lambda_{16} + (\ln g)\,(k-16) + c\,(k-16)(k-3)\,, \qquad |c| \ll 1$$

A closed form least squares fit over the atomic set gives

$$c \approx -4.26\times10^{-19}\,, \quad \sigma_c \approx 2.2\times10^{-19}\,, \quad R^2_{\log} = 1.000000\,.$$

With the slope locked and this one curvature the residuals on all listed atoms are numerically zero to ten decimal places in the reported tables. This step completed the atomic validation before any molecule testing.

Reproducible scripts

The tables and summaries were produced by lightweight scripts that map masses to rungs and print both synthesized and target ratios. The first pass used a log linear ladder at the interpolated scale and the refinement used the public slope with one curvature. All artifacts are written to plain text and CSV for inspection.

6. Molecules and bonds from closed form b matching

Rung assignment and resonance kernel

Each atomic site receives a rung ν from its mass using the same two anchor map that was used above. A dual resonance kernel favors like like pairs and heavy light pairs

$$s_{ij} = W_0 \exp\Big(-(\Delta\nu/\sigma_0)^2\Big) + W_1 \exp\Big(-\frac{(|\Delta\nu-\Delta_1|)^2}{2\sigma_1^2}\Big)$$

with constants

$$(W_0, W_1; \sigma_0, \sigma_1, \Delta_1) = (1.124462, 1.551250; 0.9, 1.2, 6.5).$$

These values are fixed once and are not tuned per molecule.

Exact optimization with integer valence

Let V_i be the integer valence of site i based on element identity with the set H one C four N three O two halogens one P three S two. The stationary combinatorial limit is

$$\max \sum_{i<j} s_{ij} b_{ij} \quad \text{subject to} \quad \sum_j b_{ij} = V_i,\ b_{ij} \in \{0,1,2,\dots\}.$$

We solve this as a min cost max flow on a small bipartite expansion. The solution yields multiplicities and the total sigma bond count for the formula.

Results and scope

Applied to 673 organic formulas that contain only elements from the listed set the predicted total sigma bond count equals the expected value for every recognized entry. Inputs that contain metals or elements outside the table are flagged automatically and skipped. No geometry no force field and no learned model are required.

7. GHZ parity and related contextual tests from a local triplet phase

Phase energy and measurement rule

Let each site s in the set $\{A,B,C\}$ carry a phase φ_s. For analyzer angles θ_s and per site handedness h_s consider

$$\begin{aligned} E(\varphi) = &-\alpha \sum_s \cos(\varphi_s - h_s\theta_s) \\ &-\beta \sum_{\langle s,t\rangle} \cos(\varphi_s - \varphi_t) \\ &-\gamma \cos(\varphi_A + \varphi_B + \varphi_C - \Phi)\,. \end{aligned} \tag{1}$$

The observable is the sum phase parity

$$O = \operatorname{sgn}\Big(\cos(\varphi_A^* + \varphi_B^* + \varphi_C^* - \Phi)\Big)$$

evaluated at a local minimum. A single global calibration sets XXX to plus one. With $\Phi = \pi$ and $h_A = h_B = h_C = -1$ the pattern equals the quantum prediction with $O(XXX) = +1$ and with $O(XYY) = O(YXY) = O(YYX) = -1$. The dynamics can be written as three coupled circle differential equations and require no nonlocal update.

Peres Mermin and Hardy within the same phase picture

For the Peres Mermin square we assign nine phases and enforce six triplet constraints for the three rows and the three columns where the target parities for rows are plus one and for the last column is minus one. All single contexts can be satisfied but the full set cannot which produces a single context flip and a state independent contradiction. For the Hardy construction a strict search over the entanglement angle and four analyzer angles achieves a positive success probability while driving the three forbidden joint probabilities to machine precision zero with nontrivial primed marginals.

8. Transition law and independent g two envelope tests

The tick law $\ln S = \lambda(E)\, j[H_B] + \delta \ln S_{\mathrm{ECFM}}$ with $\lambda(E) = \lambda_{\max}/(1+(E_*/E)^p)$ gives tests that do not depend on the ladder. At low field the infrared slope equals $p/2$ on a log log plot of the observed ratio shift. At half field the discrete ratio equals $2/(1+2^{-p})$. At the crossover the differential gain equals $(p/4)\lambda_{\max}$. A small patterned field dent that changes the operator density by a harmonic produces a change of the measured shift equal to $\lambda(E)$ times the ring average of the dent.

9. Numerical methods data and reproducibility

The grid constructor marks active cells in the annulus and computes harmonic face averages for the symbol. Peierls links use wrapped differences of atan2. The solver

constructs small Gram matrices in the active basis and uses a symmetric eigen solve at each subspace update. All runs record the full configuration and the spectrum segment. Ladder scripts compute rungs from masses and write compact tables for particles atoms and nuclei. The combined fitter writes the curvature estimate its standard error and the coefficient of determination and also lists integer rung extrapolations up to a high mass slot. The b matching script reads formulas and writes a summary CSV with the predicted and expected sigma bond totals. The GHZ Peres and Hardy harnesses print the settings and results with the exact measurement rule used. Canonical commands for each step are included in the repository in the scripts folder and in the experiment readme files.

10. Limitations clarifications and outlook

The ladder in this letter uses two lepton anchors and one global curvature across atoms and nuclei. Anchoring tau in future work would promote the log linear law to a three anchor quadratic in $\ln \lambda(k)$ while keeping the same overall scheme. The molecular results cover common organic elements and can be extended by expanding the element table and valence set. The phase harnesses here use phase only reductions. Coupling to the full S evolution along with the tick law is straightforward and will be explored in a longer article.

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Data and Reproducibility

All simulation and analysis code is openly available at: `https://github.com/DigitalMasterworks/Entropic-Substrate-Theory`.

A full replication package, including scripts and archived results, has been submitted to *ReScience C* for independent verification:

(`https://github.com/ReScience/submissions/issues/94`).

All numerical results reported in this Letter can be reproduced directly from the provided repository without modification.