高周波電圧位相誤差に対する一考察

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1　数式整理

- 説明変数の導入

$$\boldsymbol{R}(\theta) \equiv \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} g_{\mathrm{pi}} & g_{\mathrm{pm}} \\ g_{\mathrm{ni}} & g_{\mathrm{nm}} \end{bmatrix} = A_{\mathrm{pn}}(\bar{\omega}_{\mathrm{h}}) \begin{bmatrix} (1+K)L_{\mathrm{i}} & -(1-K)L_{\mathrm{m}} \\ (1-K)L_{\mathrm{i}} & -(1+K)L_{\mathrm{m}} \end{bmatrix} \tag{2}$$

$$A_{\mathrm{pn}}(\bar{\omega}_{\mathrm{h}}) = \frac{V_{\mathrm{h}} T_{\mathrm{s}}}{4 L_{\mathrm{d}} L_{\mathrm{q}} \sin(\bar{\omega}_{\mathrm{h}}/2)} \tag{3}$$

$$\bar{\omega}_{\mathrm{h}} \equiv \frac{\omega_{\mathrm{h}} T_{\mathrm{s}}}{2} \tag{4}$$

- c_{pn}, s_{pn} について

$$\begin{bmatrix} c_{\mathrm{p}} & s_{\mathrm{p}} \\ c_{\mathrm{n}} & -s_{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} g_{\mathrm{pi}} + g_{\mathrm{pm}} \cos(2\theta_\gamma) & g_{\mathrm{pm}} \sin(2\theta_\gamma) \\ g_{\mathrm{ni}} + g_{\mathrm{nm}} \cos(2\theta_\gamma) & -g_{\mathrm{nm}} \sin(2\theta_\gamma) \end{bmatrix} \tag{5}$$

$$= A_{\mathrm{pn}}(\bar{\omega}_{\mathrm{h}}) \begin{bmatrix} (1+K)L_{\mathrm{i}} - (1-K)L_{\mathrm{m}} \cos(2\theta_\gamma) & -(1-K)L_{\mathrm{m}} \sin(2\theta_\gamma) \\ (1-K)L_{\mathrm{i}} - (1+K)L_{\mathrm{m}} \cos(2\theta_\gamma) & (1+K)L_{\mathrm{m}} \sin(2\theta_\gamma) \end{bmatrix} \tag{6}$$

- $c_{\gamma\delta}$, $s_{\gamma\delta}$ について

$$\begin{bmatrix} c_\gamma & s_\gamma \\ c_\delta & -s_\delta \end{bmatrix} = (\boldsymbol{I} - \boldsymbol{J}) \begin{bmatrix} c_{\mathrm{p}} & s_{\mathrm{p}} \\ c_{\mathrm{n}} & -s_{\mathrm{n}} \end{bmatrix} \tag{7}$$

$$= A_{\mathrm{pn}}(\bar{\omega}_{\mathrm{h}}) \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} (1+K)L_{\mathrm{i}} - (1-K)L_{\mathrm{m}} \cos(2\theta_\gamma) & -(1-K)L_{\mathrm{m}} \sin(2\theta_\gamma) \\ (1-K)L_{\mathrm{i}} - (1+K)L_{\mathrm{m}} \cos(2\theta_\gamma) & (1+K)L_{\mathrm{m}} \sin(2\theta_\gamma) \end{bmatrix} \tag{8}$$

$$= A_{\mathrm{pn}}(\bar{\omega}_{\mathrm{h}}) \begin{bmatrix} 2L_{\mathrm{i}} - 2L_{\mathrm{m}} \cos(2\theta_\gamma) & 2KL_{\mathrm{m}} \sin(2\theta_\gamma) \\ -2KL_{\mathrm{i}} - 2KL_{\mathrm{m}} \cos(2\theta_\gamma) & 2L_{\mathrm{m}} \sin(2\theta_\gamma) \end{bmatrix} \tag{9}$$

$$= 2A_{\mathrm{pn}}(\bar{\omega}_{\mathrm{h}}) \begin{bmatrix} L_{\mathrm{i}} - L_{\mathrm{m}} \cos(2\theta_\gamma) & KL_{\mathrm{m}} \sin(2\theta_\gamma) \\ -K(L_{\mathrm{i}} + L_{\mathrm{m}} \cos(2\theta_\gamma)) & L_{\mathrm{m}} \sin(2\theta_\gamma) \end{bmatrix} \tag{10}$$

- $\tilde{c}_{\mathrm{pn}}$, $\tilde{s}_{\mathrm{pn}}$ について

$$\begin{bmatrix} \tilde{c}_{\mathrm{p}} \\ \tilde{s}_{\mathrm{p}} \end{bmatrix} = [c_{\mathrm{p}} \boldsymbol{I} + s_{\mathrm{p}} \boldsymbol{J}] \begin{bmatrix} \cos(\theta_{\mathrm{he}}) \\ \sin(\theta_{\mathrm{he}}) \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} \tilde{c}_{\mathrm{n}} \\ \tilde{s}_{\mathrm{n}} \end{bmatrix} = [c_{\mathrm{n}} \boldsymbol{I} + s_{\mathrm{n}} \boldsymbol{J}] \begin{bmatrix} \cos(\theta_{\mathrm{he}}) \\ -\sin(\theta_{\mathrm{he}}) \end{bmatrix} \tag{12}$$

$$\Leftrightarrow \begin{bmatrix} \tilde{c}_{\mathrm{p}} & \tilde{c}_{\mathrm{n}} \\ \tilde{s}_{\mathrm{p}} & -\tilde{s}_{\mathrm{n}} \end{bmatrix} = \boldsymbol{R}(\theta_{\mathrm{he}}) \begin{bmatrix} c_{\mathrm{p}} & c_{\mathrm{n}} \\ s_{\mathrm{p}} & -s_{\mathrm{n}} \end{bmatrix} \tag{13}$$

$$\Leftrightarrow \begin{bmatrix} \tilde{c}_{\mathrm{p}} & \tilde{s}_{\mathrm{p}} \\ \tilde{c}_{\mathrm{n}} & -\tilde{s}_{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} c_{\mathrm{p}} & s_{\mathrm{p}} \\ c_{\mathrm{n}} & -s_{\mathrm{n}} \end{bmatrix} \boldsymbol{R}^{\mathrm{T}}(\theta_{\mathrm{he}}) \tag{14}$$

- $\tilde{c}_{\mathrm{pn}}$, $\tilde{s}_{\mathrm{pn}}$ から計算される $\tilde{c}_{\gamma\delta}$, $\tilde{s}_{\gamma\delta}$ の導入

$$\begin{bmatrix} \tilde{c}_\gamma & \tilde{s}_\gamma \\ \tilde{c}_\delta & -\tilde{s}_\delta \end{bmatrix} \equiv (\boldsymbol{I} - \boldsymbol{J}) \begin{bmatrix} \tilde{c}_{\mathrm{p}} & \tilde{s}_{\mathrm{p}} \\ \tilde{c}_{\mathrm{n}} & -\tilde{s}_{\mathrm{n}} \end{bmatrix} \tag{15}$$

$$= (\boldsymbol{I} - \boldsymbol{J}) \begin{bmatrix} c_{\mathrm{p}} & s_{\mathrm{p}} \\ c_{\mathrm{n}} & -s_{\mathrm{n}} \end{bmatrix} \boldsymbol{R}^{\mathrm{T}}(\theta_{\mathrm{he}}) \tag{16}$$

$$= \begin{bmatrix} c_\gamma & s_\gamma \\ c_\delta & -s_\delta \end{bmatrix} \boldsymbol{R}^{\mathrm{T}}(\theta_{\mathrm{he}}) \tag{17}$$

2 偏角の導出

- θ_{he} を未知変数とする連立方程式

$$\boldsymbol{X}_{\mathrm{pn}} \equiv \begin{bmatrix} c_{\mathrm{p}} & s_{\mathrm{p}} \\ c_{\mathrm{n}} & -s_{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} (1+K)L_{\mathrm{i}} - (1-K)L_{\mathrm{m}}\cos(2\theta_\gamma) & -(1-K)L_{\mathrm{m}}\sin(2\theta_\gamma) \\ (1-K)L_{\mathrm{i}} - (1+K)L_{\mathrm{m}}\cos(2\theta_\gamma) & (1+K)L_{\mathrm{m}}\sin(2\theta_\gamma) \end{bmatrix} \tag{18}$$

$$\boldsymbol{X}_{\gamma\delta} \equiv \begin{bmatrix} c_\gamma & s_\gamma \\ c_\delta & -s_\delta \end{bmatrix} = 2A_{\mathrm{pn}}(\bar{\omega}_{\mathrm{h}}) \begin{bmatrix} L_{\mathrm{i}} - L_{\mathrm{m}}\cos(2\theta_\gamma) & KL_{\mathrm{m}}\sin(2\theta_\gamma) \\ -K(L_{\mathrm{i}} + L_{\mathrm{m}}\cos(2\theta_\gamma)) & L_{\mathrm{m}}\sin(2\theta_\gamma) \end{bmatrix} \tag{19}$$

$$\tilde{\boldsymbol{X}}_{\mathrm{pn}} \equiv \begin{bmatrix} \tilde{c}_{\mathrm{p}} & \tilde{s}_{\mathrm{p}} \\ \tilde{c}_{\mathrm{n}} & -\tilde{s}_{\mathrm{n}} \end{bmatrix} = \boldsymbol{X}_{\mathrm{pn}}\boldsymbol{R}^{\mathrm{T}}(\theta_{\mathrm{he}}) \tag{20}$$

$$\tilde{\boldsymbol{X}}_{\gamma\delta} \equiv \begin{bmatrix} \tilde{c}_\gamma & \tilde{s}_\gamma \\ \tilde{c}_\delta & -\tilde{s}_\delta \end{bmatrix} = \boldsymbol{X}_{\gamma\delta}\boldsymbol{R}^{\mathrm{T}}(\theta_{\mathrm{he}}) \tag{21}$$

- 求解

$$\boldsymbol{R}^{\mathrm{T}}(\theta_{\mathrm{he}}) = \boldsymbol{X}_{\gamma\delta}^{-1}\tilde{\boldsymbol{X}}_{\gamma\delta} \tag{22}$$

$$\Leftrightarrow \begin{bmatrix} \cos(\theta_{\mathrm{he}}) & \sin(\theta_{\mathrm{he}}) \\ -\sin(\theta_{\mathrm{he}}) & \cos(\theta_{\mathrm{he}}) \end{bmatrix} = \frac{1}{\det(\boldsymbol{X}_{\gamma\delta})} \begin{bmatrix} L_{\mathrm{m}}\sin(2\theta_\gamma) & -KL_{\mathrm{m}}\sin(2\theta_\gamma) \\ K(L_{\mathrm{i}} + L_{\mathrm{m}}\cos(2\theta_\gamma)) & L_{\mathrm{i}} - L_{\mathrm{m}}\cos(2\theta_\gamma) \end{bmatrix} \begin{bmatrix} \tilde{c}_\gamma & \tilde{s}_\gamma \\ \tilde{c}_\delta & -\tilde{s}_\delta \end{bmatrix} \tag{23}$$

$$\det(\boldsymbol{X}_{\gamma\delta}) = 2A_{\mathrm{pn}}(\bar{\omega}_{\mathrm{h}}) \left((k^2+1)L_{\mathrm{i}} + (k^2-1)L_{\mathrm{m}}\cos(2\theta_\gamma) \right) L_{\mathrm{m}}\sin(2\theta_\gamma) \tag{24}$$

$$\Leftrightarrow \begin{bmatrix} \cos(\theta_{\mathrm{he}}) \\ \sin(\theta_{\mathrm{he}}) \end{bmatrix} = \frac{L_{\mathrm{m}}}{\det(\boldsymbol{X}_{\gamma\delta})}\sin(2\theta_\gamma) \begin{bmatrix} \tilde{c}_\gamma - K\tilde{c}_\delta \\ \tilde{s}_\gamma + K\tilde{s}_\delta \end{bmatrix} \tag{25}$$

$$\therefore \tan(\theta_{\mathrm{he}}) = \frac{\tilde{s}_\gamma + K\tilde{s}_\delta}{\tilde{c}_\gamma - K\tilde{c}_\delta} \tag{26}$$

3 検証

- SymPy による検証
  - https://github.com/digital-servo/analyze-high-freq-voltage-phase-error

```
import sympy as sy

# original simplify atan
def simplify_atan(sympy, expr_y, expr_x):
    fx = sympy.simplify(expr_x)
    fy = sympy.simplify(expr_y)
    tan = fy / fx
    atan = sympy.atan(tan)
    theta = sympy.simplify(atan)
    return sympy.factor(theta)

# original simplify atan
def simplify_collect(sympy, expr_input, collect_expr):
    expr = expr_input
    # expr = sympy.factor(expr)
    expr = sympy.simplify(expr)
    # expr = sympy.factor(expr)
    expr = sympy.collect(expr, collect_expr)
    return expr

#motor params
Li = sy.Symbol("Li", real=True)
Lm = sy.Symbol("Lm", real=True)
```

```
A = sy.Symbol("A", real=True)
K = sy.Symbol("K", real=True)
theta_gamma = sy.Symbol("theta_gamma", real=True)
theta_re = sy.Symbol("theta_re", real=True)
theta_e = sy.Symbol("theta_e", real=True)

#gain
g_pi = sy.Symbol("g_pi", real=True)
g_pm = sy.Symbol("g_pm", real=True)
g_ni = sy.Symbol("g_ni", real=True)
g_nm = sy.Symbol("g_nm", real=True)
g_pi = A*(1+K)*Li
g_pm = A*(-(1-K)*Lm)
g_ni = A*(1-K)*Li
g_nm = A*(-(1+K)*Lm)

#complex amp
c_p = sy.Symbol("c_p", real=True)
s_p = sy.Symbol("s_p", real=True)
c_n = sy.Symbol("c_n", real=True)
s_n = sy.Symbol("s_n", real=True)
c_p = g_pi + g_pm * sy.cos(2*theta_gamma)
s_p = g_pm * sy.sin(2*theta_gamma)
c_n = g_ni + g_nm * sy.cos(2*theta_gamma)
s_n = g_nm * sy.sin(2*theta_gamma)

#variable num
C_2p = sy.Symbol("C_2p", real=True)
S_2p = sy.Symbol("S_2p", real=True)
C_2p = c_p*c_n - s_p*s_n
S_2p = s_p*c_n + c_p*s_n
theta_re = sy.atan(C_2p / S_2p)

#hosooka
#complex amp
c_pt = sy.Symbol("c_pt", real=True)
s_pt = sy.Symbol("s_pt", real=True)
c_nt = sy.Symbol("c_nt", real=True)
s_nt = sy.Symbol("s_nt", real=True)
c_pt = c_p * sy.cos(theta_e) - s_p * sy.sin(theta_e)
s_pt = s_p * sy.cos(theta_e) + c_p * sy.sin(theta_e)
c_nt = c_n * sy.cos(theta_e) - s_n * (-sy.sin(theta_e))
s_nt = s_n * sy.cos(theta_e) + c_n * (-sy.sin(theta_e))

#original
#gamma delta tilde complex amp
c_gammat = sy.Symbol("c_gammat", real=True)
s_gammat = sy.Symbol("s_gammat", real=True)
c_deltat = sy.Symbol("c_deltat", real=True)
s_deltat = sy.Symbol("s_deltat", real=True)
c_gammat = c_pt + c_nt
s_gammat = s_pt - s_nt
c_deltat = -c_pt + c_nt
s_deltat = -(-s_pt - s_nt)

```

```
print("@calculate 220827-2 my expr")
y = s_gammat + K*s_deltat
x = c_gammat - K*c_deltat
print(simplify_collect(sy,x,sy.cos(theta_e)))
print(simplify_collect(sy,y,sy.sin(theta_e)))
print("@calculate atan my expr")
print(simplify_atan(sy, y, x))
```

- 出力