

## Lösungen zum 12. Übungsblatt

### 1. Aufgabe. a)

$$\int \sqrt[m]{x^n} dx = \frac{m \cdot \sqrt[m]{x^{n+m}}}{n+m} + C$$

b)

$$\int a^x e^x dx = \frac{(ae)^x}{1 + \ln a} + C$$

c)

$$\int \frac{(1-t)^2}{t\sqrt{t}} dt = \frac{2t^2 - 12t - 6}{3\sqrt{t}} + C$$

d)

$$\int \frac{1 + \cos^2(t)}{1 + \cos(2t)} dt = \frac{1}{2}(t + \tan t) + C.$$

### 2. Aufgabe. a)

$$\int_0^1 x^2 dx = \frac{1}{3},$$

b)

$$\int_0^1 e^x dx = e - 1,$$

c)

$$\int_0^1 x^k dx = \frac{1}{k+1}, \quad k \in \mathbb{N},$$

d)

$$\int_1^2 \frac{1}{x} dx = \ln 2.$$

### 3. Aufgabe.

a)

$$\int_0^1 \sqrt{1+x} dx = \frac{4\sqrt{2}-2}{3}$$

b)

$$\int_0^\pi \sin(x) dx = 2$$

c)

$$\int_0^1 (e^x - 1)^4 e^x dx = \frac{1}{5}(e - 1)^5$$

d)

$$\int_1^2 \frac{1}{x+x^3} dx = \ln\left(\frac{2\sqrt{2}}{\sqrt{5}}\right)$$

e)

$$\int_0^1 x e^{-x} dx = 1 - 2e^{-1}$$

f)

$$\int_0^2 x \log_2(x) dx = -\frac{2+\ln 2}{2\ln 2}$$

$$\text{g)} \int_0^{e-1} \ln(x+1) dx = 1$$

$$\text{h)} \int_4^9 \frac{\sqrt{x}}{\sqrt{x-1}} dx = 7 + 2 \ln 2$$

$$\text{i)} \int_0^1 \frac{\sqrt{e^x}}{\sqrt{e^x+e^{-x}}} dx = \ln \frac{e+\sqrt{e^2+1}}{1+\sqrt{2}}$$

#### 4. Aufgabe.

a)

$$\int \frac{dx}{1+\sqrt{1+x}} = 2(\sqrt{x+1} - \ln(1+\sqrt{1+x})) + C$$

b)

$$\int \frac{dx}{\sqrt{1+e^x}} = \ln \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1} + C$$

c)

$$\int \sqrt{1+\cos^2(t)} \sin(2t) \cos(2t) dt = 0, 4(\sqrt{1+\cos^2 t})^3(3-2\cos^2 t) + C$$

d)

$$\int \frac{t^2}{\sqrt{a^2-t^2}} dt = \frac{a^2}{2} \arcsin \frac{t}{a} - \frac{t}{2} \sqrt{a^2-t^2} + C$$

e)

$$\int \frac{y^5 dy}{(y^2-4)^2} = \frac{1}{2}(y^2-4) + 4 \ln |y^2-4| - \frac{8}{y^2-4} + C$$

f)

$$\int \frac{\sqrt{1+\ln(y)}}{y \ln(y)} dy = 2\sqrt{1+\ln y} + \ln |\ln y| - 2 \ln |\sqrt{1+\ln y} + 1| + C$$