Lösungen zum 12. Übungsblatt

1. Aufgabe.

a) $\begin{cases} y_1 = e^{-6t}(\cos t + 2\sin t) \\ y_2 = e^{-6t}(3\cos t + \sin t) \end{cases}$

b)
$$\begin{cases} y_1 = -2e^t \sin(3t) \\ y_2 = 2e^t \cos(3t) \end{cases}$$

c)
$$\begin{cases} y_1 = -e^{-2t} + e^{4t} \\ y_2 = e^{-2t} + e^{4t} \end{cases}$$

d)
$$\begin{cases} y_1 = c_1 e^{3t} + c_2 e^t \\ y_2 = -\frac{4}{3} c_1 e^{3t} - c_2 e^t \end{cases}$$

e)
$$\begin{cases}
y_1 = e^{2t}(c_1 \cos(3t) + c_2 \sin(3t)) \\
y_2 = \frac{1}{2}e^{2t}((c_1 + c_2)\cos(3t) + (c_2 - c_1)\sin(3t))
\end{cases}$$

f)
$$\begin{cases} y_1 = c_1 e^{2t} + c_2 t e^{2t} \\ y_2 = -4c_1 e^{2t} + c_2 e^{2t} - 4c_2 t e^{2t} \end{cases}$$

2. Aufgabe.

a) $\begin{cases} y_1 = c_1 e^{-6t} + c_2 e^{-t} - 15\cos t + 16\sin t \\ y_2 = 4c_1 e^{-6t} - c_2 e^{-t} + 14\cos t - 10\sin t \end{cases}$

b)
$$\begin{cases} y_1 = e^{-t}(c_1 \sin(4t) + c_2 \cos(4t)) + e^{7t} \\ y_2 = \frac{1}{2}e^{-t}((3c_1 + c_2)\sin(4t) + (3c_2 + c_1)\cos(4t)) + e^{7t} \end{cases}$$

c)
$$\begin{cases} y_1 = c_1 + c_2 e^{3t} + 2t^2 + 3t - \frac{2}{3} \\ y_2 = -c_1 - 2c_2 e^{3t} - 2t^2 - 4t \end{cases}$$

3. Aufgabe.

a)
$$\begin{cases} y_1 = (c_1 + c_2 t)e^{-t} + 7e^{2t} + 4t^2 - 16t + 28 \\ y_2 = \frac{1}{4}(c_2 - 6c_1 - 6c_2 t)e^{-t} - 7e^{2t} - 5t^2 + 22t - 35 \end{cases}$$

b)
$$\begin{cases} y_1 = c_1 + c_2 e^{2t} - 4t e^{2t} - 2t^2 - 2t \\ y_2 = 6t e^{2t} - c_1 - \frac{3}{2}c_2 e^{2t} + \frac{3}{2}e^{2t} + 2t^2 + 3t \end{cases}$$

c)
$$\begin{cases} y_1 = c_1 e^t + c_2 e^{-t} + \frac{1}{2} t e^t - \frac{1}{2} t e^{-t} \\ y_2 = c_1 e^t - c_2 e^{-t} + \frac{1}{2} e^t + \frac{1}{2} t e^t - \frac{1}{2} e^{-t} + \frac{1}{2} t e^{-t} \end{cases}$$