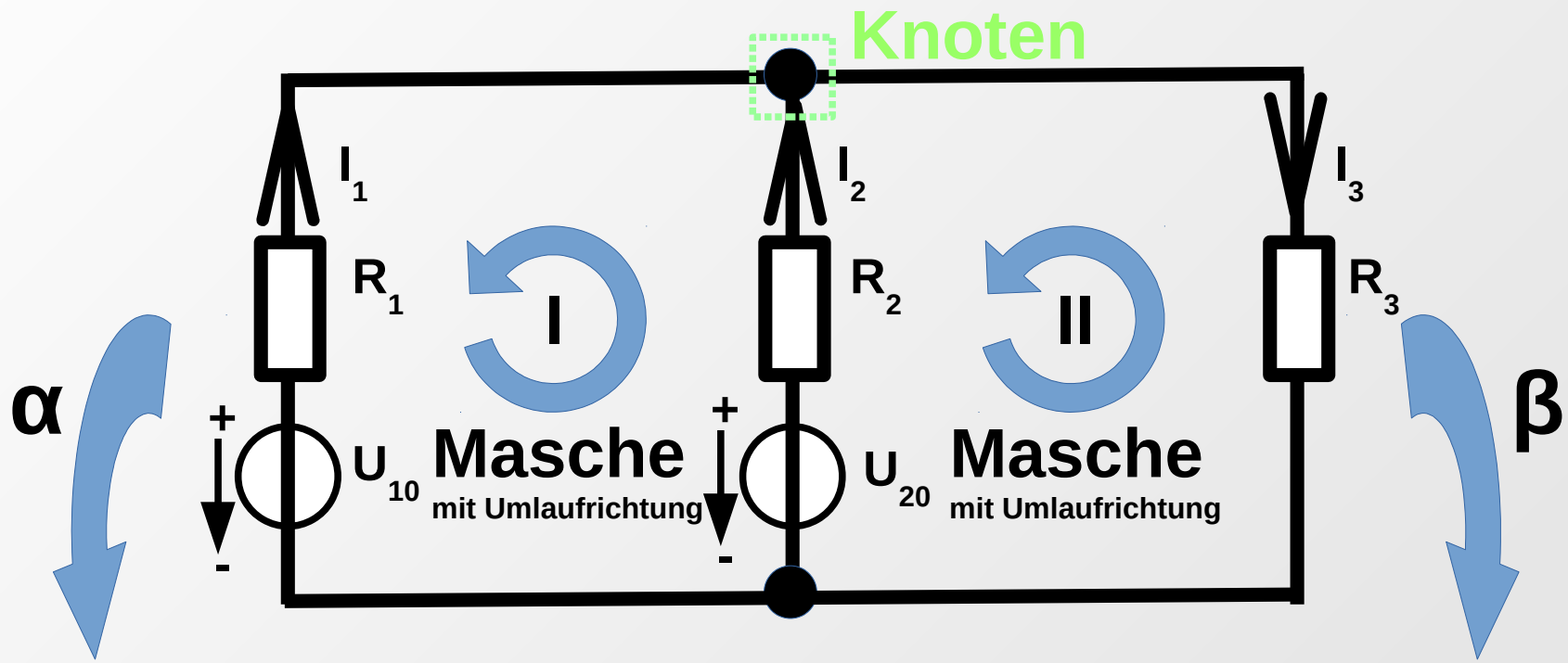
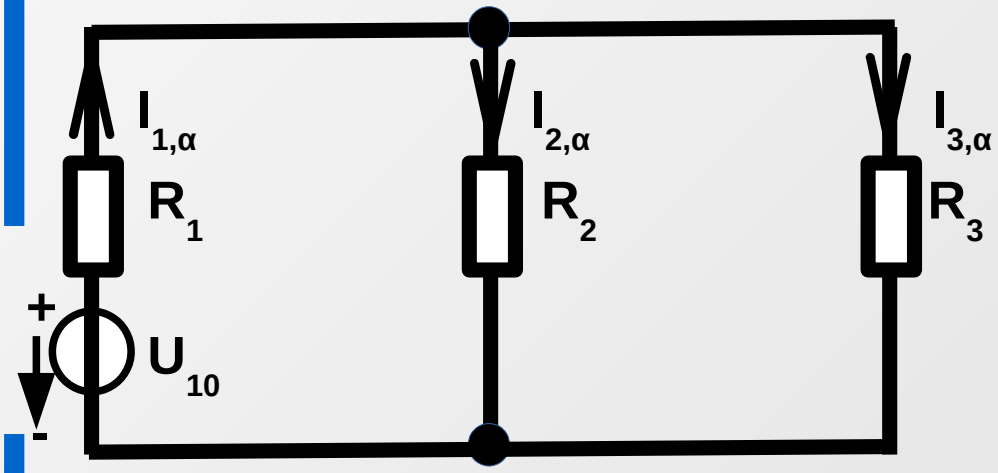
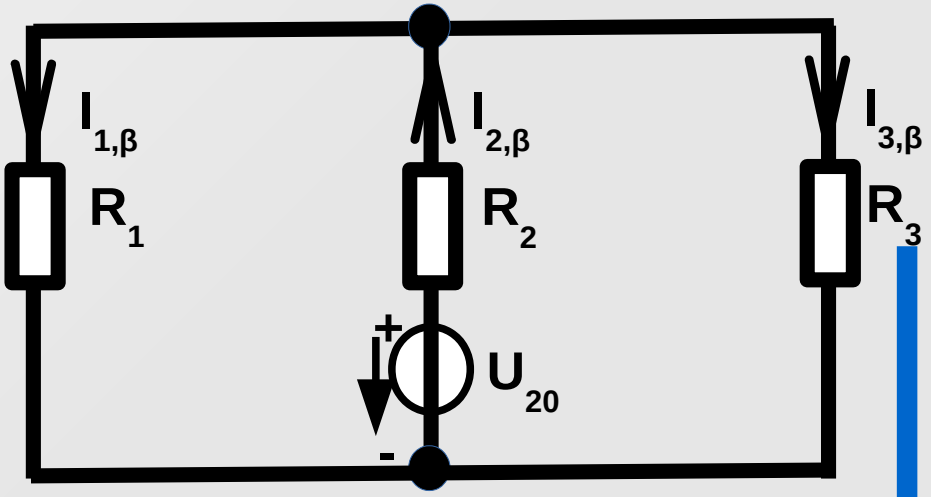
Gleichstromnetzwerk und seine Zerlegung nach Helmholtz







Aufstellen Maschen- und Knotengleichung

Gesucht ist I_3

$$\text{MI}: -U_1 + U_{10} - U_{20} + U_2 = 0 \qquad -U_1 + U_2 = U_{20} - U_{10}$$

$$\text{MII}: -U_2 + U_{20} - U_3 = 0 \qquad -U_2 - U_3 = -U_{20}$$

$$\text{KN}: I_1 + I_2 - I_3 = 0 \qquad I_1 + I_2 - I_3 = 0$$

Umschreiben

$$\begin{aligned} -U_1 + U_2 &= U_{20} - U_{10} \\ -U_2 - U_3 &= -U_{20} \\ I_1 + I_2 - I_3 &= 0 \end{aligned} \quad \rightarrow \quad \begin{aligned} -R_1 I_1 + R_2 I_2 &= U_{20} - U_{10} \\ -R_2 I_2 - R_3 I_3 &= -U_{20} \\ I_1 + I_2 - I_3 &= 0 \end{aligned}$$

Berechnen I

$$-R_1 I_1 + R_2 I_2 = U_{20} - U_{10} \;//\quad //\cdot\left(\frac{-1}{R_2}\right)$$

$$-R_2 I_2 - R_3 I_3 = -U_{20} \quad +$$

$$I_1 + I_2 - I_3 = 0 \quad +$$

$$-R_1 I_1 - R_3 I_3 = -U_{10} \qquad //\cdot\left(\frac{-1}{R_3}\right)$$

$$\left(\frac{R_1}{R_2}\right) I_1 + I_1 - I_3 = \left(\frac{-1}{R_2}\right)\cdot\left(U_{20} - U_{10}\right) \quad + \quad \equiv \text{GLI3}$$

$$\left(\frac{R_1}{R_3}\right) I_1 + \left(\frac{R_1}{R_2}\right) I_1 + I_1 = \left(\frac{U_{10}}{R_3}\right) - \left(\frac{1}{R_2}\right)\cdot\left(U_{20} - U_{10}\right)$$

Berechnen II

$$\left(\frac{R_1}{R_3}\right) I_1 + \left(\frac{R_1}{R_2}\right) I_1 + I_1 = \left(\frac{U_{10}}{R_3}\right) - \left(\frac{1}{R_2}\right) \cdot \left(U_{20} - U_{10}\right)$$

$$\rightarrow \quad I_1 \left(\frac{R_1}{R_3} + \frac{R_1}{R_2} + 1\right) = \frac{U_{10}}{R_3} - \frac{1}{R_2} \cdot \left(U_{20} - U_{10}\right)$$

$$\rightarrow I_1 \left(\frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 R_3}\right) = \frac{1}{R_2 R_3} \left(U_{10} R_2 - U_{20} R_3 + U_{10} R_3\right)$$

$$\rightarrow I_1 = \frac{U_{10} R_2 - U_{20} R_3 + U_{10} R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Berechnen III

$$I_1 = \frac{U_{10} R_2 - U_{20} R_3 + U_{10} R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Mit GLI3

$$\left(\frac{R_1}{R_2}\right) I_1 + I_1 - I_3 = \left(\frac{-1}{R_2}\right) \cdot \left(U_{20} - U_{10}\right)$$

$$\rightarrow \quad I_3 = I_1 \left(\frac{R_1}{R_2} + 1\right) \left(\frac{1}{R_2}\right) \cdot \left(U_{20} - U_{10}\right)$$

Es sei gegeben: $U_{10} = 100 \text{ V}; \; U_{20} = 110 \text{ V}$

$R_1 = 10 \ \Omega; \; R_2 = 10 \ \Omega; \; R_3 = 200 \ \Omega$

$$\rightarrow \; I_3 = 0.512195 \text{ A}$$

Nach dem Helmholtzverfahren

Schaltbild α U_{20} kurzgeschlossen:

$$\frac{I_{3,\alpha}}{I_{1,\alpha}} = \frac{R_2}{R_2 + R_3} \quad \rightarrow \quad I_{3,\alpha} = I_{1,\alpha} \cdot \frac{R_2}{R_2 + R_3}$$

$$I_{1,\alpha} = \frac{U_{10}}{R_{ges,\alpha}} = \frac{U_{10}}{R_1 + R_2 \| R_3} = \frac{U_{10}}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

$$= \frac{U_{10}}{\frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}} = U_{10} \cdot \frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_{3,\alpha} = I_{1,\alpha} \cdot \frac{R_2}{R_2 + R_3} = U_{10} \cdot \frac{R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Nach dem Helmholtzverfahren

Schaltbild β U_{10} kurzgeschlossen:

$$\frac{I_{3,\beta}}{I_{2,\beta}} = \frac{R_1}{R_1 + R_3} \quad \rightarrow \quad I_{3,\beta} = I_{2,\beta} \cdot \frac{R_1}{R_1 + R_3}$$

$$I_{2,\beta} = \frac{U_{20}}{R_{ges,\beta}} = \frac{U_{20}}{R_2 + R_1 \| R_3} = U_{20} \cdot \frac{R_1 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_{3,\beta} = I_{2,\beta} \cdot \frac{R_1}{R_1 + R_3} = U_{20} \cdot \frac{R_1}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_3 = I_{3,\alpha} + I_{3,\beta} = \left(U_{10} R_2 + U_{20} R_1\right) \cdot \frac{1}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Es sei gegeben: $U_{10} = 100\ \text{V};\ U_{20} = 110\ \text{V}$

$R_1 = 10\ \Omega;\ R_2 = 10\ \Omega;\ R_3 = 200\ \Omega$

$$\rightarrow\ I_3 = 0.512195\ \text{A}$$