Rechnerische Kontrolle

Resonanzfrequenzen

c)

$$\text{Im}\{Z(\omega_0)\} = \omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{(LC)}}$$

$$\omega = 2\pi f \Rightarrow f_0 = \frac{1}{2\pi\sqrt{(LC)}}$$

$$f_0 = \frac{1}{2\pi\sqrt{(9*10^{-3}H*230*10^{-9}F)}} = 3498,1 Hz$$

d) Im folgenden eine grobe Herleitung und Lösungen. Gerne zeigen wir Ihnen auch unsere per Hand geschriebene, ausführliche Herleitung.

$$Komplexer Leitwert der Schaltung$$

$$Y = \frac{R}{(R^2 + (\omega L - \frac{1}{\omega C_1}))} - j\frac{(\omega L - \frac{1}{\omega C_1})}{R^2 + (\omega L - \frac{1}{\omega C_1})^2} + j\omega C_2$$

$$\text{Im}\{Y(\omega_0)\} = 0$$

$$0 = \omega C_2 - \frac{(\omega L - \frac{1}{\omega C_1})}{R^2 + (\omega L - \frac{1}{\omega C_1})^2} \Leftrightarrow 0 = \omega^4 + \omega^2(\frac{R^2}{L^2} - \frac{2}{LC_1} - \frac{1}{LC_2}) + \frac{C_2 + C_1}{L^2 C_1^2 C_2}$$

$$Substituiert mit x = \omega^2$$

$$0 = x^2 + x*(\frac{R^2}{L^2} - \frac{2}{LC_1} - \frac{1}{LC_2}) + \frac{C_2 + C_1}{L^2 C_1^2 C_2}$$

$$Wir ersetzen durch$$

$$\Rightarrow p = \frac{R^2}{L^2} - \frac{2}{LC_1} - \frac{1}{LC_2}$$

$$\Rightarrow q = \frac{C_2 + C_1}{L^2 C_1^2 C_2}$$

$$under halten mit der p - q Formel die Gleichungen$$

$$f_{r1} = \frac{\sqrt{-\frac{p}{2} - \sqrt{\frac{p^2}{4} - q}}}{2\pi}$$

$$f_{r2} = \frac{\sqrt{-\frac{p}{2} + \sqrt{\frac{p^2}{4} - q}}}{2\pi}$$

$$under haltend a mit folgende Lösungen$$
$$f_{r1} = 3637,3 Hz$$
$$f_{r2} = 7124,6 Hz$$

Kontrollrechnungen

a)

$$1000 Hz:$$

$$\underline{Z} = R + j\omega L\omega = 2\pi f = 2\pi * 1000 = 6283,2\frac{1}{s}$$

$$\underline{Z} = 100\Omega + (j * 6283,2\frac{1}{s} * 0,009H) = (100 + j56,55)\Omega$$

$$\underline{Z} = |Z| = \sqrt{(100^2 + 56,55^2)} = 114,88\Omega$$

$$Y = \frac{1}{Z} = 0,0087046S = 8,70mS$$

$$4000 Hz:$$

$$\underline{Z} = R + j\omega L\omega = 2\pi f = 2\pi * 4000 = 25133\frac{1}{s}$$

$$\underline{Z} = 100\Omega + (j * 25133\frac{1}{s} * 0,009H) = (100 + j226,19)\Omega$$

$$\underline{Z} = |Z| = \sqrt{(100^2 + 226,19^2)} = 247,31\Omega$$

$$Y = \frac{1}{Z} = 0,0040434S = 4,04mS$$

b)

$$3000 Hz:$$

$$\underline{Z} = R + \frac{1}{(j\omega C)}\omega = 2\pi f = 2\pi * 3000 = 18850\frac{1}{s}$$

$$\underline{Z} = 100\Omega + \frac{1}{(j * 18850\frac{1}{s} * 230 * 10^{-9}F)} = (100 - j230,66)\Omega$$

$$\underline{Z} = |Z| = \sqrt{(100^2 + 230,66^2)} = 251,40\Omega$$

$$Y = \frac{1}{Z} = 0,0039777S = 3,97mS$$

$$10000 Hz:$$

$$\underline{Z} = R + \frac{1}{(j\omega C)}\omega = 2\pi f = 2\pi * 10000 = 62832\frac{1}{s}$$

$$\underline{Z} = 100\Omega + \frac{1}{(j * 62832\frac{1}{s} * 230 * 10^{-9}F)} = (100 - j69,20)\Omega$$

$$\underline{Z} = |Z| = \sqrt{(100^2 + 69,20)} = 121,61\Omega$$

$$Y = \frac{1}{Z} = 0,0082232S = 8,22mS$$

c)

$$2000Hz:$$

$$\underline{Z} = R + \frac{1}{(j\omega C)} + j\omega L\omega = 2\pi f = 2\pi * 2000 = 12566\frac{1}{s}$$

$$\underline{Z} = 100\Omega + \frac{1}{(j * 12566\frac{1}{s} * 230 * 10^{-9}F)} + (j * 12566\frac{1}{s} * 0,009H) = (100 - j232,89)\Omega$$

$$Z = |\underline{Z}| = \sqrt{(100^2 + 232,89^2)} = 253,45\Omega$$

$$Y = \frac{1}{Z} = 0,0039455S = 3,94mS$$

$$7500Hz:$$

$$\underline{Z} = R + \frac{1}{(j\omega C)} + j\omega L\omega = 2\pi f = 2\pi * 7500 = 47124\frac{1}{s}$$

$$\underline{Z} = 100\Omega + \frac{1}{(j * 47124\frac{1}{s} * 230 * 10^{-9}F)} + (j * 47124\frac{1}{s} * 0,009H) = (100 + j331,85)\Omega$$

$$Z = |\underline{Z}| = \sqrt{(100^2 + 331,85^2)} = 346,59\Omega$$

$$Y = \frac{1}{Z} = 0,0028852S = 2,88mS$$

d)

$$2000Hz:$$

$$\underline{Y} = \frac{1}{R + j(\omega L - \frac{1}{\omega C_1})} + j\omega C_2\omega = 2\pi f = 2\pi * 2000 = 12566\frac{1}{s}$$

$$\underline{Y} = \frac{1}{100\Omega + j(12566\frac{1}{s} * 0,009H - \frac{1}{12566\frac{1}{s} * 230 * 10^{-9}F})} + j12566\frac{1}{s} * 66 * 10^{-9}F$$

$$= (0,0043877 - j0,0041330)S$$

$$Y = |\underline{Y}| = \sqrt{(0,0043877^2 + 0,0041330^2)} = 0,0060277S = 6,02mS$$

$$Z = \frac{1}{Y} = 165,9\Omega$$

$$6000Hz:$$

$$\underline{Y} = \frac{1}{R + j(\omega L - \frac{1}{\omega C_1})} + j\omega C_2 \quad \omega = 2\pi f = 2\pi * 6000 = 37699\frac{1}{s}$$

$$\underline{Y} = \frac{1}{100\Omega + j(37699\frac{1}{s} * 0,009H - \frac{1}{37699\frac{1}{s} * 230 * 10^{-9}F})} + j37699\frac{1}{s} * 66 * 10^{-9}F$$

$$= (0,00079924 - j0,00022361)S$$

$$\underline{Y} = |Y| = \sqrt{(0,00079924^2 + 0,00022361^2)} = 0,00082993S = 0,82mS$$

$$Z = \frac{1}{Y} = 1204,92\Omega$$

$$8500Hz:$$

$$\underline{Y} = \frac{1}{R + j(\omega L - \frac{1}{\omega C_1})} + j\omega C_2 \quad \omega = 2\pi f = 2\pi * 8500 = 53407\frac{1}{s}$$

$$\underline{Y} = \frac{1}{100\Omega + j(53407\frac{1}{s} * 0,009H - \frac{1}{53407\frac{1}{s} * 230 * 10^{-9}F})} + j53407\frac{1}{s} * 66 * 10^{-9}F$$

$$= (0,00041487 + j0,0015307)S$$

$$\underline{Y} = |Y| = \sqrt{(0,00041487^2 + 0,00041487^2)} = 0,0015859S = 1,58mS$$

$$Z = \frac{1}{Y} = 630,54\Omega$$