Assignment 4

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Download all python codes from

https://github.com/Digjoy12/Signal−Processing/blob/main/Assignment_4/Codes/linear_form.py

and latex codes from

https://github.com/Digjoy12/Signal−Processing/blob/main/Assignment_4/main.tex

Problem

(Linearforms - Q2.55) Prove that the function f(x) = 5x–3 is continuous at x = 0, at x = –3 and at x = 5.

Solution

A function f(x) is defined to be continuous at x = a if

$$\lim_{h\to 0} f(a+h) = f(a) = \lim_{h\to 0} f(a-h) \tag{0.0.1}$$

1) For x=0,

$$\begin{aligned} \lim_{h\to 0} f(0+h) &= \lim_{h\to 0} f(h) && (0.0.2)\\ &= \lim_{h\to 0} 5h-3 && (0.0.3)\\ &= -3 && (0.0.4) \end{aligned}$$

and,

$$\begin{aligned} \lim_{h\to 0} f(0-h) &= \lim_{h\to 0} f(-h) && (0.0.5)\\ &= \lim_{h\to 0} -5h-3 && (0.0.6)\\ &= -3 && (0.0.7) \end{aligned}$$

Since,

$$\lim_{h\to 0} f(0+h) = \lim_{h\to 0} f(0-h) = f(0) = -3 \tag{0.0.8}$$

Therefore, f(x) is continuous at x=0.

2) For x = -3,

$$\begin{aligned} \lim_{h\to 0} f(-3+h) &= \lim_{h\to 0} 5(-3+h)-3 && (0.0.9)\\ &= \lim_{h\to 0} -15+5h-3 && (0.0.10)\\ &= -18 && (0.0.11) \end{aligned}$$

and,

$$\begin{aligned} \lim_{h\to 0} f(-3-h) &= \lim_{h\to 0} 5(-3-h)-3 && (0.0.12)\\ &= \lim_{h\to 0} -15-5h-3 && (0.0.13)\\ &= -18 && (0.0.14) \end{aligned}$$

Since,

$$\lim_{h\to 0} f(-3+h) = \lim_{h\to 0} f(-3-h) = f(-3) = -18 \tag{0.0.15}$$

Therefore, f(x) is continuous at x=-3.

3) For x = 5,

$$\begin{aligned} \lim_{h\to 0} f(5+h) &= \lim_{h\to 0} 5(5+h)-3 && (0.0.16)\\ &= \lim_{h\to 0} 25+5h-3 && (0.0.17)\\ &= 22 && (0.0.18) \end{aligned}$$

and,

$$\begin{aligned} \lim_{h\to 0} f(5-h) &= \lim_{h\to 0} 5(5-h)-3 && (0.0.19)\\ &= \lim_{h\to 0} 25-5h-3 && (0.0.20)\\ &= 22 && (0.0.21) \end{aligned}$$

Since,

$$\lim_{h\to 0} f(5+h) = \lim_{h\to 0} f(5-h) = f(5) = 22 \tag{0.0.22}$$

Therefore, f(x) is continuous at x=5.

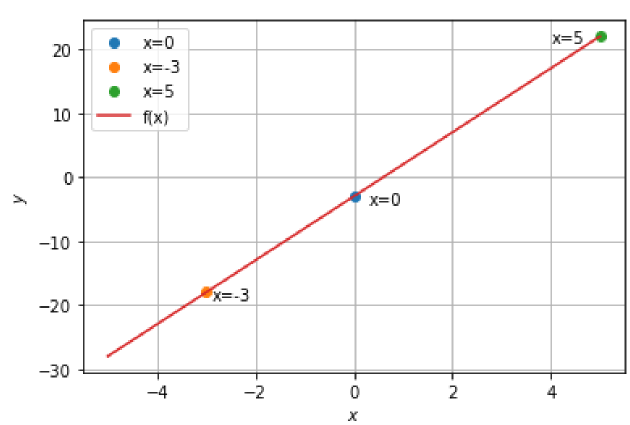

Fig. 3: Plot of the graph