Assignment 2

Digjoy Nandi - AI20BTECH11007

Download all python codes from

https://github.com/Digjoy12/Signal−Processing/tree/main/Assignment_2/Code

and latex codes from

https://github.com/Digjoy12/Signal−Processing/blob/main/Assignment_2/main.tex

Problem

(Matrix - Q2.60) Find the matrix $\mathbf{X}$ so that

$$\mathbf{X}\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix} = \begin{pmatrix}-7 & -8 & -9\\2 & 4 & 6\end{pmatrix}$$

Solution

Let,

$$\mathbf{A} = \begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix} \tag{0.0.1}$$

$$\mathbf{B} = \begin{pmatrix}-7 & -8 & -9\\2 & 4 & 6\end{pmatrix} \tag{0.0.2}$$

Now, multiplying $\mathbf{A}^\top$ on both sides,

$$\mathbf{X}\mathbf{A}\mathbf{A}^\top = \mathbf{B}\mathbf{A}^\top \tag{0.0.3}$$

$$\implies \mathbf{X} = \mathbf{B}\mathbf{A}^\top(\mathbf{A}\mathbf{A}^\top)^{-1} \tag{0.0.4}$$

Therefore,

$$\mathbf{X} = \begin{pmatrix}-7 & -8 & -9\\2 & 4 & 6\end{pmatrix}\begin{pmatrix}1 & 4\\2 & 5\\3 & 6\end{pmatrix}(\mathbf{A}\mathbf{A}^\top)^{-1} \tag{0.0.5}$$

$$= \begin{pmatrix}-50 & -122\\28 & 64\end{pmatrix}\left[\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix}\begin{pmatrix}1 & 4\\2 & 5\\3 & 6\end{pmatrix}\right]^{-1} \tag{0.0.6}$$

$$= \begin{pmatrix}-50 & -122\\28 & 64\end{pmatrix}\left[\begin{pmatrix}14 & 32\\32 & 77\end{pmatrix}\right]^{-1} \tag{0.0.7}$$

$$= \begin{pmatrix}-50 & -122\\28 & 64\end{pmatrix}\begin{pmatrix}\frac{77}{54} & \frac{-16}{27}\\\frac{-16}{27} & \frac{7}{27}\end{pmatrix} \tag{0.0.8}$$

$$= \begin{pmatrix}1 & -2\\2 & 0\end{pmatrix} \tag{0.0.9}$$

Alternate Solution:-
Given,

$$\mathbf{X}\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix}_{2\times 3} = \begin{pmatrix}-7 & -8 & -9\\2 & 4 & 6\end{pmatrix}_{2\times 3} \tag{0.0.10}$$

Therefore, $\mathbf{X}$ is a 2×2 matrix.
Let,

$$\mathbf{X} = \begin{pmatrix}a & b\\c & d\end{pmatrix} \tag{0.0.11}$$

Now our equation (0.0.10) becomes,

$$\begin{pmatrix}a & b\\c & d\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix} = \begin{pmatrix}-7 & -8 & -9\\2 & 4 & 6\end{pmatrix} \tag{0.0.12}$$

$$\implies \begin{pmatrix}a+4b & 2a+5b & 3a+6b\\c+4d & 2c+5d & 3c+6d\end{pmatrix} = \begin{pmatrix}-7 & -8 & -9\\2 & 4 & 6\end{pmatrix} \tag{0.0.13}$$

Since, the matrices are equal, therefore the corresponding elements are equal too.

$$a + 4b = -7 \tag{0.0.14}$$
$$2a + 5b = -8 \tag{0.0.15}$$
$$3a + 6b = -9 \tag{0.0.16}$$
$$c + 4d = 2 \tag{0.0.17}$$
$$2c + 5d = 4 \tag{0.0.18}$$
$$3c + 6d = 6 \tag{0.0.19}$$

Now, (0.0.14) and (0.0.15) are two equation with a and b variables, which can be expressed in vector form as

$$\begin{pmatrix}1 & 4\\2 & 5\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}-7\\-8\end{pmatrix} \tag{0.0.20}$$

The corresponding augmented matrix is

$$\left(\begin{array}{cc|c}1 & 4 & -7\\2 & 5 & -8\end{array}\right) \tag{0.0.21}$$

We use the Guass Jordan Elimination method as:

$$\left(\begin{array}{cc|c} 1 & 4 & -7 \\ 2 & 5 & -8 \end{array}\right) \tag{0.0.22}$$

$$\xleftrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & 4 & -7 \\ 0 & -3 & 6 \end{array}\right) \tag{0.0.23}$$

$$\xleftrightarrow{R_2 \to \frac{-1}{3} R_2} \left(\begin{array}{cc|c} 1 & 4 & -7 \\ 0 & 1 & -2 \end{array}\right) \tag{0.0.24}$$

$$\xleftrightarrow{R_1 \to R_1 - 4R_2} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -2 \end{array}\right) \tag{0.0.25}$$

Therefore, the values of a and b are:

$$a = 1 \tag{0.0.26}$$

$$b = -2 \tag{0.0.27}$$

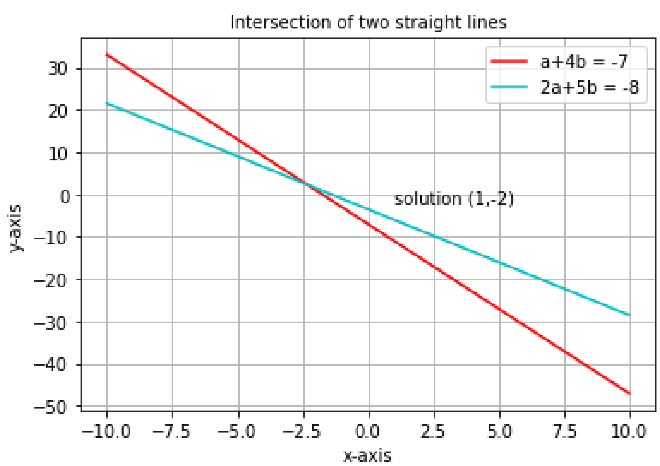

Fig. 0: Plot of the lines

Now, (0.0.14) and (0.0.15) are two equation with c and d variables, which can be expressed in vector form as

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{0.0.28}$$

The corresponding augmented matrix is

$$\left(\begin{array}{cc|c} 1 & 4 & 2 \\ 2 & 5 & 4 \end{array}\right) \tag{0.0.29}$$

We use the Guass Jordan Elimination method as:

$$\left(\begin{array}{cc|c} 1 & 4 & 2 \\ 2 & 5 & 4 \end{array}\right) \tag{0.0.30}$$

$$\xleftrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & -3 & 0 \end{array}\right) \tag{0.0.31}$$

$$\xleftrightarrow{R_2 \to \frac{-1}{3} R_2} \left(\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & 1 & 0 \end{array}\right) \tag{0.0.32}$$

$$\xleftrightarrow{R_1 \to R_1 - 4R_2} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \end{array}\right) \tag{0.0.33}$$

Therefore, the values of c and d are:

$$a = 2 \tag{0.0.34}$$

$$b = 0 \tag{0.0.35}$$

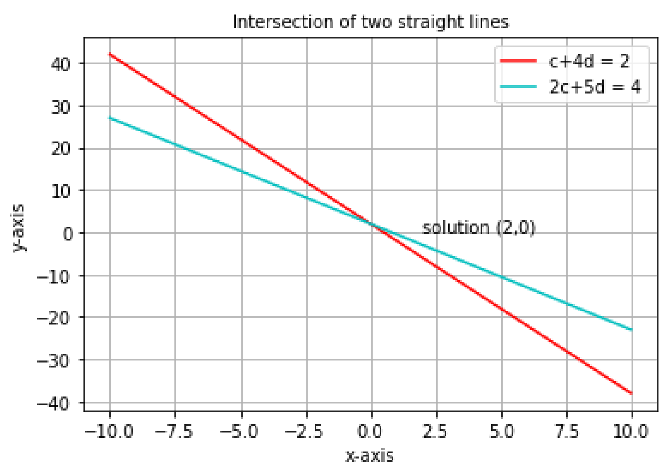

Fig. 0: Plot of the lines

Therefore, the matrix $\mathbf{X}$ is

$$\mathbf{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{0.0.36}$$

$$= \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix} \tag{0.0.37}$$