Quiz 2

Digjoy Nandi - AI20BTECH11007

Download all python codes from

https://github.com/Digjoy12/Signal−Processing/tree/main/Quiz_2/Codes

and latex codes from

https://github.com/Digjoy12/Signal−Processing/blob/main/Quiz_2/main.tex

Problem

(Q 3.20(a)) For each of the following pairs of input and output z-transforms X(z) and Y(z), determine the region of convergence for the system function H(z):

$$X(z) = \frac{1}{1-\frac{3}{4}z^{-1}},\ |z| > \frac{3}{4}$$

$$Y(z) = \frac{1}{1-\frac{2}{3}z^{-1}},\ |z| > \frac{2}{3}$$

Solution

Theorem 1 (Convolution Theorem)**.** *Let f and g be two functions with convolution $f * g$. Let F be the Fourier transform operator. Then*

$$F(f * g) = F(f) \cdot F(g) \tag{0.0.1}$$

$$F(f \cdot g) = F(f) * F(g) \tag{0.0.2}$$

For a LTI system, we know that,

$$y[n] = h[n] * x[n] \tag{0.0.3}$$

Using theorem 1,

$$Y(z) = H(z)X(z) \tag{0.0.4}$$

$$H(z) = \frac{Y(z)}{X(z)} \tag{0.0.5}$$

$$H(z) = \frac{1-\frac{3}{4}z^{-1}}{1-\frac{2}{3}z^{-1}} \tag{0.0.6}$$

Therefore,

$$\left|\frac{2}{3}z^{-1}\right| < 1 \tag{0.0.7}$$

$$\iff \left|\frac{2}{3z}\right| < 1 \tag{0.0.8}$$

$$\iff |z| > \frac{2}{3} \tag{0.0.9}$$

Hence, the ROC of H(z) is $|z| < \frac{2}{3}$.

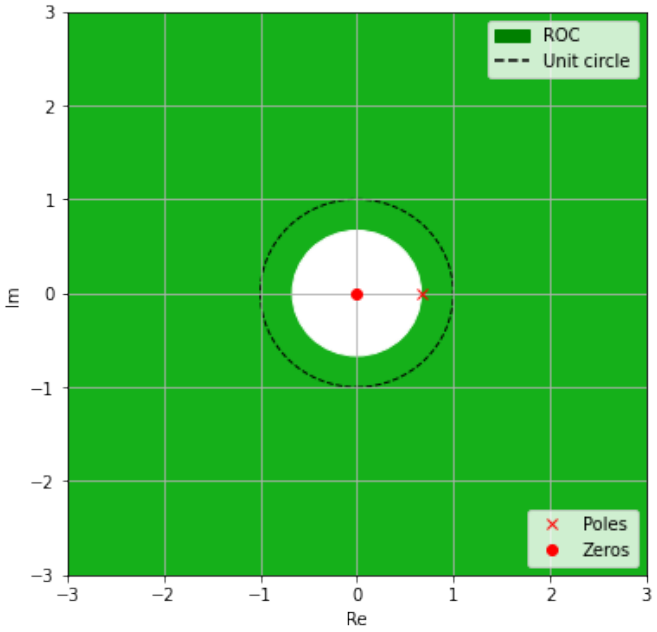

Fig. 0: ROC for H(z)