GATE Assignment 4

Digjoy Nandi - AI20BTECH11007

Download all latex codes from

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https://github.com/Digjoy12/Signal−Processing/
    blob/main/GATE_4/main.tex
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Problem

(GATE-EC 1999- Q 1.18) A signal x(t) has a Fourier transform x(ω). If x(t) is a real and odd function of t, then x(ω) is

(a) a real and even function of ω
(b) an imaginary and odd function of ω
(c) an imaginary and even function of ω
(d) a real and odd function of ω

Solution

Definition 1. **(Even function)**. For a real-valued function f(x) is even function when,

$$f(-x) = f(x) \tag{0.0.1}$$

for all values of x in the domain of f.

Definition 2. **(Odd function)**. For a real-valued function f(x) is odd function when,

$$f(-x) = -f(x) \tag{0.0.2}$$

for all values of x in the domain of f.

Corollary 0.1. The product of two odd function is even.
Let f(x) and g(x) be two odd function, then

$$f(x)g(x) = -f(x) - g(x) = f(x)g(x) \tag{0.0.3}$$

Corollary 0.2. The product of one odd and one even function is odd.
Let f(x) be an even and g(x) be an odd function, then

$$f(x)g(x) = f(x) - g(x) = -f(x)g(x) \tag{0.0.4}$$

The Fourier transform of real x(t) is given by

$$\mathcal{F}[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\exp(-i\omega x)dx \tag{0.0.5}$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)(\cos(\omega x) - i\sin(\omega x))dx \tag{0.0.6}$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cos(\omega x)dx - \frac{i}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\sin(\omega x)dx \tag{0.0.7}$$

Here,
$\cos(\omega x)$ is even function and f(x) is an odd function, therefore using (0.1),

$$f(x)\cos(\omega x) \text{ is an odd function.} \tag{0.0.8}$$

Again,
$\sin(\omega x)$ is odd function and f(x) is an odd function, therefore using (0.2),

$$f(x)\sin(\omega x) \text{ is an even function} \tag{0.0.9}$$

Now, equation (0.0.7) can be written as,

$$\mathcal{F}[f(x)] = \frac{2i}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\sin(\omega x)dx \tag{0.0.10}$$

$$= i\sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} f(x)\sin(\omega x)dx \tag{0.0.11}$$

Therefore, Fourier transform of real odd function is **imaginary**.

Now, The Fourier transform of an odd function is

$$\mathcal{F}[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\exp(-i\omega x)dx \tag{0.0.12}$$

Substituting f(x) for f(x) yields:

$$\mathcal{F}[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(-x) \exp(-i\omega x) dx \tag{0.0.13}$$

Substituting u for x and du for dx yields:

$$\mathcal{F}[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{u=\infty}^{u=-\infty} -f(x) \exp(-i\omega(-x)) dx \tag{0.0.14}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \exp(-i\omega(-x)) dx \tag{0.0.15}$$

$$= \mathcal{F}(-x) \tag{0.0.16}$$

Hence, Fourier transform of real odd function is **odd**.
Example of real and odd function of x(t) is

$$x(t) = A \sin \omega_0 t \tag{0.0.17}$$

then Fourier transform of x(t) is given by

$$\mathcal{F}[x(t)] = AJ\pi[\delta(\omega + \omega_0 - \delta(\omega + \omega_0)] \tag{0.0.18}$$

which is an imaginary and odd function.
Hence, **the correct option is (b)**.