

Assignment 2

Digjoy Nandi - AI20BTECH11007

Download all python codes from

https://github.com/Digjoy12/Signal-Processing/tree/main/Assignment_2/Code

and latex codes from

https://github.com/Digjoy12/Signal-Processing/blob/main/Assignment_2/main.tex

PROBLEM

(Matrix - Q2.60) Find the matrix \mathbf{X} so that

$$\mathbf{X} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$

SOLUTION

Given,

$$\mathbf{X} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}_{2 \times 3} \quad (0.0.1)$$

Therefore, \mathbf{X} is a 2×2 matrix.

Let,

$$\mathbf{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad (0.0.2)$$

Now our equation (0.0.1) becomes,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix} \quad (0.0.3)$$

$$\Rightarrow \begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix} \quad (0.0.4)$$

Since, the matrices are equal, therefore the corresponding elements are equal too.

$$a + 4b = -7 \quad (0.0.5)$$

$$2a + 5b = -8 \quad (0.0.6)$$

$$3a + 6b = -9 \quad (0.0.7)$$

$$c + 4d = 2 \quad (0.0.8)$$

$$2c + 5d = 4 \quad (0.0.9)$$

$$3c + 6d = 6 \quad (0.0.10)$$

Now, (0.0.5) and (0.0.6) are two equations with a and b variables, which can be expressed in vector form as

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -7 \\ -8 \end{pmatrix} \quad (0.0.11)$$

The corresponding augmented matrix is

$$\left(\begin{array}{cc|c} 1 & 4 & -7 \\ 2 & 5 & -8 \end{array} \right) \quad (0.0.12)$$

We use the Gauss Jordan Elimination method as:

$$\left(\begin{array}{cc|c} 1 & 4 & -7 \\ 2 & 5 & -8 \end{array} \right) \quad (0.0.13)$$

$$\xleftrightarrow{R_2 \rightarrow R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & 4 & -7 \\ 0 & -3 & 6 \end{array} \right) \quad (0.0.14)$$

$$\xleftrightarrow{R_2 \rightarrow -\frac{1}{3}R_2} \left(\begin{array}{cc|c} 1 & 4 & -7 \\ 0 & 1 & -2 \end{array} \right) \quad (0.0.15)$$

$$\xleftrightarrow{R_1 \rightarrow R_1 - 4R_2} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -2 \end{array} \right) \quad (0.0.16)$$

Therefore, the values of a and b are:

$$a = 1 \quad (0.0.17)$$

$$b = -2 \quad (0.0.18)$$

Now, (0.0.8) and (0.0.9) are two equations with c and d variables, which can be expressed in vector form as

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (0.0.19)$$

The corresponding augmented matrix is

$$\left(\begin{array}{cc|c} 1 & 4 & 2 \\ 2 & 5 & 4 \end{array} \right) \quad (0.0.20)$$

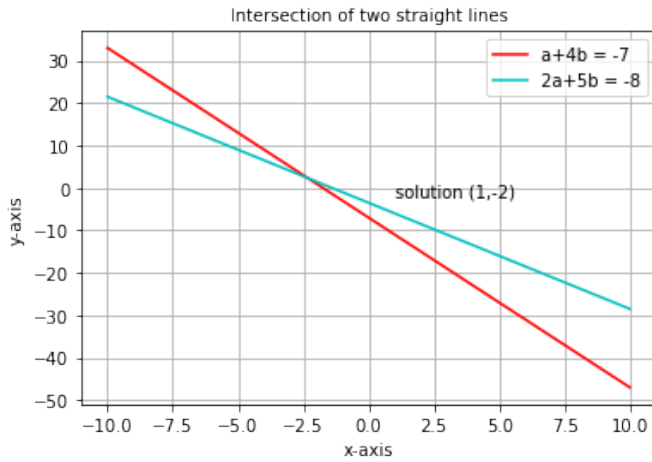


Fig. 0: Plot of the lines

Therefore, the matrix \mathbf{X} is

$$\mathbf{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad (0.0.27)$$

$$= \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix} \quad (0.0.28)$$

We use the Gauss Jordan Elimination method as:

$$\left(\begin{array}{cc|c} 1 & 4 & 2 \\ 2 & 5 & 4 \end{array} \right) \quad (0.0.21)$$

$$\xleftrightarrow{R_2 \rightarrow R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & -3 & 0 \end{array} \right) \quad (0.0.22)$$

$$\xleftrightarrow{R_2 \rightarrow -\frac{1}{3}R_2} \left(\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & 1 & 0 \end{array} \right) \quad (0.0.23)$$

$$\xleftrightarrow{R_1 \rightarrow R_1 - 4R_2} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \end{array} \right) \quad (0.0.24)$$

Therefore, the values of c and d are:

$$a = 2 \quad (0.0.25)$$

$$b = 0 \quad (0.0.26)$$

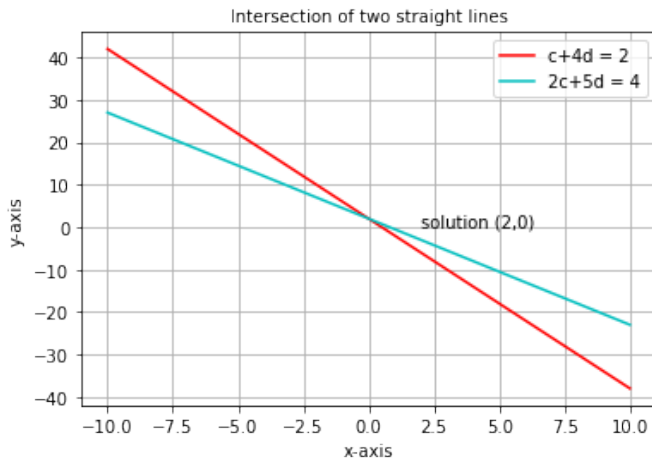


Fig. 0: Plot of the lines