# GATE Assignment

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Download all python codes from

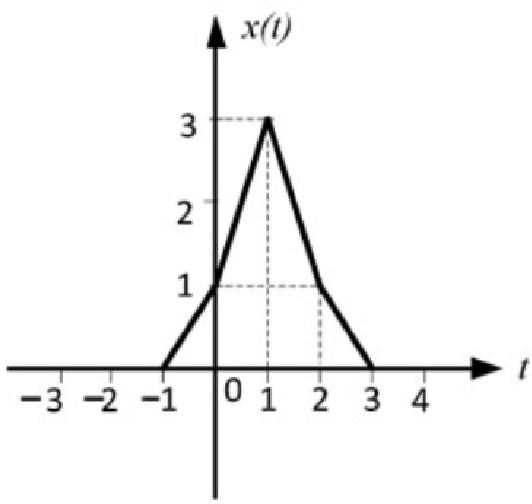
```
https://github.com/Digjoy12/Signal-Processing/
    blob/main/Assignment%202-GATE/Codes/
    code.py
```

and latex codes from

```
https://github.com/Digjoy12/Signal-Processing/
    blob/main/Assignment%202-GATE/main.tex
```

## Problem

**(GATE EC 2020 - Q52)** X($\omega$) is the Fourier Transform of x(t) shown below. The value of $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ (**rounded off to two decimal places**) is ..................




## Solution

By Parseval's theorem, we know that

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega \tag{0.0.1}$$

$$\implies 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega \tag{0.0.2}$$

$$\implies 2\pi \int_{-\infty}^{\infty} x(t)^2 dt = \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega \tag{0.0.3}$$

[*Since, x(t) is real*]

Since, shifting of a signal does not change the energy of the signal, therefore left shifting the signal by 1, we get the signal as y(t) = x(t+1)

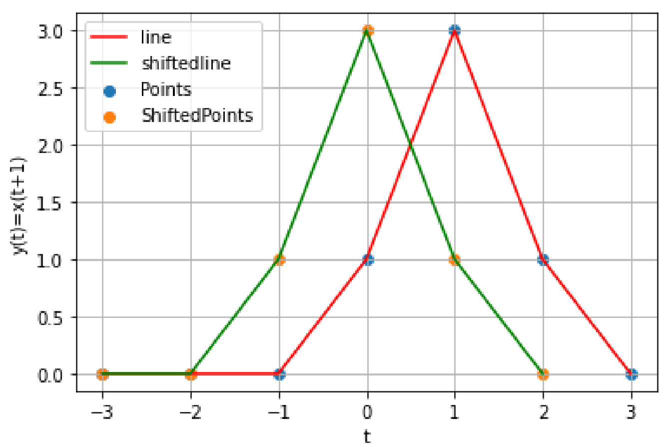



Fig. 0: Graphical Transformation

Now, y(t) is an even function since it is symmetric around the origin.
Therefore $y^2(t)$ is also an even signal.

$$\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} y(t)^2 dt \tag{0.0.4}$$

$$= 4\pi \int_{0}^{\infty} y(t)^2 dt \tag{0.0.5}$$

$$= 4\pi \left[ \int_{0}^{1} y(t)^2 dt + \int_{1}^{2} y(t)^2 dt \right] \tag{0.0.6}$$

For t=0 to t=1,

$$y(t) = x(t+1) = -2t + 3$$

For t=1 to t=2,

$$y(t) = x(t+1) = -t + 2$$

Therefore,

$$\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 4\pi \left[ \int_{0}^{1} y(t)^2 dt + \int_{1}^{2} y(t)^2 dt \right] \tag{0.0.7}$$

$$= 4\pi \left[ \int_{0}^{1} (-2t+3)^2 dt + \int_{1}^{2} (-t+2)^2 dt \right] \tag{0.0.8}$$

$$= 4\pi \left[ \int_{0}^{1} (4t^2 - 12t + 9) dt + \int_{1}^{2} (t^2 - 4t + 4) dt \right] \tag{0.0.9}$$

$$= 4\pi \left[ \left\{ \frac{4t^3}{3} - 6t^2 + 9t \right\}_0^1 + \left\{ \frac{t^3}{3} - 2t^2 + 4t \right\}_1^2 \right] \tag{0.0.10}$$

$$= 4\pi \left[ \frac{13}{3} + \frac{1}{3} \right] \tag{0.0.11}$$

$$= 4\pi \left[ \frac{14}{3} \right] \tag{0.0.12}$$

$$= 58.61\% \tag{0.0.13}$$

Hence,

$$\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \mathbf{58.61\%}$$