Quiz 1

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Download all python codes from

https://github.com/Digjoy12/Signal−Processing/blob/main/Quiz_1/Codes/quiz_1_code.py

and latex codes from

https://github.com/Digjoy12/Signal−Processing/blob/main/Quiz_1/main.tex

Problem

(Q 2.7) Determine whether each of the following signals is periodic. If the signal is periodic, state its period.

$$(a) x[n] = \exp(j(\pi n/6))$$
$$(b) x[n] = \exp(j(3\pi n/4))$$
$$(c) x[n] = [sin(\pi n/5)]/(\pi n)$$

Solution

We know that, x[n] is periodic with period N if x[n] = x[n+N] for some integer N.

1) Let,

$$x[n] = x[n+N] \tag{0.0.1}$$
$$\implies \exp\left(\frac{j(\pi n)}{6}\right) = \exp\left(\frac{j(\pi(n+N))}{6}\right) \tag{0.0.2}$$

Now,

$$\exp\left(\frac{j(\pi(n+N))}{6}\right) = \exp\left(j(\frac{\pi}{6}n + 2\pi k)\right) \tag{0.0.3}$$
$$\implies 2\pi k = \frac{\pi}{6}N, \text{for integers k,N} \tag{0.0.4}$$
$$\implies N = 12k \tag{0.0.5}$$

Hence, for k = 1, x[n] is a **periodic function** which have a period of 12.

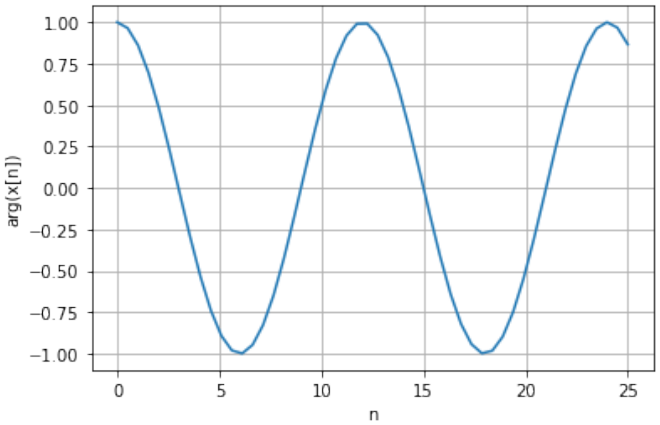

Fig. 1: Plot of x[n] = exp($j(\pi n/6)$)

2) Let,

$$x[n] = x[n+N] \tag{0.0.6}$$
$$\implies \exp j(\frac{3\pi n}{4}) = \exp j(\frac{3\pi}{4})(n+N) \tag{0.0.7}$$

Now,

$$\exp j(\frac{3\pi}{4})(n+N) = \exp j(\frac{3\pi n}{4} + 2\pi k) \tag{0.0.8}$$
$$\implies 2\pi k = \frac{3\pi}{4}N, \text{for integers k,N} \tag{0.0.9}$$
$$\implies N = \frac{8}{3}k \tag{0.0.10}$$

Hence, for k = 3, x[n] is a **periodic function** which have a period of 8.

3) Let,

$$x[n] = x[n+N] \tag{0.0.11}$$
$$\frac{[sin(\pi n/5)]}{\pi n} = \frac{[sin(\pi(n+N)]}{\pi(n+N)} \tag{0.0.12}$$
$$= \frac{[sin(\pi n/5 + N/5)]}{\pi n + \pi N} \tag{0.0.13}$$

Since, the denominator term is linear in n. Hence, x[n] is **not a periodic function**.

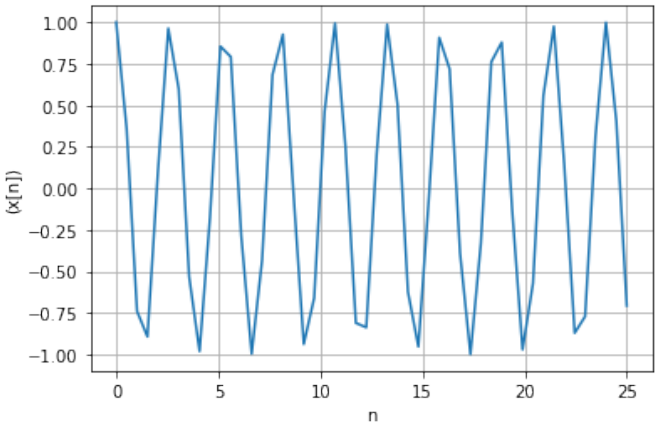

Fig. 2: Plot of x[n] = exp($j(3\pi n/4)$)

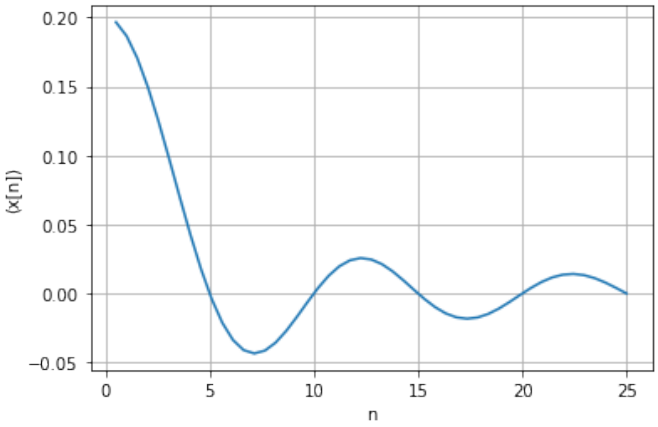

Fig. 3: Plot of x[n] = [sin($\pi n/5$)]/(πn)