

# Assignment 4

Digjoy Nandi - AI20BTECH11007

Download all python codes from

[https://github.com/Digjoy12/Signal-Processing/blob/main/Assignment\\_4/Codes/linear\\_form.py](https://github.com/Digjoy12/Signal-Processing/blob/main/Assignment_4/Codes/linear_form.py)

and latex codes from

[https://github.com/Digjoy12/Signal-Processing/blob/main/Assignment\\_4/main.tex](https://github.com/Digjoy12/Signal-Processing/blob/main/Assignment_4/main.tex)

## PROBLEM

**(Ramsey/1.2 Loci - Q5)** A point moves so that the sum of the squares of its distance from the points  $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$  and  $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$  is constant. Find the equation of the locus.

## SOLUTION

Let  $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$  be the point and  $\mathbf{A} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ .

Since, the sum of the squares of its distance from the points  $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$  and  $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$  is constant.

Therefore,

$$\mathbf{x}\mathbf{A} = \|\mathbf{x} - \mathbf{A}\|^2 \quad (0.0.1)$$

$$\mathbf{x}\mathbf{B} = \|\mathbf{x} - \mathbf{B}\|^2 \quad (0.0.2)$$

Now,

$$\Rightarrow \mathbf{x}\mathbf{A}^2 + \mathbf{x}\mathbf{B}^2 = c \quad (0.0.3)$$

$$\Rightarrow \|\mathbf{x} - \mathbf{A}\|^2 + \|\mathbf{x} - \mathbf{B}\|^2 = c \quad (0.0.4)$$

$$\begin{aligned} \Rightarrow \|\mathbf{x}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{x}^\top \mathbf{A} + \\ \|\mathbf{x}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{x}^\top \mathbf{B} = c \end{aligned} \quad (0.0.5)$$

$$\begin{aligned} \Rightarrow \|\mathbf{x}\|^2 + \|\mathbf{A}\|^2 - 2 \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \\ \|\mathbf{x}\|^2 + \|\mathbf{B}\|^2 - 2 \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = c \end{aligned} \quad (0.0.6)$$

$$\begin{aligned} \Rightarrow \|\mathbf{x}\|^2 + \mathbf{A}^\top \mathbf{A} - 2 \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \\ \|\mathbf{x}\|^2 + \mathbf{B}^\top \mathbf{B} - 2 \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = c \end{aligned} \quad (0.0.7)$$

$$\begin{aligned} \Rightarrow \|\mathbf{x}\|^2 + \begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} - 2 \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \\ \|\mathbf{x}\|^2 + \begin{pmatrix} 4 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} - 2 \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = c \end{aligned} \quad (0.0.8)$$

$$\Rightarrow 2\|\mathbf{x}\|^2 - 2 \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 7 \\ 7 \end{pmatrix} + 50 = c \quad (0.0.9)$$

$$\Rightarrow \|\mathbf{x}\|^2 - \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 7 \\ 7 \end{pmatrix} + 25 = c' \quad (0.0.10)$$

Here  $c' = c/2$

$$\Rightarrow \|\mathbf{x}\|^2 - \mathbf{x}^\top (\mathbf{A} + \mathbf{B}) + \|\mathbf{A}\|^2 = c' \quad (0.0.11)$$

Since,  $\mathbf{A} + \mathbf{B} = \begin{pmatrix} 7 \\ 7 \end{pmatrix}$  and  $\|\mathbf{A}\|^2 = \|\mathbf{B}\|^2 = 25$ .

Therefore, the equation of the locus is

$$\Rightarrow \|\mathbf{x}\|^2 - \mathbf{x}^\top (\mathbf{A} + \mathbf{B}) + \|\mathbf{A}\|^2 = c'$$

or,

$$\Rightarrow \|\mathbf{x}\|^2 - \mathbf{x}^\top (\mathbf{A} + \mathbf{B}) + \|\mathbf{B}\|^2 = c'$$