Assignment 3

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Download all python codes from

```
https://github.com/Digjoy12/probability/tree/main/
    Assignment%203/code
```

and latex codes from

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https://github.com/Digjoy12/probability/blob/main/
    Assignment%203/main.tex
```

1 Problem

(Gate MA - 2016 Q.47) Let X_1 be an exponential random variable with mean 1 and X_2 a gamma random variable with mean 2 and variance 2. If X_1 and X_2 are independently distributed, then $\Pr(X_1 < X_2)$ is equal to ....

2 Solution

We know that,

$$f(X_1) = \begin{cases} 0 & x < 0 \\ \frac{\exp\left(\frac{-X_1}{\beta}\right)}{\beta} & 0 \leq x < \infty \end{cases} \tag{2.0.1}$$

Given, $E(X_1) = \beta = 1$
Therefore,

$$f(X_1) = \begin{cases} 0 & x < 0 \\ \exp(-X_1) & 0 \leq x < \infty \end{cases} \tag{2.0.2}$$

We know that,

$$f(X_2) = \begin{cases} 0 & x < 0 \\ \frac{X_2^{\alpha-1}\exp\left(\frac{-X_2}{\beta}\right)}{\beta^{\alpha}\Gamma(\alpha)} & 0 \leq x < \infty \end{cases} \tag{2.0.3}$$

Given,

$$E(X_2) = \alpha\beta = 2 \tag{2.0.4}$$

$$V(X_2) = \alpha\beta^2 = 2 \tag{2.0.5}$$

Solving 2.0.4 and 2.0.5, we get, $\alpha = 2$, $\beta = 1$ and $\Gamma(2) = 1$
Therefore,

$$f(X_2) = \begin{cases} 0 & x < 0 \\ X_2\exp(-X_2) & 0 \leq x < \infty \end{cases} \tag{2.0.6}$$

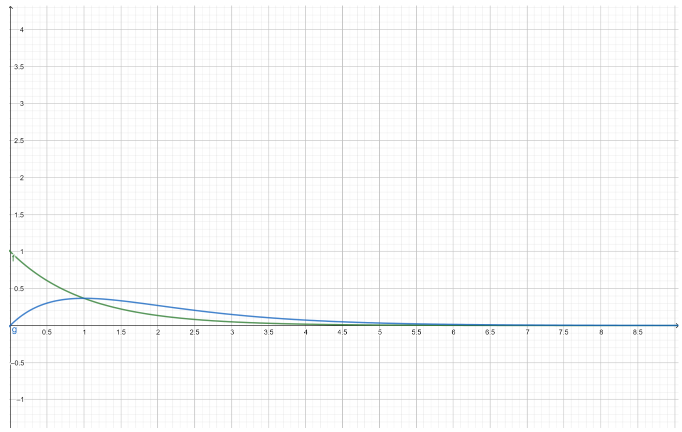

Fig. 0: Probability Distribution of (X_1, X_2)

Alternately, we have CDF of X_1 and X_2 given by

$$F_{X_1}(x) = \begin{cases} 0 & x < 0 \\ 1 - \exp(-x) & 0 \leq x < \infty \end{cases} \tag{2.0.7}$$

$$F_{X_2}(x) = \begin{cases} 0 & x < 0 \\ 1 - (x+1)\exp(-x) & 0 \leq x < \infty \end{cases} \tag{2.0.8}$$

Thus

$$\Pr(X_1 \leq X_2) = \int_{-\infty}^{\infty} F_{X_1}(x) f(X_2) dX \tag{2.0.9}$$

$$= \int_{0}^{\infty} -\exp(-x)x\exp(-x)dx \tag{2.0.10}$$

$$= \frac{3}{4} \tag{2.0.11}$$