

Density functional theory (DFT)

Modules: `dft`, `pbc.dft`

Introduction

Kohn-Sham density functional theory (KS-DFT) has been implemented through derived classes of the `pyscf.scf.hf.SCF`

[../pyscf_api_docs/pyscf.scf.html#pyscf.scf.hf.SCF] parent class. As such, the methods and capabilities introduced in [Self-consistent field \(SCF\) methods](#) [scf.html#user-scf] are also available to the `dft` module, e.g., the efficient second-order Newton-Raphson algorithm.

A minimal example of using the `dft` module reads, cf. [dft/22-newton.py](#)

[<https://github.com/pyscf/pyscf/blob/master/examples/dft/22-newton.py>]:

```
>>> from pyscf import gto, dft
>>> mol_hf = gto.M(atom = 'H 0 0 0; F 0 0 1.1', basis = 'ccpvdz',
symmetry = True)
>>> mf_hf = dft.RKS(mol_hf)
>>> mf_hf.xc = 'lda,vwn' # default
>>> mf_hf = mf_hf.newton() # second-order algorithm
>>> mf_hf.kernel()
```

This will run a restricted, closed-shell Kohn-Sham DFT calculation with the default LDA functional.

Theory

In KS-DFT, as first proposed by Kohn and Sham [1] [#kohsha1965], the electron density of a reference noninteracting system is used to represent the density of the true interacting system. As a result, the computational formulation of KS-DFT resembles that of Hartree-Fock (HF) theory, but with a different effective Fock potential. In KS-DFT, the total electronic energy is defined as follows:

$$E = T_s + E_{\text{ext}} + E_J + E_{\text{xc}} .$$

Here, T_s is the noninteracting kinetic energy, E_{ext} is the energy due to the external potential, E_J is the Coulomb energy, and E_{xc} is the exchange-correlation (xc) energy. In practice, E_{xc} is approximated by a density functional approximation, which themselves may be divided into several classes along different rungs of Jacob's ladder [2] [#perdew-jacobs-ladder-aip-conf-proc-2001]:

- local density approximations (e.g. LDA; xc energy depends only on the electron density, ρ),
- generalized gradient approximations (GGA; xc energy also depends on the density gradient, $|\nabla\rho|$),
- meta-GGAs (xc energy also depends on the kinetic energy density and/or the density Laplacian, $\sum_i |\nabla\psi_i|^2$, $\nabla^2\rho$; the latter is not supported in PySCF at the moment),
- non-local correlation functionals (xc energy involves a double integral)
- hybrid density functionals (a fraction of exact exchange is used), and
- long-range corrected density functionals (exact exchange is used with a modified interaction kernel)

Variationally minimizing the total energy with respect to the density yields the KS equations for the non-interacting reference orbitals, on par with HF theory, and these have the same general form as the Fock equations in `theory.scf`. However, the exact exchange, \hat{K} , is replaced by the xc potential, $\hat{v}_{\text{xc}} = \delta E_{\text{xc}} / \delta \rho$. For hybrid and meta-GGA calculations, PySCF uses the generalized KS formalism [3] [#gks], in which the so-called generalized KS equations minimize the total energy with respect to the orbitals themselves.

Predefined `xc` Functionals and Functional Aliases

The choice of `xc` functional is assigned via the attribute `DFT.xc`. This is a comma separated string (precise grammar discussed [below](#) [`#user-dft-custom-func`], e.g., `xc = 'pbe, pbe'` denotes PBE exchange plus PBE correlation. In common usage, a single name (alias) is often used to refer to the combination of a particular exchange and correlation approximation instead. To support this, PySCF will first examine a lookup table to see if `DFT.xc` corresponds to a common compound name, and if so, the implementation dispatches to the appropriate exchange and correlation forms, e.g., `xc = 'pbe'` directly translates to `xc = 'pbe, pbe'`. However, if the name is not found in the compound functional table, and only a single string is given, it will be treated as an exchange functional only, e.g., `xc = 'b86'` leads to B86 exchange only (without correlation). Please note that earlier PySCF versions (1.5.0 or earlier) did not support compound functional aliases, and both exchange and correlation always had to be explicitly assigned.

PySCF supports two independent libraries of `xc` functional implementations, namely [Libxc](https://www.tddft.org/programs/libxc/) [<https://www.tddft.org/programs/libxc/>] and [XCFun](https://xcfun.readthedocs.io/en/latest/) [<https://xcfun.readthedocs.io/en/latest/>]. The former of these is the default, but the latter may be selected by setting `DFT._numint.libxc = dft.xcfun`, cf. [dft/32-xcfun_as_default.py](#) [https://github.com/pyscf/pyscf/blob/master/examples/dft/32-xcfun_as_default.py]. For complete lists of the available density functional approximations, the user is referred to the `XC_CODES` dictionaries in [pyscf/dft/libxc.py](#) [<https://github.com/pyscf/pyscf/blob/master/pyscf/dft/libxc.py>] and [pyscf/dft/xcfun.py](#) [<https://github.com/pyscf/pyscf/blob/master/pyscf/dft/xcfun.py>], respectively. The user can choose the library at runtime in order to leverage any of its exclusive features.

The constant maintenance and development of density functional libraries is hard work at little personal benefit, while everyone benefits from having a huge variety of density functionals in numerically stable form for use in applications. If you use Libxc in your calculations, please cite the most up-to-date work on Libxc in your

paper. You can see the most up-to-date citation on the [Libxc web page](https://www.tddft.org/programs/libxc/) [https://www.tddft.org/programs/libxc/]; at the moment, this is [4] [#lehtola-libxc-softwarex-2018]. Likewise, if you use XCFun in your calculations, please cite the most up-to-date work on XCFun in your paper. You can find recent citations on the [XCFun web page](https://github.com/dftlibs/xcfun/) [https://github.com/dftlibs/xcfun/]; at present, this is [5] [#ekstroem-xcfun-jctc-2010]. Please check your log files for the library used in your calculation (you may need to increase the `DFT.verbose` setting of your calculation to see this).

Customizing xc functionals

The xc functional of choice can be customized. The simplest way to customize to achieve this is to assign a string expression to the `DFT.xc` attribute:

```
>>> HF_X, LDA_X = .6, .08
>>> B88_X = 1. - HF_X - LDA_X
>>> LYP_C = .81
>>> VWN_C = 1. - LYP_C
>>> mf_hf.xc = f'{HF_X:} * HF + {LDA_X:} * LDA + {B88_X:} * B88,
{LYP_C:} * LYP + {VWN_C:} * VWN'
>>> mf_hf.kernel()
>>> mf_hf.xc = 'hf'
>>> mf_hf.kernel()
```

The XC functional string is parsed against a set of rules, as described below.

- The given functional description must be a one-line string
- The functional description is case-insensitive
- The functional description string has two parts, separated by a `,`. The first part describes the exchange functional, the second part sets the correlation functional (as for [aliases](#) [#user-dft-predef-func])
 - If a `,` does not appear in the string, the entire string is treated as the name of a compound functional (containing both the exchange and the correlation functional) which should be in the list of functional aliases.

Again, if the string is not found in the aliased functional list, it is treated as an exchange functional

- To input only an exchange functional (without a correlation functional), one should leave the second part blank. E.g., `slater`, implies a functional with the LDA contribution only
- Correspondingly, to neglect the contribution of the exchange functional (i.e. to just use a correlation functional), one should leave the first part blank, e.g., `' , vwn '` means a functional with VWN only
- If a compound xc functional is specified, no matter whether it is in the exchange part (the string in front of the comma) or the correlation part (the string behind the comma), both exchange and correlation functionals of the compound xc functional will be used
- The functional name can be placed in an arbitrary order. Two names need be separated by operators `+` or `-`. Blank spaces are ignored. NOTE the parser only reads the operators `+`, `-`, `*`, while `/` is not supported
- A functional name can have at most one factor. If a factor is not given, it is set to 1. Compound functionals can be scaled as a unit. For example, `.5 * b3lyp` is equivalent to `.1 * HF + .04 * LDA + .36 * B88, .405 * LYP + .095 * VWN`
- The string `HF` stands for exact exchange (HF K matrix). `HF` can be put in the correlation functional part (after the comma). Putting `HF` in the correlation part is the same as putting `HF` in the exchange part
- The special string `RSH` means a range-separated operator. Its format is `RSH(omega, alpha, beta)`. Another way to input range separation is to use keywords `SR_HF` and `LR_HF`, e.g., `SR_HF(.1) * alpha_plus_beta` and `LR_HF(.1) * alpha` where the number in the parenthesis is the value of `omega`
- The `RSH` kernel in PySCF is based on the error function kernel; Yukawa kernels are not supported at present
- One need in general be careful with the Libxc convention of GGA functionals, in which the LDA contribution is included

For completeness, it's worth mentioning that yet another way to customize xc functionals exists, which uses the `eval_xc()` method of the numerical integral class:

```
>>> def eval_xc(xc_code, rho, spin=0, relativity=0, deriv=1,
    verbose=None):
>>>     # A fictitious functional to demonstrate the usage
>>>     rho0, dx, dy, dz = rho
>>>     gamma = (dx ** 2 + dy ** 2 + dz ** 2)
>>>     exc = .01 * rho0 ** 2 + .02 * (gamma + .001) ** .5
>>>     vrho = .01 * 2 * rho0
>>>     vgamma = .02 * .5 * (gamma + .001) ** (-.5)
>>>     vlapl = None
>>>     vtau = None
>>>     vxc = (vrho, vgamma, vlapl, vtau)
>>>     fxc = None # 2nd-order functional derivative
>>>     kxc = None # 3rd-order functional derivative
>>>     return exc, vxc, fxc, kxc
>>> dft.libxc.define_xc_(mf_hf._numint, eval_xc, xctype='GGA')
>>> mf_hf.kernel()
```

By calling the `dft.libxc.define_xc_()` function, the customized `eval_xc()` function is patched to the numerical integration class `DFT._numint` dynamically.

For more examples of DFT xc functional customization, cf. [dft/24-custom_xc_functional.py](#)

[https://github.com/pyscf/pyscf/blob/master/examples/dft/24-custom_xc_functional.py] and [dft/24-define_xc_functional.py](#)

[https://github.com/pyscf/pyscf/blob/master/examples/dft/24-define_xc_functional.py].

Numerical integration grids

PySCF implements several numerical integration grids, which can be tuned in KS-DFT calculations following the examples in [dft/11-grid_scheme.py](#)

[https://github.com/pyscf/pyscf/blob/master/examples/dft/11-grid_scheme.py].

For instance, predefined grids (identical to those used in [TURBOMOLE](#)

[<https://www.turbomole.org/>]) may be set by using levels from 0 (very sparse) to 9 (very dense), with a default values of 3, cf. [pyscf/dft/gen_grid.py](#) [https://github.com/pyscf/pyscf/blob/master/pyscf/dft/gen_grid.py] for more details. Likewise, the default integration grids use Bragg radii for atoms, Treutler-Ahlrichs radial grids, Becke partitioning for grid weights, the pruning scheme of NWChem, and mesh grids, which are all setting that may be overwritten:

```
>>> mf_hf.grids.level = 5
>>> mf_hf.radi_method = dft.gauss_chebeshev
>>> mf_hf.grids.prune = None # disabling pruning of grids near core regions
```

In addition, these grids can be used for the general numerical evaluation of basis functions, electron densities, and integrals. Some examples of these functionalities can be found in [dft/30-ao_value_on_grid.py](#)

[https://github.com/pyscf/pyscf/blob/master/examples/dft/30-ao_value_on_grid.py] and [dft/31-xc_value_on_grid.py](#)

[https://github.com/pyscf/pyscf/blob/master/examples/dft/31-xc_value_on_grid.py]. For instance, the electron density may be readily obtained:

```
>>> mf_hf.xc = 'b3lyp'
>>> coords = mf_hf.grids.coords
>>> weights = mf_hf.grids.weights
>>> ao_value = numint.eval_ao(mol_hf, coords, deriv=1) # AO value and its gradients
>>> rho = numint.eval_rho(mol_hf, ao_value, dm, xctype='GGA') # density & density gradients
```

From `rho`, the energy density and xc potential can be computed by calling into `dft.libxc.eval_xc()`.

A more specialized example is the following on computing the kinetic energy from the nonnegative kinetic energy density according to the formulas:

$$t_s(\mathbf{r}) = \frac{1}{2} \sum_{i \in occ} |\nabla \psi_i(\mathbf{r})|^2,$$

$$T_s = \int d\mathbf{r} t_s(\mathbf{r}) .$$

In PySCF, the code boils down to:

```
>>> import numpy as np
>>> occ_orbs = mf_hf.mo_coeff[:, mf_hf.mo_occ > 0.]
>>> grids = dft.gen_grid.Grids(mol_hf)
>>> grids.build(with_non0tab=True)
>>> weights = grids.weights
>>> ao1 = dft.numint.eval_ao(mol_hf, grids.coords, deriv=1,
non0tab=grids.non0tab)
>>> ts = 0.5 * np.einsum('xgp,pi,xgq,qi->g', ao1[1:], occ_orbs,
ao1[1:], occ_orbs)
>>> Ts = np.einsum('g,g->', weights, ts)
```

or - as an alternative - the same may be achieved in the following way:

```
>>> Ts_ao = mol_hf.intor('int1e_kin')
>>> Ts_analyt = np.einsum('ui,uv,vi->', occ_orbs, Ts_ao, occ_orbs)
```

Dispersion corrections

Two main ways exist for adding dispersion (van der Waals) corrections to KS-DFT calculations. One is to augment mean-field results by Grimme's D3 corrections [6] [#dftd3], which can be added through an interface to the external library `libdftd3` [https://github.com/cuanto/libdftd3], cf. `dftd3/00-hf_with_dftd3.py` [https://github.com/pyscf/pyscf/blob/master/examples/dftd3/00-hf_with_dftd3.py]:

```
>>> from pyscf import dftd3
>>> mf_hf_d3 = dftd3.dftd3(dft.RKS(mol_hf))
>>> mf_hf_d3.kernel()
```

Alternatively, non-local correlation may be added through the VV10 functional [7] [#vydrov-voorhis-vv10-functional-jcp-2010], cf. `dft/33-nlc_functionals.py` [https://github.com/pyscf/pyscf/blob/master/examples/dft/33-nlc_functionals.py]:


```
>>> mf_hf.xc = 'wb97m_v'  
>>> mf_hf.nlc = 'vv10'  
>>> mf_hf.grids.atom_grid = {'H': (99, 590), 'F': (99, 590)}  
>>> mf_hf.grids.prune = None  
>>> mf_hf.nlcgrids.atom_grid = {'H': (50, 194), 'F': (50, 194)}  
>>> mf_hf.nlcgrids.prune = dft.gen_grid.sg1_prune  
>>> mf_hf.kernel()
```

It's important to keep in mind that the evaluation of the VV10 functional involves a double grid integration, so differences in grid size can make an enormous difference in time.

Periodic Boundary Conditions

Besides finite-sized systems, PySCF further supports KS-DFT calculations with PBCs for performing solid-state calculations. The APIs for molecular and crystalline KS-DFT calculations have deliberately been made to align to the greatest extent possible, and an all-electron KS-DFT calculation for an initialized `Cell` object at either the Γ -point or with k-point sampling may be run through `dft` and `pbc.dft`, respectively. For more details on PBC functionalities, please see the dedicated sections on [PBC-KS-DFT](#) [[pbc.html#user-pbc](#)].

References

[1 [#id1]]

W. Kohn and L. J. Sham. Self-Consistent Equations Including Exchange and Correlation Effects. *Phys. Rev.*, 140:A1133, 1965.

[2 [#id2]]

J. P. Perdew and K. Schmidt. Jacob's Ladder of Density Functional Approximations for the Exchange-Correlation Energy. *AIP Conf. Proc.*, 577:1, 2001.

[3 [#id3]]

A Seidl, A. Görling, P. Vogl, J. A. Majewski, and M. Levy. Generalized Kohn-Sham Schemes and the Band-Gap Problem. *Phys. Rev. B*, 53:3764, 1996.

[4 [id4]]

S. Lehtola, C. Steigemann, M. J. T. Oliveira, and M. A. L. Marques. Recent Developments in LIBXC — a Comprehensive Library of Functionals for Density Functional Theory. *SoftwareX*, 7:1, 2018.

[5 [id5]]

U. Ekström, L. Visscher, R. Bast, A. J. Thorvaldsen, and K. Ruud. Arbitrary-Order Density Functional Response Theory from Automatic Differentiation. *J. Chem. Theory Comput.*, 6:1971, 2010.

[6 [id6]]

S. Grimme, J. Antony, S. Ehrlich, and H. Krieg. A Consistent and Accurate *ab initio* Parametrization of Density Functional Dispersion Correction (DFT-D) for the 94 Elements H-Pu. *J. Chem. Phys.*, 132:154104, 2010.

[7 [id7]]

O. A. Vydrov and T. Van Voorhis. Nonlocal van der Waals Density Functional: The Simpler the Better. *J. Chem. Phys.*, 133:244103, 2010.