Classification of Irrigated/Rainfed In Madison Country using DecisionTreeClassifier

In this project, we are trying to solve the problem of classifying the fields as irrigated or rainfed using DecisionTreeClassifier.

Libraries used:

- 1. Numpy
- 2. Pandas
- 3. Matplotlib for visualization and plotting

The classification problem was solved following the steps given below:

- 1. Data Preprocessing and Cleaning
- 2. Split data into 3 chunks using stratified sampling
- 3. Handle skewness using smote algorithm in the training set
- 4. Apply decision tree algorithm
- 5. Model selection using validation set and accuracy-depth plot
- Evaluation of result on test dataset.

1. Data Preprocessing and Cleaning:

The following steps were carried out for data preprocessing and cleaning:

- i. Missing values were handled using linear interpolation.
- ii. The data was standardized using z-score normalization.
- iii. PCA was used for dimensionality reduction. (As using principal components with more than 95% resulted in overfitting from even a small decision tree, 91% variance were preserved while performing the dimensionality reduction with only 10 principal components.)
- iv. As the dataset was imbalanced (containing {1: 1288, 0: 5053}), it was handled using an oversampling algorithm: Smote on training set.

2. Split data into 3 chunks using stratified sampling

After data preprocessing and cleaning, the data were splitted into three chunks: train, validation and test containing 70%, 20% and 10% of total data respectively using stratified sampling. Thereafter, the data were shuffled in each of these sets.

3. Apply decision tree algorithm:

```
import numpy as np
class DecisionTree:
    """
    A class to construct decision tree
```

```
Attributes
             min sample size: int
                   Minimum sample size required to split the data
             max_depth: int
                   Maximum possible depth
             mode: string
                   To compute "gini" or "entropy" as a measure of impurity
      .....
      def __init__(self, min_sample_size=2, max_depth=1000, mode="gini"):
             self.min_sample_size = min_sample_size
             self.max depth = max depth
             self.mode = mode
      def check_purity(self, data):
             """Check if a data contains only a single class label.
             Parameters
                    data: DataFrame
                         Dataset with features and target
             if len(np.unique(data[:, -1])) == 1:
                   return True
             else:
                    return False
      def classify data(self, data):
             """Classify data into classes based on the maximum occurence
             Parameters
                   data: DataFrame
                          Dataset with features and target
             Returns
             _____
                  class label
             unique classes, count unique classes = np.unique(data[:, -1],
return_counts = True)
             return unique classes[count unique classes.argmax()]
      def get_potential_splits(self, data):
             """Get all the unique value of a particular column of a
             dataframe
             Parameters
                    data: DataFrame
                         Dataset with features and target
             Returns
                   Dictionary of potential splits
             11 11 11
```

```
potential_splits = {}
             _, n_columns = data.shape
             for column in list(range(n columns -1)):
                    values = data[:, column]
                    unique values = np.unique(values)
                    if len(unique values) == 1:
                           potential splits[column] = unique values
                    else:
                          potential splits[column] = []
                           for i in range(len(unique values)):
                                 if i != 0:
                                        current value = unique values[i]
                                        previous value = unique values[i - 1]
                                        potential_splits[column].append((current_value
+ previous value) / 2)
             return potential_splits
      def split data(self, data, split column, split value):
             """Split data in left and right branch
             Parameters
             _____
                    data: DataFrame
                          Dataset with features and target
                    split_column: str
                          Column to split
                    split_value: float
                          Value on the basis of which to split
             .. .. ..
             split column values = data[:, split column]
             return data[split_column_values <= split_value], data[split_column_values</pre>
> split value]
      def calculate_impurity_parent(self, data):
             """Calculate impurity of a parent class
             Parameters
                   data: DataFrame
                          Dataset with features and target
             Returns
                    Impurity of a parent class (for both gini and entropy)
             , unique classes = np.unique(data[:, -1], return counts=True)
             probability = unique classes/unique classes.sum()
             if self.mode == "entropy":
                    return np.sum(probability * -np.log2(probability))
             else:
                    return 1- np.sum(np.square(probability))
```

```
def calculate_impurity_children(self, data_below, data_above):
             """Calculate impurity of a parent class
             Parameters
             _____
                   data: DataFrame
                          Dataset with features and target
             Returns
                    Impurity of a child class (for both gini and entropy)
             prob_data_below = len(data_below) / (len(data_below) + len(data_above))
             prob_data_above = len(data_above) / (len(data_below) + len(data_above))
             return prob data below * self.calculate impurity parent(data below) +
prob data above * self.calculate impurity parent(data above)
      def calculate information gain(self, data, data below, data above):
             """Calculate information gain by subtracting the impurity of a parent
class
             with that of a child class.
             Parameters
                    data: DataFrame
                          Dataset with features and target
                    data below: DataFrame
                          Child in the left position of tree
                    data above: DataFrame
                          Child in the right position of tree
             Returns
                   Information gain
             return self.calculate impurity parent(data) -
self.calculate_impurity_children(data_below, data_above)
      def determine best split(self, data, potential splits):
             """Determine best split column and split value based on information gain
             Parameters
                    data: DataFrame
                          Dataset with features and target
                   potential splits: dict
                          Dictionary of all potential splits
             Returns
                   best split column, best split value
             11 11 11
             max_info_gain = -float("inf")
             best split column = 0
             best split value = 0
             for split_column in potential_splits:
                    for split_value in potential_splits[split_column]:
```

```
data_below, data_above = self.split_data(data,
split column, split value)
                           current info gain = self.calculate information gain(data,
data below, data above)
                           if current info gain >= max info gain:
                                  max info gain = current info gain
                                  best split column = split column
                                  best_split_value = split_value
             return best_split_column, best_split_value
      def build decision tree(self, dataframe, current depth=0):
             """Build decision tree by checking purity of data, calculating potential
splits and
             determining best split
             Parameters:
                    dataframe: pd.DataFrame
                           Dataset with features and target
                    current depth: int
                          Depth of a decision tree
             Returns:
                    Decision Tree
             if current_depth == 0:
                    global COLUMNS
                    COLUMNS = dataframe.columns
                    data = dataframe.values
             else:
                    data = dataframe
             if self.check purity(data) or len(data) < self.min sample size or
current depth == self.max depth:
                    return self.classify_data(data)
             else:
                    current depth += 1
                    potential_splits = self.get_potential_splits(data)
                    split column, split value = self.determine best split(data,
potential_splits)
                    data below, data above = self.split data(data, split column,
split_value)
                    if len(data_below) == 0 or len(data_above) == 0:
                           return self.classify data(data)
                    else:
                           question = str(COLUMNS[split column]) + " <= " +</pre>
str(split value)
                           decision tree = {question: []}
                           positive = self.build_decision_tree(data_below,
current depth)
                           negative = self.build_decision_tree(data_above,
current depth)
                           if positive == negative:
                                  decision tree = positive
                           else:
```

```
decision tree[question].append(negative)
                    return decision tree
def classify(self, sample, tree):
      """Classify the sample
      Parameters:
      sample: pd.DataFrame
             Features
      tree: Decision tree which is to be used during classification
      Returns
            classification result
      if not isinstance(tree, dict):
             return tree
      question = list(tree.keys())[0]
      attribute, value = question.split(" <= ")</pre>
      if sample[attribute] <= float(value):</pre>
             answer = tree[question][0]
      else:
             answer = tree[question][1]
      return self.classify(sample, answer)
def predictions(self, dataframe, tree):
      """Predict the result of entire dataframe
      Parameters
      _____
      dataframe: pd.DataFrame
             Features
      tree: dict
             Decision tree which is to be used during classification
      Returns
            prediction result
      return dataframe.apply(self.classify, axis = 1, args = (tree,))
def calculate_accuracy(self, predicted, actual):
      """Calculates Accuracy
      Parameters
       _____
             predicted: numpy.ndarray
                   Prediction result
             acutal: numpy.ndarray
                    Actual result
      ,, ,, ,,
      correct = predicted == actual
      return correct.mean()
```

decision tree[question].append(positive)

Evaluation of Result:

While applying the above decision tree algorithm, we obtained the following result on different values of maximum depth:

```
maxDepth = 1: accTest = 61.04%, accTrain = 61.99%, accVal = 61.75%, buildTime = 68.03s
maxDepth = 2: accTest = 64.35%, accTrain = 65.40%, accVal = 65.30%, buildTime =
119.99s
maxDepth = 3: accTest = 66.09%, accTrain = 68.33%, accVal = 65.77%, buildTime =
163.53s
maxDepth = 4: accTest = 70.50%, accTrain = 71.59%, accVal = 67.51%, buildTime =
189.19s
maxDepth = 5: accTest = 70.19%, accTrain = 73.21%, accVal = 68.69%, buildTime =
219.62s
maxDepth = 6: accTest = 72.56%, accTrain = 75.38%, accVal = 67.74%, buildTime = 6
maxDepth = 7: accTest = 75.08%, accTrain = 79.34%, accVal = 68.69%, buildTime =
245.81s
maxDepth = 8: accTest = 76.97%, accTrain = 82.27%, accVal = 69.32%, buildTime =
240.14s
maxDepth = 9: accTest = 77.13\%, accTrain = 85.02\%, accVal = 69.16\%, buildTime =
maxDepth = 10: accTest = 79.97%, accTrain = 87.44%, accVal = 69.32%, buildTime =
268.94s
maxDepth = 11: accTest = 80.91%, accTrain = 89.91%, accVal = 69.24%, buildTime =
269.37s
maxDepth = 12: accTest = 80.76%, accTrain = 92.09%, accVal = 68.14%, buildTime =
289.53s
maxDepth = 13: accTest = 82.33%, accTrain = 94.22%, accVal = 67.74%, buildTime =
284.75s
maxDepth = 14: accTest = 84.07%, accTrain = 95.92%, accVal = 68.45%, buildTime = 95.92%, accVal = 68.45%, accVal 
292.29s
maxDepth = 15: accTest = 85.02%, accTrain = 97.39%, accVal = 68.14%, buildTime =
maxDepth = 16: accTest = 86.28%, accTrain = 98.42%, accVal = 68.22%, buildTime =
282.71s
maxDepth = 17: accTest = 86.44%, accTrain = 99.04%, accVal = 68.22%, buildTime =
299.08s
maxDepth = 18: accTest = 86.59%, accTrain = 99.50%, accVal = 68.22%, buildTime =
301.04s
```

```
maxDepth = 19: accTest = 86.28%, accTrain = 99.76%, accVal = 68.14%, buildTime =
289.62s
maxDepth = 20: accTest = 86.59%, accTrain = 99.91%, accVal = 67.98%, buildTime =
293.78s
maxDepth = 21: accTest = 86.75%, accTrain = 99.96%, accVal = 67.82%, buildTime =
270.45s
maxDepth = 22: accTest = 86.75%, accTrain = 99.99%, accVal = 67.82%, buildTime =
252.66s
maxDepth = 23: accTest = 86.75%, accTrain = 100.00%, accVal = 67.82%, buildTime =
277.94s
```

Then, the accuracy-depth and error-depth graphs were plotted from the results obtained:

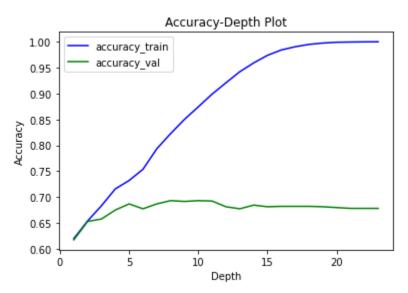


Figure 1a. Plot between accuracy and depth

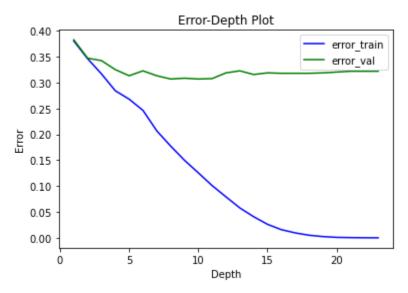


Figure 1b. Plot between error and depth

Therefore, visualizing the training plot, max-depth=15 seems to be the correct model whereas while looking at the validation plot, max-depth=5 seems to be the correct model.

A 2-dimensional decision tree boundary plot around max-depth=5 and max-depth=15 was plotted to help select the best model.

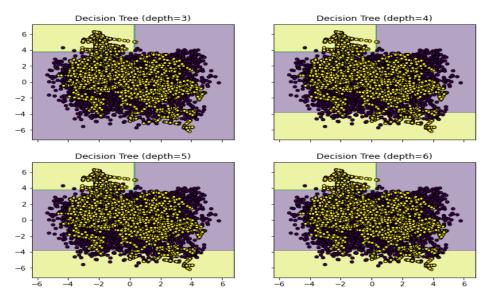


Figure 2a. Decision boundary for max-depth (3,4,5 and 6) in training set

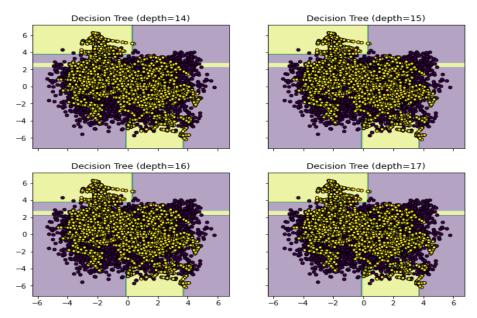


Figure 2b. Decision boundary for max-depth (14,15,16 and 17) in training set

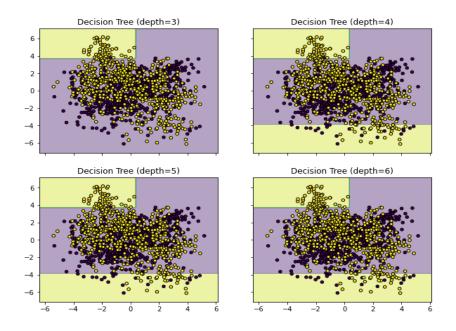


Figure 2c. Decision boundary for max-depth (3, 4, 5 and 6) in validation set

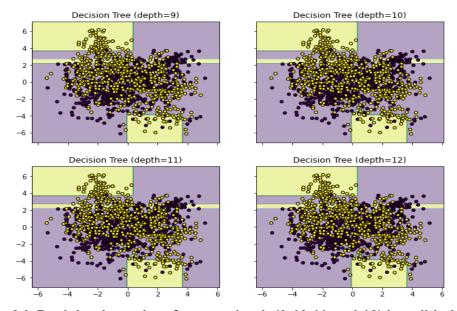


Figure 2d. Decision boundary for max-depth (9,10,11 and 12) in validation set

Pruning the decision tree:

For solving the problem of overfitting, we also pruned the decision tree trained at depth=5 and depth=15. The decision boundaries obtained are as follows:

When depth=5:

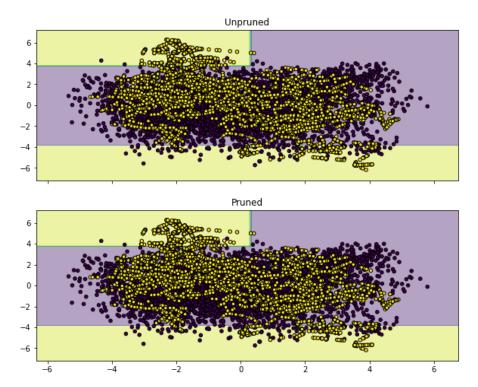


Figure 3a. Effect of pruning on training dataset when max_depth = 5

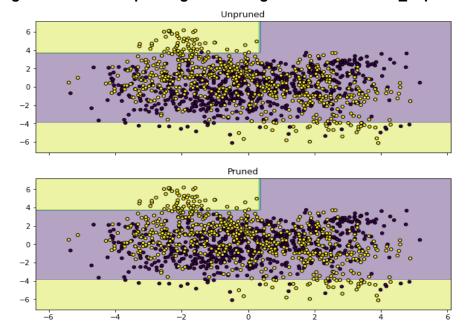


Figure 3b. Effect of pruning on validation dataset when max_depth = 5

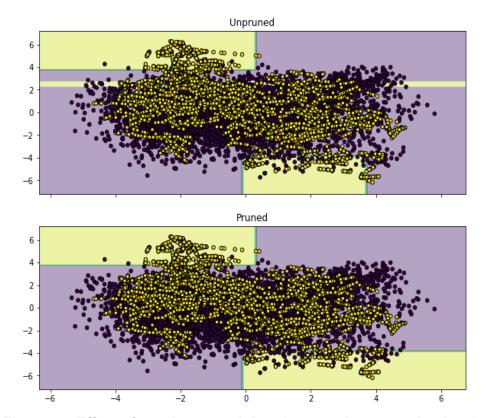


Figure 3c. Effect of pruning on training dataset when max_depth = 15

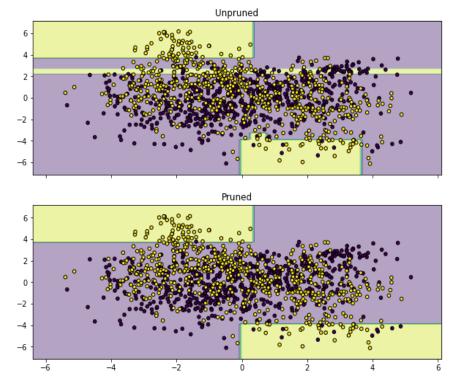


Figure 3d. Effect of pruning on training dataset when max_depth = 15

```
Unpruned
                                            Pruned
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                                            {'PC3 <= 2.01698085272447': [{'PC2 <=
3.724320189213132': [{'PC8 <=
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        0.0]},
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        1.0]}]},
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[1.0, 0.0]}]}]}]}
```

<u>Decision tree trained at max_depth = 15</u>

Unpruned	Pruned
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```
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```
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Evaluation of model on validation dataset

	Unpruned	Pruned	Unpruned	Pruned
	(depth=5)	(depth=5)	(depth=15)	(depth=15)
Confusion matrix	[[506 128]	[[506 128]	[[537 97]	[[575 59]
	[269 365]]	[254 380]]	[307 327]]	[262 372]]
Accuracy	0.69	0.7	0.68	0.75

Precision (1)	0.74	0.75	0.77	0.86
Precision (0)	0.65	0.67	0.64	0.69
Recall (1)	0.58	0.6	0.52	0.59
Recall (0)	0.8	0.8	0.85	0.91
F1-score (1)	0.65	0.67	0.62	0.7
F1-score (0)	0.72	0.73	0.73	0.78

As decision tree with max-depth 15 produced better f1-score on validation dataset, we have selected decision-tree with max-depth 15 as the final model.

Result on test dataset:

Confusion matrix:

[[297 20] [102 215]]

Accuracy: 0.81 Precision (1): 0.91 Precision (0): 0.74 Recall (1): 0.68 Recall (0): 0.94 F1-score (1): 0.78 F1-score (0): 0.83

Conclusion:

Firstly, the data was pre-processed and cleaned, standardized and pca was applied. After training with the decision tree classifier, we plotted the graph and found that there was a wide gap between training and validation accuracy. To handle that, we pruned the decision tree, and evaluated the result on a test (unseen) dataset.