Problem Set - MLE

September 25, 2021

1 Maximum Likelihood Estimator

1.1 Gaussian Distribution

Given a Gaussian distribution $f(y, \mu, \sigma = 1) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{(y-\mu)^2}{2})$ and a set of observations y_1, y_2 ..., y_n

1. Write down the likelihood function $L(y_i, \mu)$ for the ith observation y_i. What is the total likelihood function for all n observation $L(y_1, y_2, ..., y_n, \mu)$?
2. What is the total log likelihood function $\ell(y_1, y_2, ..., y_n, \mu)$
3. Prove that ℓ is maximized when $\mu = 1$, given one observation $y_1 = 1$.
4. Prove that ℓ is maximized when $\mu = \sum_{i=1}^{n} \frac{y_i}{n}$, given n observations y_1, y_2 ..., y_n

1.2 Bernoulli Distribution

Given a Bernoulli distribution $f(y, p) = p^y(1-p)^{1-y}$ and a set of observations y_1, y_2 ..., y_n. Assume that k out of the n observations are 1, the rest are 0. We then have the total likelihood function $L(y_1, y_2, ...y_n, p) = \prod_{i=1}^{n} p^k(1-p)^{k-n}$.

1. What is the total log likelihood function $\ell(y_1, y_2, ..., y_n, \mu)$?
2. Prove that ℓ is maximized when $p = \frac{k}{n}$.