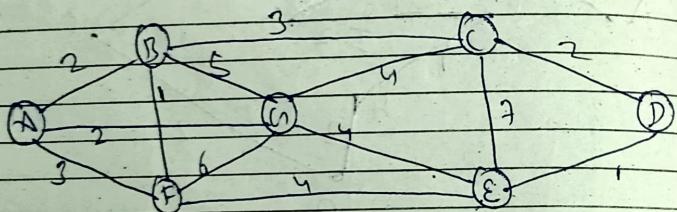


## Kruskal's algorithm

- \* It uses the concept of disjoint set.
- \* It is also a greedy algorithm.
- \* Everytime there is a choice it chooses the cheapest edge.
- \* In this algorithm we start with the cheapest edge.

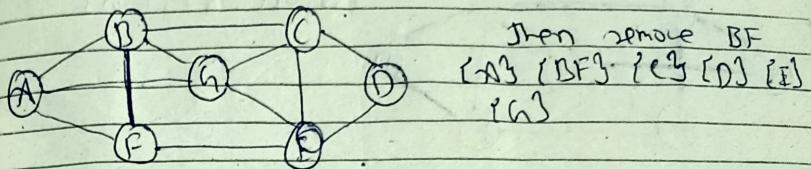
## Kruskal's algorithm for MST :-



Edges	Weight	Disjoint Set
AB	2	A {B}
AG	2	A {B} {C} {D} {E} {F} {G}
AF	3	A {B} {C} {D} {E} {F} {G}
BF	1	A {B} {C} {D} {E} {F} {G}
BC	3	A {B} {C} {D} {E} {F} {G}
BG	5	A {B} {C} {D} {E} {F} {G}
CG	4	A {B} {C} {D} {E} {F} {G}
CE	7	A {B} {C} {D} {E} {F} {G}
CD	2	A {B} {C} {D} {E} {F} {G}
DE	1	A {B} {C} {D} {E} {F} {G}
EG	4	A {B} {C} {D} {E} {F} {G}
EF	4	A {B} {C} {D} {E} {F} {G}

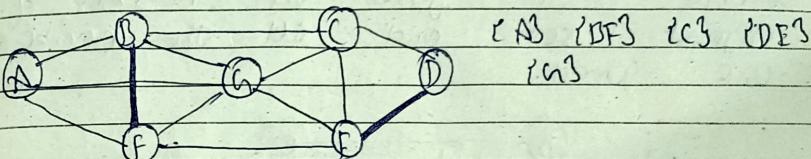
if the number of disjoint set item one + minimum. Date : / /  
Page No.

minimum weight BF = 1 Disjoint set



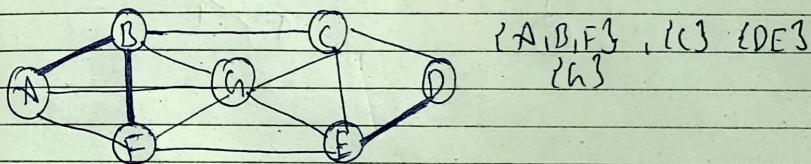
Then remove BF  
{AB} {BF} {CG} {D} {E} {F} {G}

minimum weight DF = 1



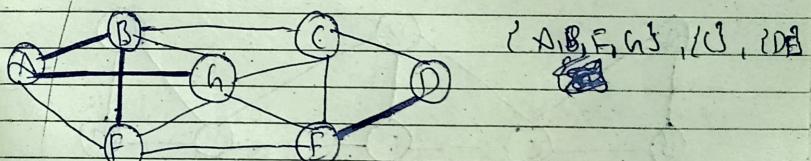
{AB} {BF} {CG} {DE} {F} {G}

minimum weight AD = 2



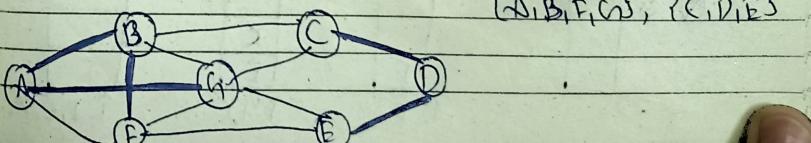
{AB, F} {CG} {DE} {F} {G}

minimum weight AG = 2



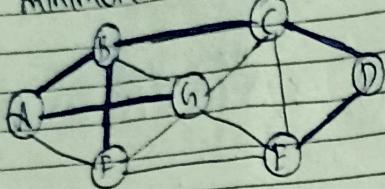
{AB, FG}, {CG}, {DE}

minimum weight CD

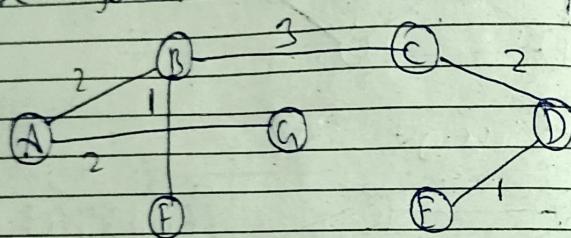


{AB, FG}, {CG, DE}

minimum weight BC  
 $\{A, B, C, D, F, E, \text{in}\}$



All the remaining edges can't be joined because we join the member of disjoint set and all the members are joined.



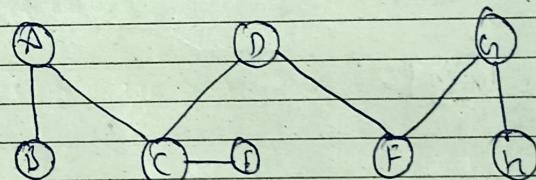
B	A	C	D	F
A	C	E	D	D
C	F	E	C	C
E			E	E
	T: F, C, A, B	T: E, G, A, B	T: E, C, A, B, D	T: E, C, A, B, F

G	H
F	F
D	D
C	C
E	E

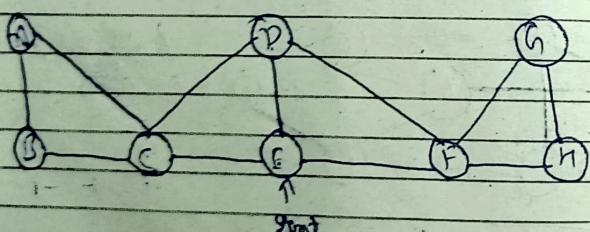
Then pop one by one

T: E, G, A, B, D, F, G, H

T: E, C, A, B, D, F, G, H



Q Apply DFS algorithm to find the spanning tree from given graph.



Ans

T:

T: E

T: E

T: F, C, A

Q Apply BFS in some question solve solved one. (queue)

Q: [ ]

T:

Q: [E]

T: E

(E)

Q: E C D F

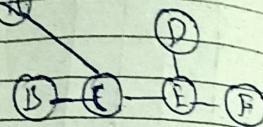
T: E, C, R, F

Date :  
Page No.



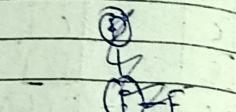
Q: I C D F A B

T: E, C, D, F, A, B



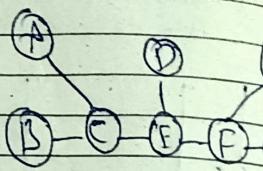
Q: D F A B C

T: F, C, D, F, A, B



Q: F A B G H

T: E, C, D, F, A, B, G, H



Q: P B H N

T:

Q: B G H

T:

Q: G H

T:

Q: H

T:

Q: D

T: E, C, D, F, A, B, G, H

all pair shortest path algorithm  
Floyd-Warshall's algorithm

Step 1 : Find the weight matrix for given graph ( $W$ )

Step 2 : Obtain the cost matrix ( $O$ ) from ( $W$ ) using depth following rules :

if  $[W[i, j]] = 0$

then  $[O[i, j]] = \infty$

(else) otherwise  $[O[i, j]] = [W[i, j]]$

Step 3 : Repeat  $k = 1$  to  $n$

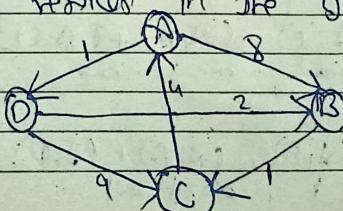
Repeat  $i = 1$  to  $n$

Repeat  $j = 1$  to  $n$

$$O^k[i, j] = \min [O^{k-1}[i, j], O^{k-1}[i, k] + O^{k-1}[k, j]]$$

$$O^k[i, j] = \min [O^{k-1}[i, j], O^{k-1}[i, k] + O^{k-1}[k, j]]$$

Use Floyd-Warshall's method to find the shortest path among all the pairs of vertices in the given graph



$$B_1 A_2 = \min [ (B_1, A_2), (B_1, A) + (A_2, A) ] \\ = \min [ \infty, \infty + \infty ] \\ = \infty$$

	A	B	C	D
A	0	8	0	1
B	0	0	0	1
C	4	0	0	0
D	0	2	9	0

	A	B	C	D
A	$\infty$	8	$\infty$	1
B	$\infty$	$\infty$	0	$\infty$
C	4	$\infty$	$\infty$	$\infty$
D	$\infty$	2	9	$\infty$

	A	B	C	D
A	$\infty$	8	$\infty$	1
B	$\infty$	$\infty$	1	$\infty$
C	4	12	$\infty$	5
D	$\infty$	2	9	$\infty$

$$Q^k[i,j] = \min [ Q[i,j], Q^{k-1}[i,k] + Q^{k-1}[k,j] ]$$

$$A_1 A_2 = \min [ (A_1 A_2), (A_1 A) + (A_2 A) ] \\ = \min [ \infty, \infty + \infty ] \\ = \infty$$

$$A_1 B = \min [ (A_1 B), (A_1 A) + (A_2 B) ] \\ = \min [ 8, \infty + 8 ] \\ = 8$$

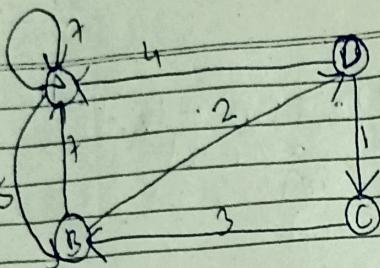
$$A_1 C = \min [ (A_1 C), (A_1 A) + (A_2 C) ] \\ = \min [ \infty, \infty, \infty ] \\ = \infty$$

$$A_1 D = \min [ (A_1 D), (A_1 A) + (A_2 D) ] \\ = \min [ 1, \infty + \infty ] \\ = \min [ 1, \infty ]$$

$$B_1 C = \min [ (B_1 C), (B_1 A) + (A_2 C) ] \\ = \min [ 1, \infty + \infty ] \\ = \min [ 1, \infty ]$$

	A	B	C	D
A	$\infty$	8	$\infty$	1
B	$\infty$	$\infty$	1	$\infty$
C	4	12	$\infty$	5
D	$\infty$	2	9	$\infty$

	A	B	C	D
A	$\infty$	8	$\infty$	1
B	$\infty$	$\infty$	1	$\infty$
C	4	$\infty$	$\infty$	$\infty$
D	$\infty$	2	9	$\infty$

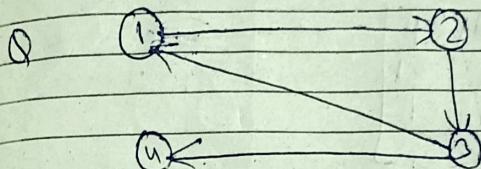


$$Q^0 = \begin{bmatrix} \infty & A & B & C & D \\ A & 7 & 5 & \infty & \infty \\ B & 7 & \infty & \infty & 2 \\ C & \infty & 3 & \infty & \infty \\ D & 0 & 4 & 2 & 1 \end{bmatrix}$$

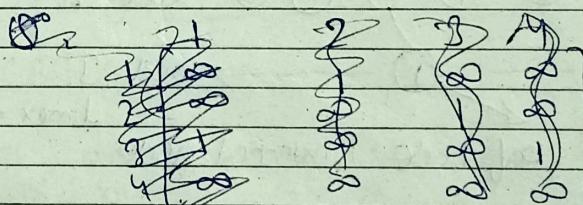
$$Q^1 = \begin{bmatrix} \infty & A & B & C & D \\ A & 7 & 5 & \infty & \infty \\ B & 7 & 12 & \infty & 2 \\ C & \infty & 3 & \infty & \infty \\ D & 4 & 9 & 1 & \infty \end{bmatrix}$$

$$Q^2 = \begin{bmatrix} \infty & A & B & C & D \\ A & 7 & 5 & 2 & 7 \\ B & 7 & 12 & \infty & 2 \\ C & 10 & 3 & \infty & 5 \\ D & 4 & 9 & 1 & 11 \end{bmatrix}$$

$$Q^3 = \begin{bmatrix} \infty & A & B & C & D \\ A & 7 & 5 & 2 & 7 \\ B & 7 & 12 & \infty & 2 \\ C & 10 & 3 & \infty & 5 \\ D & 4 & 9 & 1 & 6 \end{bmatrix}$$

$$Q^4 = \begin{bmatrix} \infty & A & B & C & D \\ A & 7 & 5 & 8 & 7 \\ B & 6 & 6 & 3 & 2 \\ C & 9 & 3 & 6 & 5 \\ D & 4 & 4 & 1 & 6 \end{bmatrix}$$


Find TC using warshall's algorithm

$$TCA = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{bmatrix}$$


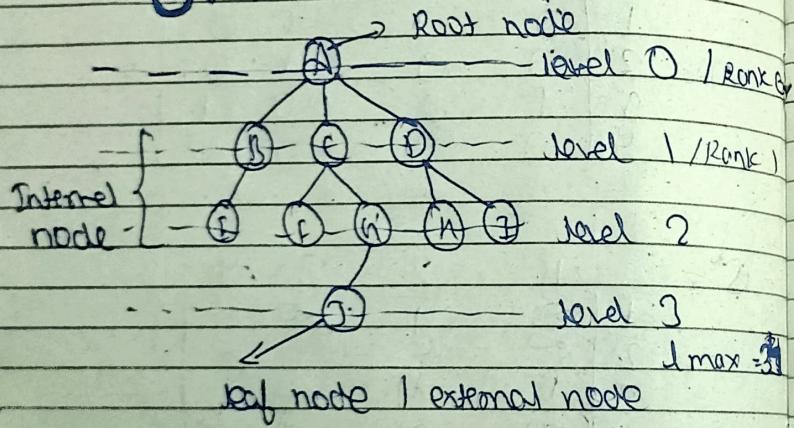
$$T^2 = A \times A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T^3 = A \times A \times A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{bmatrix}$$

	1	2	3	4
1	0	1	0	0
2	0	0	1	0
3	1	0	0	①1
4	0	0	0	0

A.	1	1	①1	1
	1	1	1	②1
	0	0	0	0

## UNIT - 4



All the nodes with degree 0 is known as leaf node.

**Height / Depth :** The maximum number of nodes that are possible in a path starting from the root node to the leaf node is known as height of the tree.

**Degree to entity :** Number of child nodes of a parent node.

level of the root node is always 0.

Height of the tree implies height of the root node.

In diagram height of tree is 4

A → C → H → J

Height = level (max) + 1

**Binary tree :** In binary tree one node to should have 2 child node.

maximum degree of binary tree is 2.

full binary tree and complete binary tree { heap }

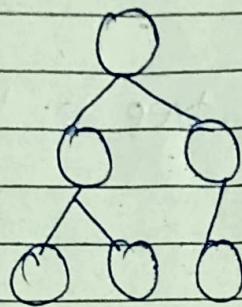
CBT

A binary tree is said CBT if all its levels (except possibly the last level) have the maximum number of possible nodes and the last level has all its nodes to the left side or less or possible.

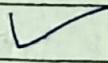
EFT

A binary is full if each node is either leaf node or possesses exactly

two child nodes and have a  
A tree which has 1 degree is known  
as FBT.



CBT



FBT



CBT



FBT

