EMHW5

YiTao Hu, Charles Rambo, Junyu(Kevin) Wu, Jin (Jane) Huangfu

08/02/2020

Problem 1: VAR implementation

Use the data on quarterly excess stock market returns, the market Dividend / Price ratio, and the di§erence between the 10-yr Treasury yield and the Fed Funds rate in the excel spreadsheet "MktRet_DP_TermSpread.xlsx". The interest rate data is from the FRED data depository, available online from the St. Louis Fed.

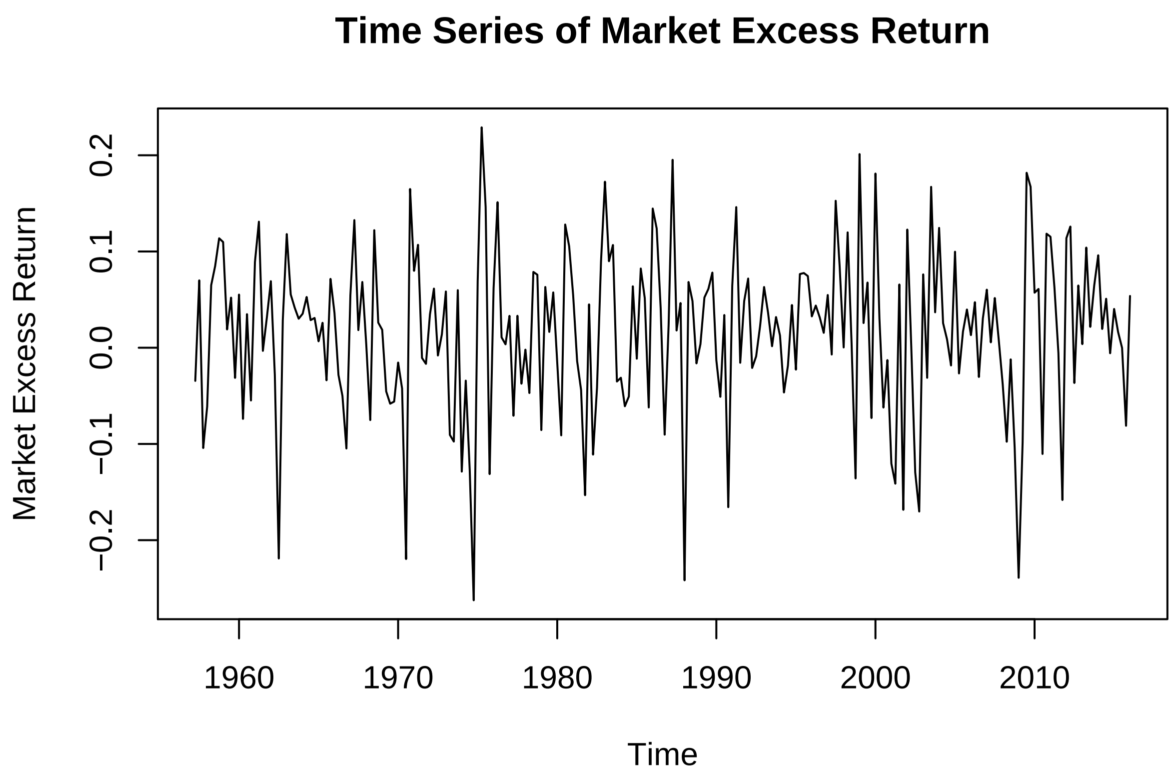
```
#import data and library
library(ggplot2)
library(expm)
library(lmtest)
library(sandwich)
library(vars)
library(DataAnalytics)
library(readxl)
HW5data = read_excel("MktRet_DP_TermSpread.xlsx")
```

1

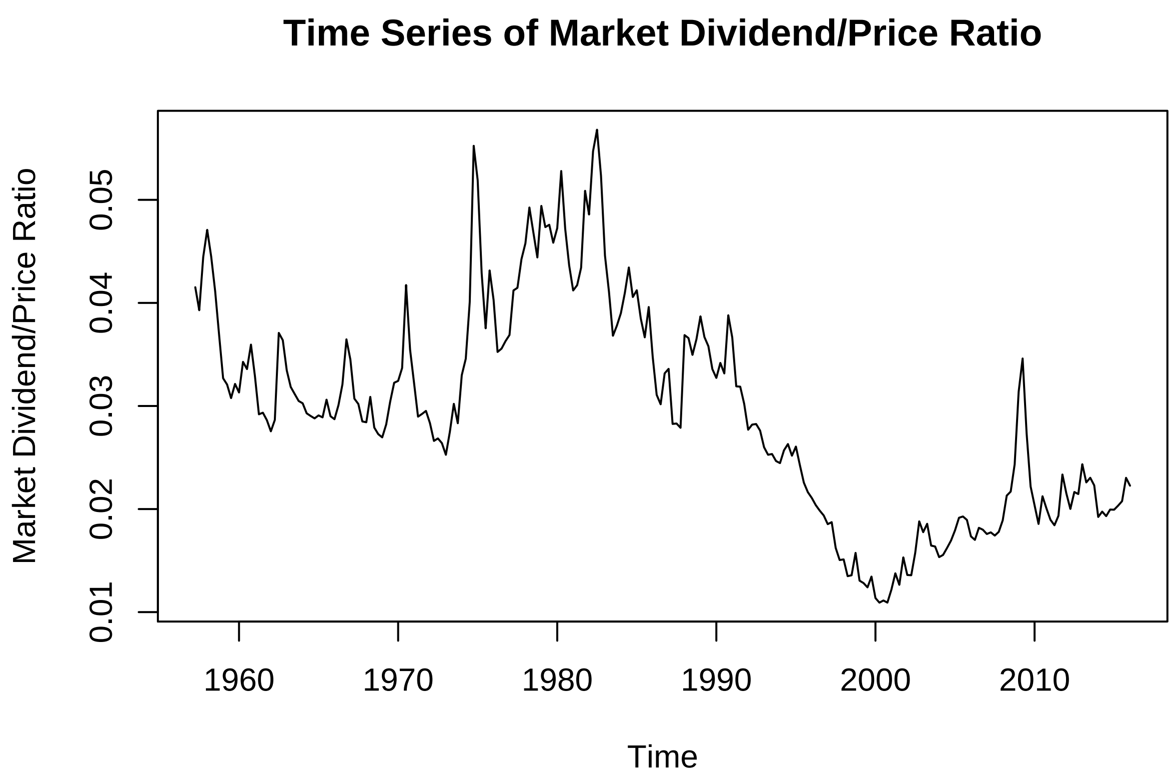
Plot each series. Give the sample mean, standard deviation, and first order autocorrelation of each series. From the first-order autocorrelation, calculate the half-life of each series (see ARMA notes for exact half-life formula).

Plot each Series.

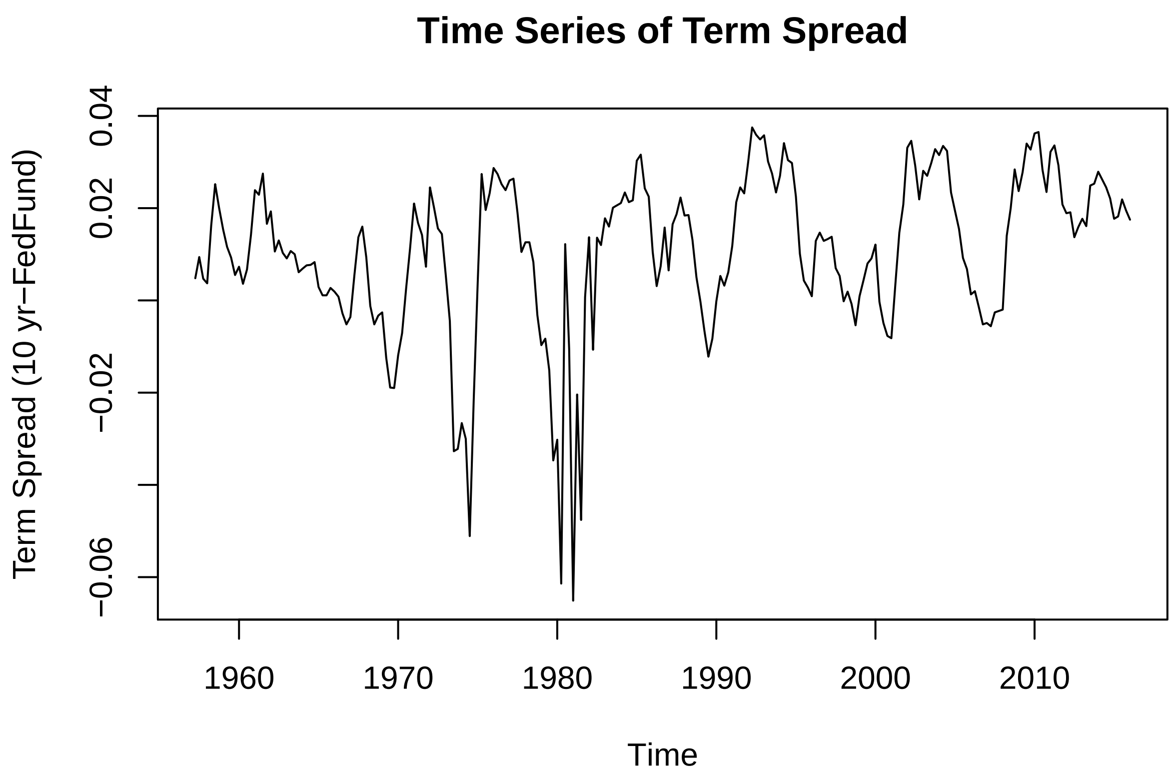
```
plot(x = HW5data$Date,y = HW5data$MktExRet,xlab = 'Time',ylab = 'Market Excess Return',main = 'Time Seri
```




```
plot(x = HW5data$Date,y = HW5data$Mkt_DP,xlab = 'Time',ylab = 'Market Dividend/Price Ratio',main = 'Tim
```




```
plot(x = HW5data$Date,y = HW5data$y10minFedFunds,xlab = 'Time',ylab = 'Term Spread (10 yr-FedFund)',mai
```



Recall the formula for half life is:

$$h = \frac{ln(0.5)}{ln(\phi_1)}$$

```
#compute descriptive stats
desrp_data=descStat(HW5data[,-1])
```

```
##                  Mean Median    SD   IQR SE Mean 95% CI-L 95% CI-U NMissing
## MktExRet        0.016  0.026 0.084 0.096   0.005    0.005    0.027        0
## Mkt_DP          0.029  0.029 0.010 0.016   0.001    0.028    0.031        0
## y10minFedFunds 0.011  0.013 0.017 0.021   0.001    0.008    0.013        0
## Number of Observations =  236
```

```
#compute 1st order acf
Phi_1=apply(X = HW5data[,-1],MARGIN = 2 , FUN = acf,lag.max=1,plot=F)
Phi_1=c(Phi_1$MktExRet$acf[2],Phi_1$Mkt_DP$acf[2],Phi_1$y10minFedFunds$acf[2])
desrp_data=cbind(desrp_data,Phi_1)
HalfLife=log(0.5)/log(desrp_data[,9])
desrp_data=cbind(desrp_data,HalfLife)
desrp_data
```

```
##                  Mean Median    SD   IQR SE Mean 95% CI-L 95% CI-U NMissing
## MktExRet        0.016  0.026 0.084 0.096   0.005    0.005    0.027        0
## Mkt_DP          0.029  0.029 0.010 0.016   0.001    0.028    0.031        0
## y10minFedFunds 0.011  0.013 0.017 0.021   0.001    0.008    0.013        0
##                    Phi_1   HalfLife
## MktExRet       0.06960754  0.2601042
## Mkt_DP         0.95807815 16.1852221
## y10minFedFunds 0.80344922  3.1673510
```

2

Recall from the slides, the VAR(1) Model would follow the following hypothesis:

$$\vec{y_t} = \vec{\phi_0} + \Phi_1 \vec{y_{t-1}} + \vec{\epsilon_t}$$

where

$$\vec{y_t} = \begin{pmatrix} R_t^E \\ DP_t \\ SPD_t \end{pmatrix}$$

$\vec{\phi_0}$ is the 3 by 1 constant coeff vector,

$$\Phi$$

the 3 by 3 slop coeff matrix, and $\vec{\epsilon_t}$ is 3 by 1 residual vector. Perform the VAR(1) regression, and the OLS coefficients and R-sq of each regression would be:

```r
Model_var1=VAR(y = HW5data[,-1],p = 1,type='const')
summary(Model_var1)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: MktExRet, Mkt_DP, y10minFedFunds
## Deterministic variables: const
## Sample size: 235
## Log Likelihood: 2262.805
## Roots of the characteristic polynomial:
## 0.9407 0.7953 0.07442
## Call:
## VAR(y = HW5data[, -1], p = 1, type = "const")
##
##
## Estimation results for equation MktExRet:
## =========================================
## MktExRet = MktExRet.l1 + Mkt_DP.l1 + y10minFedFunds.l1 + const
##
##                    Estimate Std. Error t value Pr(>|t|)
## MktExRet.l1         0.04852    0.06554   0.740  0.45987
## Mkt_DP.l1           1.45092    0.54218   2.676  0.00798 **
## y10minFedFunds.l1   1.04958    0.33379   3.144  0.00188 **
## const              -0.03827    0.01805  -2.121  0.03501 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.08239 on 231 degrees of freedom
## Multiple R-Squared: 0.06072, Adjusted R-squared: 0.04852
## F-statistic: 4.977 on 3 and 231 DF,  p-value: 0.002297
##
##
## Estimation results for equation Mkt_DP:
## =======================================
## Mkt_DP = MktExRet.l1 + Mkt_DP.l1 + y10minFedFunds.l1 + const
##
##                      Estimate Std. Error t value Pr(>|t|)
## MktExRet.l1       -0.0022822  0.0021906  -1.042 0.298577
```

```
## Mkt_DP.l1          0.9402772  0.0181216  51.887  < 2e-16 ***
## y10minFedFunds.l1 -0.0388263  0.0111566  -3.480 0.000599 ***
## const              0.0021214  0.0006032   3.517 0.000526 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.002754 on 231 degrees of freedom
## Multiple R-Squared: 0.9298,  Adjusted R-squared: 0.9289
## F-statistic:  1021 on 3 and 231 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation y10minFedFunds:
## ===============================================
## y10minFedFunds = MktExRet.l1 + Mkt_DP.l1 + y10minFedFunds.l1 + const
##
##                    Estimate Std. Error t value Pr(>|t|)
## MktExRet.l1       -0.014836   0.008060  -1.841   0.0669 .
## Mkt_DP.l1          0.010477   0.066677   0.157   0.8753
## y10minFedFunds.l1  0.821641   0.041050  20.016   <2e-16 ***
## const              0.001872   0.002220   0.844   0.3998
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01013 on 231 degrees of freedom
## Multiple R-Squared: 0.6516,  Adjusted R-squared: 0.647
## F-statistic:   144 on 3 and 231 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##                  MktExRet     Mkt_DP y10minFedFunds
## MktExRet        0.0067882 -2.058e-04      1.130e-04
## Mkt_DP         -0.0002058  7.583e-06     -4.058e-06
## y10minFedFunds  0.0001130 -4.058e-06      1.027e-04
##
## Correlation matrix of residuals:
##                MktExRet  Mkt_DP y10minFedFunds
## MktExRet         1.0000 -0.9073         0.1353
## Mkt_DP          -0.9073  1.0000        -0.1454
## y10minFedFunds   0.1353 -0.1454         1.0000
```

The HC White robust standard error would be:

```r
coeftest(x=Model_var1,vcov.=vcovHC(Model_var1))
```

```
##
## t test of coefficients:
##
##                                  Estimate  Std. Error t value  Pr(>|t|)
## MktExRet:(Intercept)           -0.03827466  0.01997553 -1.9161 0.0565896
## MktExRet:MktExRet.l1            0.04851912  0.07487925  0.6480 0.5176512
## MktExRet:Mkt_DP.l1              1.45092240  0.58947110  2.4614 0.0145720
## MktExRet:y10minFedFunds.l1      1.04958093  0.40866889  2.5683 0.0108503
```

```
## Mkt_DP:(Intercept)                0.00212138  0.00060034  3.5336 0.0004949
## Mkt_DP:MktExRet.l1               -0.00228220  0.00239071 -0.9546 0.3407722
## Mkt_DP:Mkt_DP.l1                  0.94027718  0.01990206 47.2452 < 2.2e-16
## Mkt_DP:y10minFedFunds.l1         -0.03882632  0.01785154 -2.1750 0.0306485
## y10minFedFunds:(Intercept)        0.00187225  0.00192781  0.9712 0.3324747
## y10minFedFunds:MktExRet.l1       -0.01483615  0.00807623 -1.8370 0.0674925
## y10minFedFunds:Mkt_DP.l1          0.01047743  0.07634819  0.1372 0.8909668
## y10minFedFunds:y10minFedFunds.l1  0.82164129  0.09298732  8.8361 2.557e-16
##
## MktExRet:(Intercept)             .
## MktExRet:MktExRet.l1
## MktExRet:Mkt_DP.l1               *
## MktExRet:y10minFedFunds.l1       *
## Mkt_DP:(Intercept)               ***
## Mkt_DP:MktExRet.l1
## Mkt_DP:Mkt_DP.l1                 ***
## Mkt_DP:y10minFedFunds.l1         *
## y10minFedFunds:(Intercept)
## y10minFedFunds:MktExRet.l1       .
## y10minFedFunds:Mkt_DP.l1
## y10minFedFunds:y10minFedFunds.l1 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

3

To test the stationarity of the VAR(1) Model, we need to check all the eigenvalues of the slope matrix

$$\Phi_1$$

is smaller than 1 in modulus sense.

```r
# grab the slope coeff matrix
est_coeff=coef(Model_var1)
Phi=rbind(est_coeff$MktExRet[-4,1],est_coeff$Mkt_DP[-4,1],est_coeff$y10minFedFunds[-4,1])
#compute the eigenvalues of the Phi matrix
lambdas=eigen(x=Phi,only.values = T)$values
lambdas
```

```
## [1] 0.94071594 0.79530199 0.07441967
```

All eigenvalus are smaller than 1. Therefore the VAR(1) model is stationary, which is consistent with the summary of the library function.

4

Recall our first regression would be the following hypothesis:

$$R_t^E = \phi_{0,R} + \phi_{1,R} R_{t-1}^E + \phi_{DP,R} DP_{t-1} + \phi_{SPD,R} SPD_{t-1} + \epsilon_{R,t}$$

Taking conditional Variance of above at time t-1:

$$Var_{t-1}(R_t^E) = Var(\epsilon_{R,t})$$

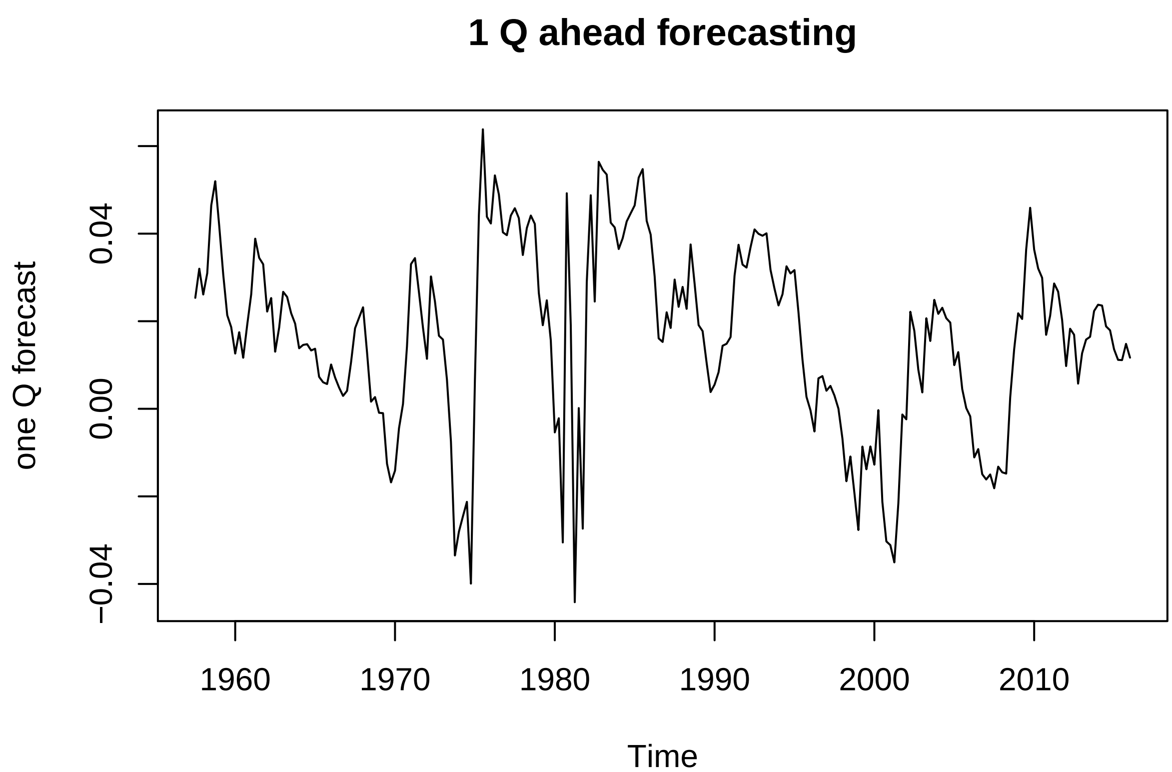
Therefore:

$$\hat{\sigma_{t-1}}(R_t^E) = \hat{\sigma}(\epsilon_{R,t}) = \hat{\sigma}(e_{R,t}) = 0.08239$$

5

Plot the one-quarter ahead expected return series

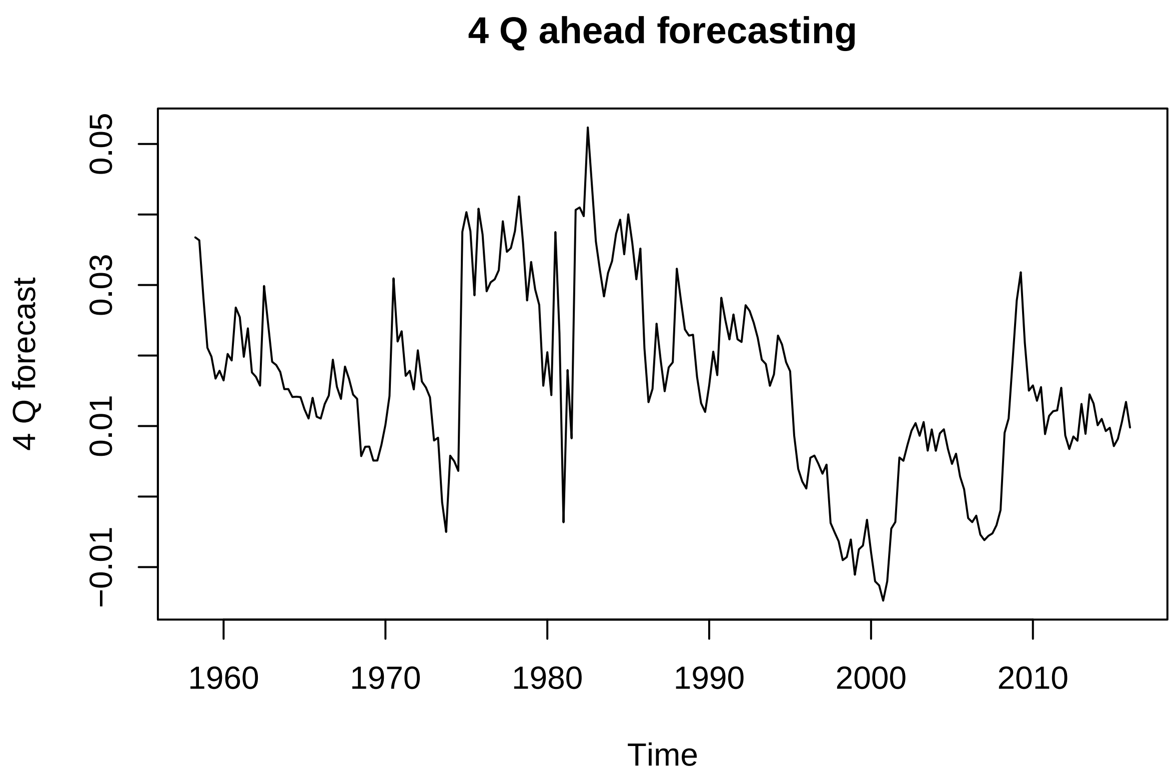
```
plot(y=Model_var1$varresult$MktExRet$fitted.values,type='l',x = HW5data$Date[-1],ylab='one Q forecast',
```



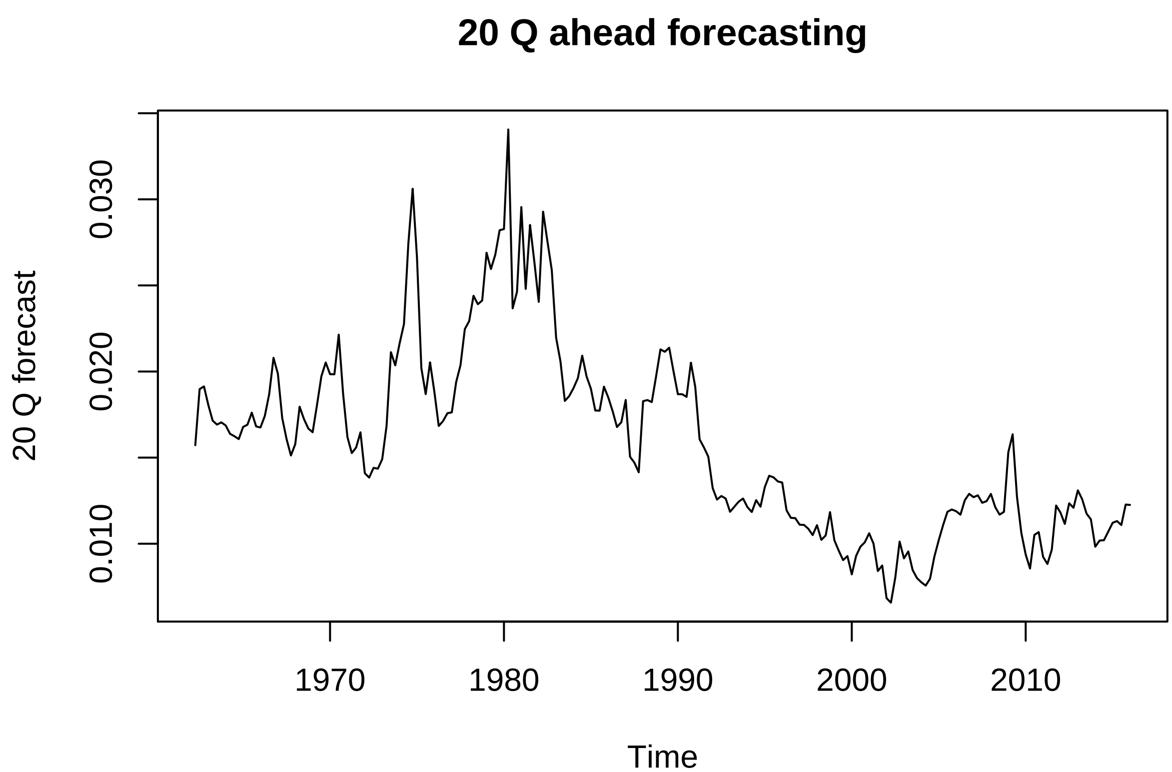
Plot the four-quarter ahead expected return series. Recall the prediction h-period formula for VAR(1) model:

$$E_t(\vec{y}_{t+h}) = (I - \Phi_1^h)\vec{\mu} + \Phi_1^h\vec{y_t}$$

```
mu=colMeans(HW5data[,-1])
Y_t4=HW5data[-1:-4,-1]
Forcast_4=t(matrix(rep((diag(3)-Phi%^%4)%*%mu,nrow(Y_t4)),nrow = 3)+(Phi%^%4)%*%t(Y_t4))
plot(y=Forcast_4[,1],type='l',x = HW5data$Date[-1:-4],ylab='4 Q forecast',xlab='Time',main='4 Q ahead f
```




```
mu=colMeans(HW5data[,-1])
Y_t20=HW5data[-1:-20,-1]
Forcast_20=t(matrix(rep((diag(3)-Phi%^%20)%*%mu,nrow(Y_t20)),nrow = 3)+(Phi%^%20)%*%t(Y_t20))
plot(y=Forcast_20[,1],type='l',x = HW5data$Date[-1:-20],ylab='20 Q forecast',xlab='Time',main='20 Q ahea
```



For longer horizons, the predictive power of term spread and the DP-ratios diminished because long horizon conditional expectation converges to unconditional expectation

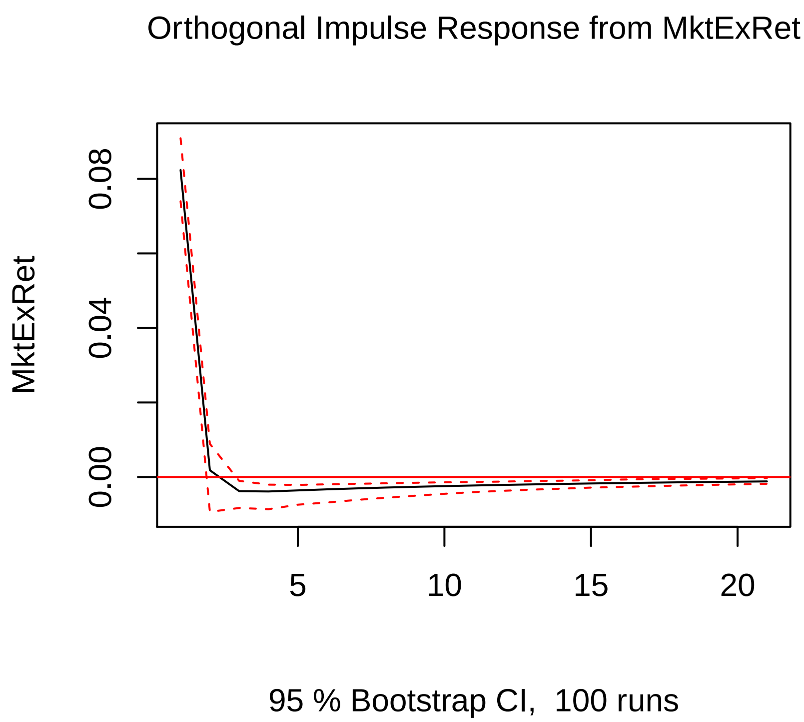
$$E(R_t^E)$$

, which is a constant.

6

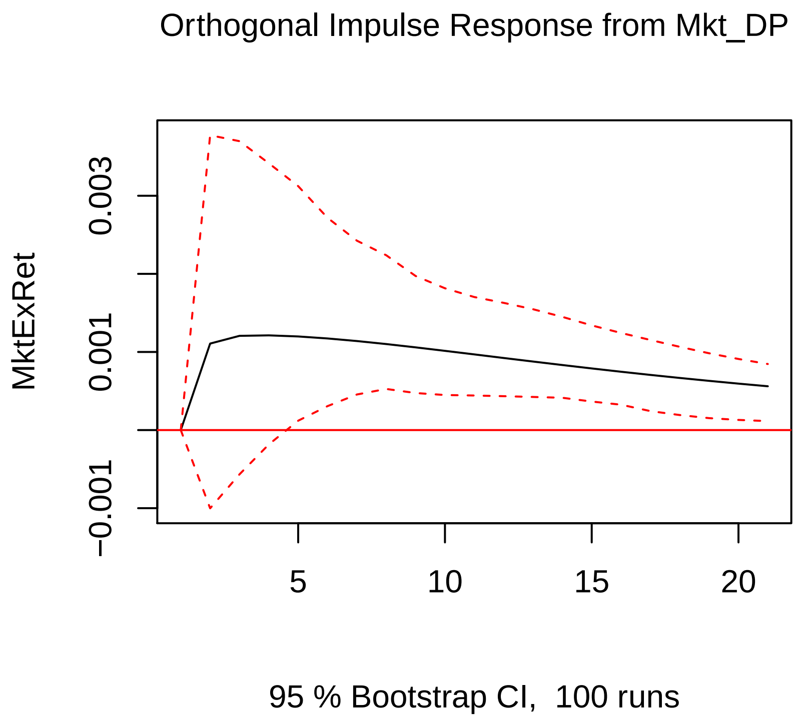
The plot of impluse-response function of Excess Return itself, with orthogonalized shock version, with 95% bootstrapping confidence bands.

```
IR_MktEX=irf(x = Model_var1,impulse = 'MktExRet',response = 'MktExRet',n.ahead = 20,ortho = T)
plot(IR_MktEX)
```



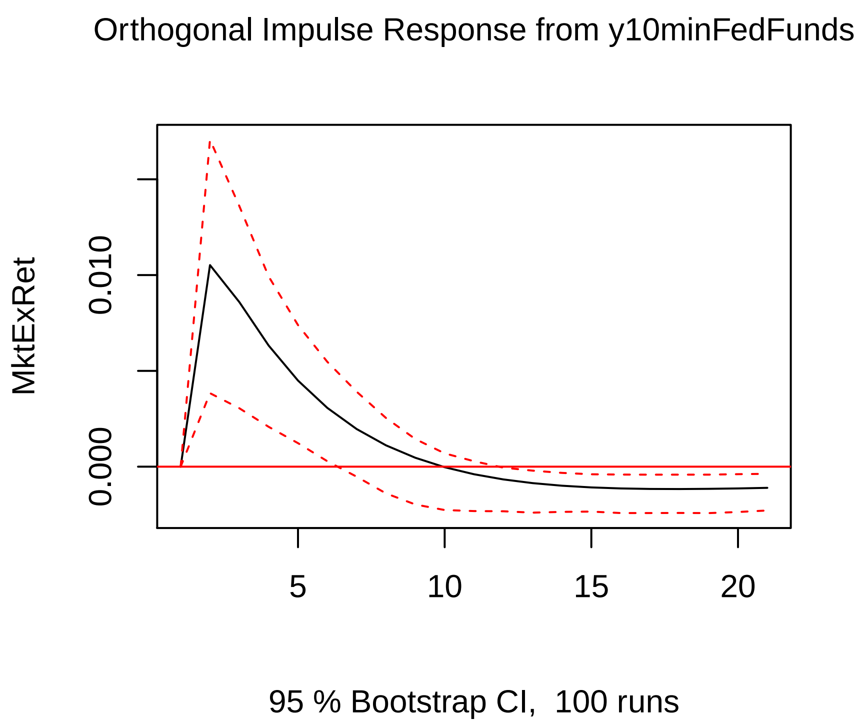
The plot of impluse-response function of Market Dividend Pirce Ratio, with orthogonalized shock version, with 95% bootstrapping confidence bands.

```
IR_MktDP=irf(x = Model_var1,impulse = 'Mkt_DP',response = 'MktExRet',n.ahead = 20,ortho = T)
plot(IR_MktDP)
```



The plot of impluse-response function of Market Dividend Pirce Ratio, with orthogonalized shock version, with 95% bootstrapping confidence bands.

```
IR_spd=irf(x = Model_var1,impulse = 'y10minFedFunds',response = 'MktExRet',n.ahead = 20,ortho = T)
plot(IR_spd)
```



7

To perform an out-of-sample test, we first need to split our data in to a 80% training set and 20% test set. Then, we fit our VAR(1) model on the training set, predict the t+1 observation, compare with the underground truth, and then roll over.

```
# compute the out-of-sample prediction and absolute error
Split_idx=round(nrow(HW5data)*0.8)
Prediction_test=data.frame(matrix(0,nrow =(nrow(HW5data)-Split_idx),ncol = 3))
colnames(Prediction_test)=c('Prediction','Observation','absError')
Prediction_test$Observation=HW5data$MktExRet[(Split_idx+1):nrow(HW5data)]
for (t in 1:(nrow(HW5data)-Split_idx)){
  #fit the model
  Model=VAR(y = HW5data[1:Split_idx,-1],p = 1,type = 'const')
  Prediction_test$Prediction[t]=predict(object = Model,n.ahead = 1)[[1]][[1]][1]
  Split_idx=Split_idx+1
}
Prediction_test$absError=abs(Prediction_test$Observation-Prediction_test$Prediction)
Prediction_test$Date=HW5data$Date[(round(nrow(HW5data)*0.8)+1):nrow(HW5data)]
```

```
Prediction_test=as.zoo(Prediction_test)
plot.zoo(Prediction_test[,-4],plot.type = 'single',col=c('blue','black','red'),main = 'VAR(1) out of sa
legend(x = 0,y = -0.1,legend = c('Prediction','Observed','AbsError'),col=c(3, 4, 6),fill = F)
```