

# Empirical Methods in Finance

## Homework 6

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### Problem 1: ARCH, GARCH, and Realized Variance

From Kenneth French's web page, download the monthly and daily returns to the investment (CMA) factor from 19630701 to 20191231. CMA is the last factor in the FF 5-factor model. Using the monthly return data:

1. Estimate an ARMA(1,1) for the return series. Report the results. What is the estimated monthly persistence of expected returns to CMA? What is the half-life of the expected return series in months?
2. Estimate an ARCH(12) and a GARCH(1,1) process for the residuals from this ARMA(1,1). Report the results and plot the time series of the conditional variance from each model on the same plot. Are the estimated variance processes stationary?
3. Plot the absolute values of the normalized residuals,  $\eta_t = \varepsilon_t / \sigma_t$ , for each model on two separate plots. Using eyeball econometrics, do the models do a good job of accounting for clustering of volatility? Plot the autocorrelation functions of  $|\eta_t|$ .
4. Using the daily data on CMA, estimate monthly realized variance for month  $t$  as

$$RV_t = \sum_{d=1}^{N_t} r_{t,d}^2, \tag{1}$$

where  $r_{t,d}^2$  is the squared return of day  $d$  in month  $t$  and where  $N_t$  is the number of days in month  $t$ . Plot the resulting monthly time series of  $RV_t$ .

5. What are the first order autocorrelations of  $RV_t$  and  $\varepsilon_t^2$ ? What is the correlation between  $RV_t$  and  $\varepsilon_t^2$ ? What is the correlation between  $RV_t$  and  $\sigma_t^2$  from the GARCH model? What is the correlation between  $\varepsilon_t^2$  and  $\sigma_t^2$  from the GARCH model?
6. Run an ARMA(1,1) on  $RV_t$  assuming normally distributed errors (which strictly speaking can't be correct). Report the results. Let  $v_t = E_{t-1} [RV_t]$  where the expectation is obtained from the estimated ARMA. What is the correlation between  $v_t$  and  $RV_t$ ? Plot on the same graph the time series of  $v_t$  and  $\sigma_t^2$  from the GARCH.