Empirical Methods in Finance
Homework 8

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Please use Matlab/R to solve these problems. [The quality of the write-up matters for your grade. Please imagine that you're writing a report for your boss at Goldman when drafting answers these questions. Try to be clear and precise.]

Working with factor models

Assume you have been given three assets to invest in, in addition to the market portfolio. From a historical regression of excess asset returns on the excess market return for $t = 1, ..., T$, you have:

$$\begin{aligned} R_{1t}^e &= 0.01 + 0.9 R_{mt}^e + \hat{\varepsilon}_{1t}, \\ R_{2t}^e &= -0.015 + 1.2 R_{mt}^e + \hat{\varepsilon}_{2t}, \\ R_{3t}^e &= 0.005 + 1.0 R_{mt}^e + \hat{\varepsilon}_{3t}. \end{aligned}$$

The sample mean excess return on the market is, $\bar{R}_m^e = 0.05$; the sample standard deviation of excess market returns is 15%. Thus, the market Sharpe ratio is 1/3. Finally, the sample variance-covariance matrix of residual returns, $\varepsilon_t = [\varepsilon_{1t} \;\; \varepsilon_{2t} \;\; \varepsilon_{3t}]'$, is:

$$var(\hat{\varepsilon}_t) = \hat{\Sigma}_\varepsilon = \begin{bmatrix} 0.1^2 & 0 & 0 \\ 0 & 0.15^2 & 0 \\ 0 & 0 & 0.05^2 \end{bmatrix}.$$

1. What is the sample mean, standard deviation, and Sharpe ratio of the excess returns these three assets?

2. For each of the three assets, construct the market-neutral versions by hedging out the market risk (R^{α}_{jt}). For each of these three hedged asset returns, give the sample average return, standard deviation, and Sharpe ratio.

3. Calculate the maximum Sharpe ratio you can obtain by optimally combining the three hedged assets. Give the math behind this calculation.

4. Given your result in (3), what is the maximum Sharpe ratio you can obtain by combining these three assets with the market portfolio?

5. You have been told to form a portfolio today, assuming the historical estimates given above are the true values also going forward, that (a) provides the maximum (expected) Sharpe ratio of returns and (b) has an (expected) volatility of 15%. You can invest in the three assets, as well as the market portfolio.

   (a) Give the portfolio weights (really, the loadings on each of these in total four assets since each asset is an excess return) that achieves objectives (a) and (b).

   Hint: Recall that the mean-variance weights are proportional to the $\Omega^{-1}\bar{R}^e$ where Ω is the 4×4 covariance matrix of all excess returns (inlcuding the market) and $\bar{R}$ is the 4×1 vector of expected asset returns. Thus, we have $w^{MVE} = k\Omega^{-1}\bar{R}^e$, for some constant k. Next, recall that the variance of the mean-variance efficient portfolio is $k^2 \left(\bar{R}^e\right)' \Omega^{-1}\bar{R}^e$. Thus, we can find k by setting $k\sqrt{\left(\bar{R}^e\right)' \Omega^{-1}\bar{R}^e} = 15\%$.

   (b) Give the expected excess return, standard deviation, and Sharpe ratio of this portfolio. When you are evaluating variance and covariances, recall that the variance of each asset includes a systemic component (relative to β_i) in addition to the residual covariance matrix given above.

6. (Optional) Next, you run Fama-MacBeth regressions of the three asset returns on their market betas and an intercept.

   (a) Give the mean return, standard deviation, and Sharpe ratio of the factor-mimicking portfolio that is implied by the regression.

(b) What is the correlation between this factor-mimicking portfolio return and the market portfolio? You are given enough information in the above to make this calculation, given your answer in (a).

(c) Do a PCA on the three assets. Again, when calculating the variance-covariance matrix of returns, remember to account for both systemic and residual return variation. What is the variance of each of the three PCs, relative to the sum of their variances?

(d) What are the portfolio weights of each PC?

(e) *Difficult:* In (b) you created a factor-mimicking portfolio that, at least intuitively, should deliver the market factor. Yet, it doesn't. The same is true in the data: a market beta sort does not give you the market factor, but something with relatively low correlation with the market factor. But, why is that? Explain your reasoning.

Hint: The simplest way to understand what's going on is to consider, for illustrative purposes, the two factor model:

$$R_{it}^e = \beta_{1i} MKT_{1t} + \beta_{2i} F_{2t} + \varepsilon_{it}, \tag{1}$$

where the first factor is the market factor, the other is a long-short factor (e.g., a long-short industry factor long auto industry and short mining). Assume the second factor is uncorrelated with the market factor. However, assume that cross-sectionally, the market betas are correlated with the long-short factor betas, i.e. $Corr(\beta_{1i}, \beta_{2i}) \neq 0$. In this case, a cross-sectional regression with only market betas (β_{1i}) on the right-hand-side suffers from an omitted variable bias.

Going back to the initial factor model considered in this homework, and given (5c), and (5d), note that all three PCs explain a non-trivial amount of total variance, and also note that the second principal component have weights that look akin to a long-short beta portfolio.