Homework 3 1152.

$$y = (x-2)^5 (2x+1)^4$$

$$(uv)' = u' \cdot v + u \cdot v'$$

$$y' = (5 \cdot (x-2))(2x+1) + (x-2)^5 \cdot 4 \cdot 2 \cdot (2x+1)^3 =$$

$$= (x-2)^4 (2x+1)^3 (5(2x+1) + 8(x-2))$$

$$= (x-2)^4 (2x+1)^3 (10x+5+8x-16)$$

$$y' = 0$$

$$(x-2)^4 (2x+1)^3 (18x-11) = 0$$

$$x = 2 \qquad x = -\frac{1}{2} \qquad x = \frac{11}{18}$$

$+ \quad - \quad + \quad +$

$\nearrow \quad -\frac{1}{2} \quad \searrow \quad \frac{11}{18} \quad \nearrow \quad 2 \quad \nearrow$

1153

$$y = \sqrt[3]{(2x-a)(a-x)^2} \quad (a>0)$$

$$y' = (2x-a)^{\frac{1}{3}} (a-x)^{\frac{2}{3}} = \left((2x-a)^{\frac{1}{3}}\right)' (a-x)' + (2x-a)^{\frac{1}{3}} \left((a-x)^{\frac{2}{3}}\right)' (a-x)' =$$

$$= \frac{2 \cdot (a-x)^{\frac{2}{3}}}{3 (2x-a)^{\frac{2}{3}}} - \frac{2(2x-a)^{\frac{1}{3}}}{3(a-x)^{\frac{1}{3}}} \Rightarrow \frac{4a-6x}{3\sqrt[3]{8ax^2-4x^3-5a^2x+a^3}}$$

$$y' = 0$$

$$4a - 6x = 0$$

$$x = \frac{2}{3}a$$

$+ \quad -$

$\nearrow \quad \frac{2}{3}a \quad \searrow$

1154.

$y = \dfrac{1+x+x^2}{1+x+x^2}$

$\left(\dfrac{u}{v}\right)' = \dfrac{u'v - u\cdot v'}{v^2}$

$y' = \dfrac{(-1+2x)(1+x+x^2) - (1-x+x^2)\cancel{(1+2x)}}{(1+x+x^2)^2} =$

$= \dfrac{-2+2x^2}{(x^2+x+1)^2}$

$\begin{array}{ccccc} + & & - & & + \\ \nearrow & -1 & \searrow & 1 & \nearrow \end{array}$

$y' = 0$

$1 - x^2 = 0$

$x = \pm 1$

1155. $y = \dfrac{10}{4x^3 - 9x^2 + 6x}$; $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$

$y' = \dfrac{0 - 10(4x^3 - 9x^2 + 6x)'}{(4x^3+9x^2+6x)^2} = -\dfrac{120x^2 - 180x + 60}{(4x^3+9x^2+6x)^2}$

$y' = 0$

$2x^2 - 3x + 1 = 0$

$x = \frac{1}{2}$ $x = 1$

$\begin{array}{ccccc} - & & + & & - \\ \searrow & \frac{1}{2} & \nearrow & 1 & \searrow \end{array}$

1165 $y = 2x^3 - 3x^2$

$y' = 6x^2 - 6x$

$6x^2 - 6x = 0$

$x - 1 = 0$

$x = 1$ $x = 0$

$\begin{array}{ccccc} + & & - & & + \\ & 0 & & 1 & \end{array}$

$+ \Rightarrow -$ $x_{max} = 0$

$x_{min} = 1$.

1166 $y = 2x^3 - 6x^2 - 18x + 7$

$y' = 6x^2 - 12x - 18$

$6x^2 - 12x - 18 = 0$

$x = -1$ $x = 3$

$\begin{array}{ccccc} + & & - & & + \\ & -1 & & 3 & \end{array}$

$x_{max} = -1$

$x_{min} = 3$

1185

$y = x^4 - 2x^2 + 5$; $[-2; 2]$

$y' = 4x^3 - 4x$

$x = 0$ $x = \pm 1$

x	y
-2	13 ← max
-1	13 ← min
0	5
1	13
2	3

1186.

$y = x + 2\sqrt{x}$; $[0; 4]$

$y' = 1 + \dfrac{1}{\sqrt{x}}$

$y' = 0$ $\cancel{x=2}$

x	y
0	0 ← min
4	8 ← max

1187. $y = x^5 - 5x^4 + 3x^3 + 1$; $[-1; 2]$

$5x^4 - 20x^3 + 15x^2 = 0$

$x = 1 \quad x = 3 \quad x = 0$

x	y
-1	-10
0	1
1	2 ← max
2	-7
3	-26 ← min

1287 $y = x^3 - 5x^2 + 3x - 5$ $\quad y' = 0$

$y' = 3x^2 - 10x + 3$ $\quad 3x^2 - 10x + 3 = 0$

$y'' = 6x - 10$ $\quad x = \frac{1}{3} \quad x = 3$

$6x - 10 = 0$

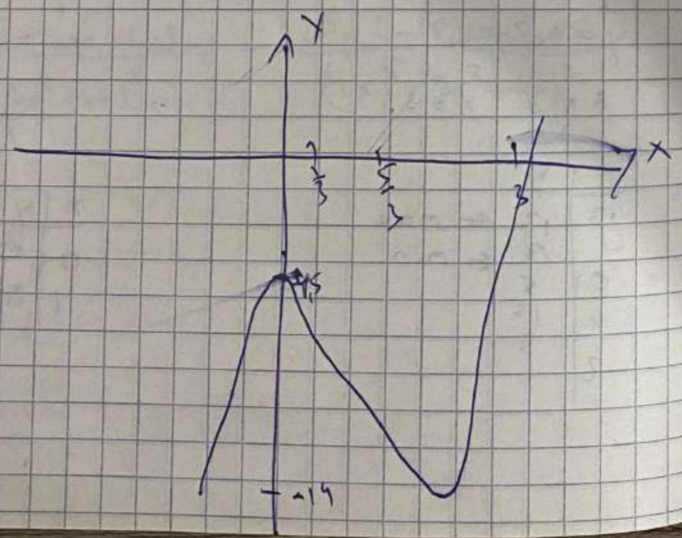
$+ \quad - \quad +$

$\nearrow \frac{1}{3} \searrow 3 \nearrow$

$x = \frac{10}{6}$

$- \quad +$

$\cap \ \frac{10}{6} \ \cup$



$f(\frac{1}{3}) \approx 4{,}5$

$f(3) = -14$

1288. $y = (x+1)^4 + e^x$

$y' = 4(x+1)^3 + e^x$

$y'' = 12(x+1)^2 + e^x$

$12(x^2 + 2x + 1) = 0$

$12x^2 + 24x + 12 = 0$

$x^2 + 2x + 1 = 0$

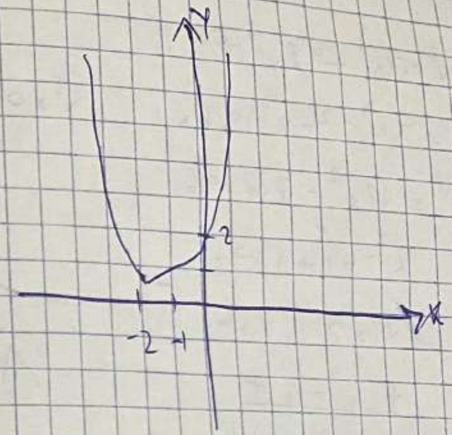
$x = -1 \ / \ x = -1$

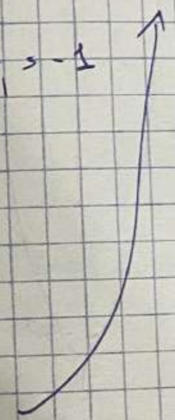
$D = 2^2 - 4 \cdot 1 = 0$

$x = -\frac{b}{2a} = -\frac{2}{2 \cdot 1} = -1$

$- \quad +$

$\cup \ -1 \ \cap$





$y' = 0$

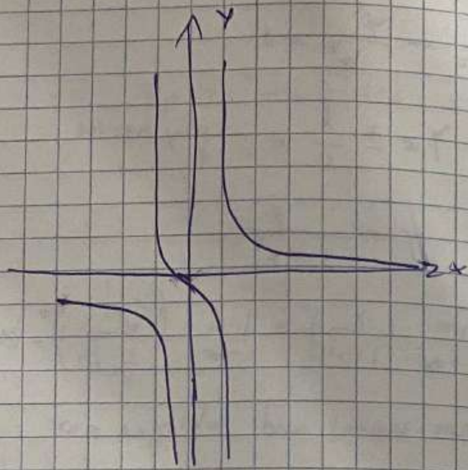
$4(x+1)^3 + e^x$

...

1300 $y = \frac{x}{x^2-1}$

$\lim_{x\to\infty} \left(\frac{x}{x^2-1}\right) = 0$

$\lim_{x\to -1} = \infty$ and $\lim_{x\to 1} = \infty$



1324.

$\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{\sqrt{x}-\sqrt{a}} = \frac{\sqrt[3]{a}-\sqrt[3]{a}}{\sqrt{a}-\sqrt{a}} = 0.$

$\lim_{x\to a} \left(\frac{\sqrt[3]{x}-\sqrt[3]{a}}{\sqrt{x}-\sqrt{a}}\right)' = \frac{2\sqrt{x}}{3\sqrt[3]{x^2}} = \frac{2}{3\sqrt[6]{a}}$

1325 $\lim_{x\to 0} \frac{\ln(\cos x)}{x} = -\frac{tg x}{1} = -tg 0 = 0$

1326 $\lim_{x\to 0} \frac{e^x-1}{\sin x} = \frac{e^x}{\cos x} = \frac{1}{1} = 1$

1743. $\int \frac{e^{2x}}{e^{2x}+a^2}dx = \int \frac{e^{2x}}{(e^{2x}+a^2)\cdot \cancel{2e^{2x}}} = \int \frac{1}{2u} = \frac{1}{2}\ln|e^{2x}+a^2| + C$

$u = e^{2x}+a^2$

$\frac{du}{dx} = \frac{1}{2e^{2x}}$

1744

$\int \tan x\, dx = \int \frac{\sin x}{\cos x} = \int \frac{\sin x}{u} \cdot -\frac{1}{\sin x} = -\ln|\cos x| + C$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

1745

$\int ctg\, x\, dx = \int \frac{\cos x}{\sin x}dx = \int \frac{\cos x}{u} \cdot \frac{1}{\cos x} = \ln|\sin x| + C$ ✓

$u = \sin x$

$\frac{du}{dx} = \cos x$

1746. $\int \tan 3x\, dx = \int \frac{\sin 3x}{\cos 3x} = \dots = -\frac{1}{3}\ln(\cos 3x) + C$

$u = \cos 3x$

1747. $\int ctg(2x+1)\,dx = \int \frac{\cos(2x+1)}{\sin(2x+1)} = \frac{1}{2}\ln(\sin(2x+1)) + C$

$u = \sin(2x+1)$

1189.

$y = x^4 - 12x^3 + 48x^2 - 50$

$y' = 4x^3 - 36x^2 + 96x$ $y' = 0$ $4x^3 - 36x^2 + 96x = 0$

$x = 0$

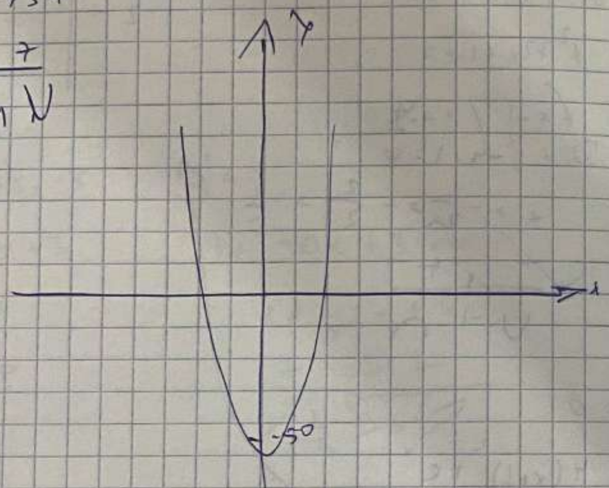
$y'' = 12x^2 - 72x + 96$

$x^2 - 6x + 8 = 0$

$x = 2$ $x = 4$

$+ \quad - \quad +$

$\cup \; 2 \; \cap \; 4 \; \cup$



1290 $y = x + 36x^2 - 2x^3 - x^4$

$y' = 1 + 72x - 8x^2 - 4x^3$ $y' = 0$ $-4x^3 - 8x^2 + 72x$

$y'' = 72 - 16x - 12x^2$ $x = 0$ $x = -1 \pm \sqrt{19}$

$x = 3$ $x = -2$

$+ \quad - \quad + \quad -$

$-1-\sqrt{19} \quad 0 \quad -1+\sqrt{19}$

$- \quad + \quad -$

$\cap \; -2 \; \cup \; 3 \; \cap$

1375

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$y = \pm \frac{b}{a} x \rightarrow$ asymptotes

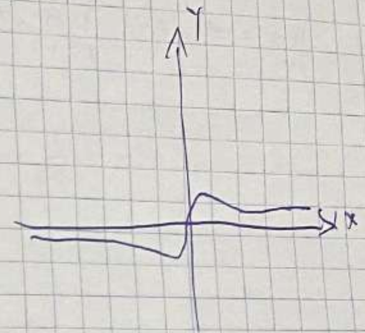
1376 $xy = 0$

$y = 0$

1398 $y = \frac{x}{1+x^2}$

$\lim_{x \to \infty} \left(\frac{x}{1+x^2} \right) = \lim_{x \to \infty} \left(\frac{\frac{1}{x}}{\frac{1}{x^2}+1} \right) = 0$

horizontal and oblique = 0



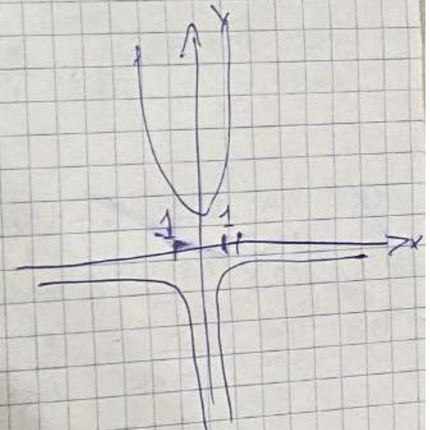
1399.

$y = \frac{1}{1-x^2}$

$\lim_{x \to \infty} \frac{1}{1-x^2} = 0$ horizontal

vertical

$\lim_{x \to -1} = \infty$ and $\lim_{x \to 1} = \infty$



1748 $\int x^3 \cdot (1+x^2)^{-\frac{1}{2}} dx = \int \frac{x^2+1-1}{2\sqrt{u}} = \frac{1}{2}\int u^{\frac{1}{2}} - \frac{1}{u^{\frac{1}{2}}} du =$

$u = 1+x^2$

$\frac{du}{dx} = 2x$

$\frac{1}{2}\left(\frac{2u \cdot \sqrt{u}}{3} - 2\sqrt{u}\right) = \frac{(1+x^2)(\sqrt{1+x^2})}{3} - \sqrt{1+x} + C$

1749.

$\int x^2 \ln(-1+x)\, dx = \frac{x^3}{3} \cdot \ln x+1 - \int \frac{x^3}{3(x+1)} =$

$\frac{x^3 \ln(x+1)}{3} + \frac{\ln(x+1)}{3} - \frac{(x+1)^3}{9} + \frac{(x+1)^2}{2} - x - 1 + C$