Assignment:- 1
AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11017

Dikshant Khandelwal
30 April, 2023

Exercise 12.13.1.10 A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution. Let X and Y be the random variables denoting the number which comes up on black and red die respectively.

Let us define cumulative frequency distribution of some random variable A,

$$F_A(i) = \Pr(A \leq i) \tag{1}$$

$$\therefore F_X(i) = F_Y(i) = \begin{cases} 0 & i < 1 \\ \frac{i}{6} & 0 < i \leq 6 \\ 1 & i > 6 \end{cases} \tag{2}$$

(a)

$$\Pr(X + Y > 9 | X = 5) = \frac{\Pr(X + Y > 9, X = 5)}{\Pr(X = 5)} \tag{3}$$

$$= \Pr(Y > 4) \tag{4}$$

$$= F_Y(6) - F_Y(4) \tag{5}$$

$$= 1 - \frac{4}{6} \tag{6}$$

$$= \frac{1}{3} \approx 0.33 \tag{7}$$

$$\therefore \Pr(X + Y > 9 | X = 5) = \frac{1}{3} \approx 0.33 \tag{8}$$

(b)

$$\Pr(X + Y = 8 | Y < 4) = \frac{\Pr(X + Y = 8, Y < 4)}{\Pr(Y < 4)} \tag{9}$$

$$= \frac{\sum_{i=2}^{i=3} \Pr(X = 8 - i, Y = i)}{F_Y(3)} \tag{10}$$

$$= \frac{\sum_{i=2}^{i=3} \Pr(X = 8 - i) \Pr(Y = i)}{F_Y(3)} \tag{11}$$

$$= \frac{\left(\frac{2}{36}\right)}{\frac{3}{6}} \tag{12}$$

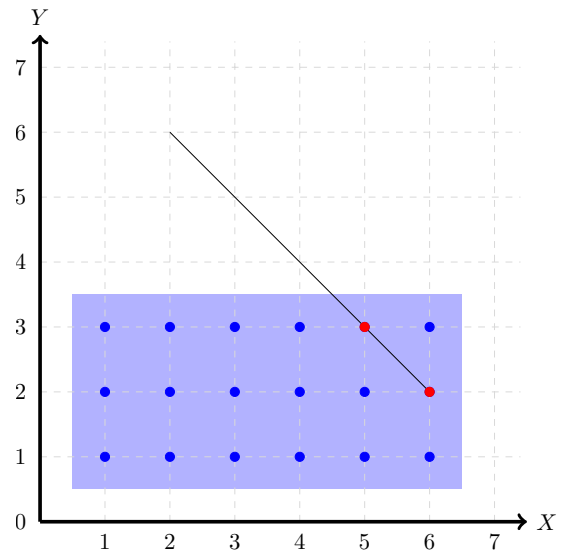$$= \frac{1}{9} \approx 0.11 \tag{13}$$

$$\therefore \Pr(X + Y = 8 | Y < 5) = \frac{1}{9} \approx 0.11 \tag{14}$$

Fig. 1. $X + Y = 8 | Y < 4$