Assignment:- 1

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11017

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Exercise 12.13.5.4 Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

(i) all the five cards are spades?
(ii) only 3 cards are spades?
(iii) none is a spade?

Solution. A deck has 52 cards among which 13 are spades. Since we are replacing drawn cards ,the probability of getting spade on any draw, $p = \frac{13}{52} = \frac{1}{4}$. This is a binomial distribution where getting a card of spades is considered success.

The probability of getting r successes in a binomial distribution having n independent Bernoulli trials and probability of success in each Bernoulli trial being p is

$$\Pr(X = r) = {}^{n}C_{r} p^{r} (1-p)^{n-r} \tag{1}$$

Here, $p = \frac{1}{4}$ and $n = 5$.

Formally, let X be the random variable denoting number of spades we get in 5 trials.

$$\therefore X \in \{0, 1, 2, 3, 4, 5\} \tag{2}$$

$$X \sim Bin\left(5, \frac{1}{4}\right) \tag{3}$$

$$\Pr(r) = {}^{5}C_{r} \left(\frac{1}{4}\right)^{r} \left(\frac{3}{4}\right)^{5-r} \tag{4}$$

(i) Probability that all 5 cards are spades is $\Pr(X = 5)$.

$$\Pr(X = 5) = {}^{5}C_{5} \left(\frac{1}{4}\right)^{5} \left(\frac{3}{4}\right)^{0} \tag{5}$$

$$= \frac{1}{1024} \approx 0.00098 \tag{6}$$

(ii) Probability that only 3 cards are spades is $\Pr(X = 3)$.

$$\Pr(X = 3) = {}^{5}C_{5} \left(\frac{1}{4}\right)^{3} \left(\frac{3}{4}\right)^{2} \tag{7}$$

$$= \frac{45}{512} \approx 0.08789 \tag{8}$$

(iii) Probability that no card is spade is $\Pr(X = 0)$.

$$\Pr(X = 0) = {}^{5}C_{0} \left(\frac{1}{4}\right)^{0} \left(\frac{3}{4}\right)^{5} \tag{9}$$

$$= \frac{243}{1024} \approx 0.23730 \tag{10}$$