

# Assignment:- 1

## AI1110: Probability and Random Variables

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CS22BTECH11017

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**Exercise 12.13.5.4** Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

- (i) all the five cards are spades?
- (ii) only 3 cards are spades?
- (iii) none is a spade?

**Solution.** A deck has 52 cards among which 13 are spades. Since we are replacing drawn cards, the probability of getting spade on any draw,  $p = \frac{13}{52} = \frac{1}{4}$ . This is a binomial distribution where getting a card of spades is considered success.

The probability of getting  $r$  successes in a binomial distribution having  $n$  independent Bernoulli trials and probability of success in each Bernoulli trial being  $p$  is

$$\Pr(X = r) = {}^nC_r p^r (1 - p)^{n-r} \quad (1)$$

Here,  $p = \frac{1}{4}$  and  $n = 5$ .

Formally, let  $X$  be the random variable denoting number of spades we get in 5 trials.

$$\therefore X \in \{0, 1, 2, 3, 4, 5\} \quad (2)$$

$$X \sim \text{Bin}\left(5, \frac{1}{4}\right) \quad (3)$$

$$\Pr(r) = {}^5C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{5-r} \quad (4)$$

- (i) Probability that all 5 cards are spades is  $\Pr(X = 5)$ .

$$\Pr(X = 5) = {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0 \quad (5)$$

$$= \frac{1}{1024} \approx 0.00098 \quad (6)$$

- (ii) Probability that only 3 cards are spades is  $\Pr(X = 3)$ .

$$\Pr(X = 3) = {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 \quad (7)$$

$$= \frac{45}{512} \approx 0.08789 \quad (8)$$

- (iii) Probability that no card is spade is  $\Pr(X = 0)$ .

$$\Pr(X = 0) = {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5 \quad (9)$$

$$= \frac{243}{1024} \approx 0.23730 \quad (10)$$