Assignment:- 1
AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11017

Dikshant Khandelwal
30 April, 2023

Exercise 12.13.1.10 A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution. Let X and Y be the random variables denoting the number which comes up on black and red die respectively.

$$\Pr(X = n) = \Pr(Y = n) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \quad (1)$$

Let $Z = X + Y$,

$$\Pr(Z = n) = \begin{cases} 0 & n \le 1 \\ \frac{n-1}{36} & 2 \le n \le 7 \\ \frac{13-n}{36} & 7 < n \le 12 \\ 0 & n > 12 \end{cases} \quad (2)$$

Let us define cumulative frequency distribution of some random variable A,

$$F_A(i) = \Pr(A \le i) \quad (3)$$

$$\therefore F_X(i) = F_Y(i) = \begin{cases} 0 & i < 1 \\ \frac{i}{6} & 0 < i \le 6 \\ 1 & i > 6 \end{cases} \quad (4)$$

Also, conditional probability of $X > r$ given $Y = k$ is:

$$\Pr(X > r|Y = k) = \frac{\Pr(X > r, Y = k)}{\Pr(Y = k)} \quad (5)$$

∵Rolling of black and red die is independent of each other,

$$\therefore \Pr(X = r, Y = k) = \Pr(X = r)\Pr(Y = k) \quad (6)$$

(a) Let us define,

$$F_{Z,X}(k, r) = \Pr(Z \le k, X = r) \quad (7)$$

Since , $Z = X + Y$,

$$F_{Z,X}(k, r) = F_Y(k - r)\Pr(X = r) \quad (8)$$

Using (8)

$$\Pr(Z > 9|X = 5) \quad (9)$$
$$= \frac{\Pr(Z > 9, X = 5)}{\Pr(X = 5)} \quad (10)$$
$$= \frac{F_{Z,X}(12, 5) - F_{Z,X}(9, 5)}{\Pr(X = 5)} \quad (11)$$
$$= \frac{F_Y(7)\Pr(X = 5) - F_Y(4)\Pr(X = 5)}{\Pr(X = 5)} \quad (12)$$
$$= F_Y(7) - F_Y(4) \quad (13)$$
$$= 1 - \frac{4}{6} \quad (14)$$
$$= \frac{1}{3} \approx 0.33 \quad (15)$$

$$\therefore \Pr(Z > 9|X = 5) = \frac{1}{3} \approx 0.33 \quad (16)$$

(b) Let us define,

$$F_{Z,Y}(k, r) = \Pr(Z = k, Y \le r) \quad (17)$$

Since , $Z = X + Y$,

$$F_{Z,Y}(k, r) = \sum_{i=1}^{r-1} \Pr(Y = i)\Pr(X = k - i) \quad (18)$$

Using (18)

$$\Pr\left(Z = 8 | Y < 4\right) = \frac{\Pr\left(Z = 8, Y < 4\right)}{\Pr\left(Y < 4\right)} \tag{19}$$

$$= \frac{F_{Z,Y}(8, 3)}{F_Y(3)} \tag{20}$$

$$= \frac{\left(\frac{2}{36}\right)}{\frac{3}{6}} \tag{21}$$

$$= \frac{1}{9} \approx 0.11 \tag{22}$$

$$\therefore \Pr\left(Z = 8 | Y < 5\right) = \frac{1}{9} \approx 0.11 \tag{23}$$