Assignment:- 1

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11017

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26 April, 2023

12.13.5.4: Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

(i) all the five cards are spades?
(ii) only 3 cards are spades?
(iii) none is a spade?

Answer:

A deck has 52 cards among which 13 are spades. Since we are replacing drawn cards ,the probability of getting spade on any draw, $p = \frac{13}{52} = \frac{1}{4}$. This is a binomial distribution where getting a card of spades is considered success.

The probability of getting r successes in a binomial distribution having n independent Bernoulli trials and probability of success in each Bernoulli trial being p is

$$P(r) = \binom{n}{r} p^r (1-p)^{n-r}$$

Here, $p = \dfrac{1}{4}$ and $n = 5$.

(i) Probability that all 5 cards are spades is $P(5)$.

$$\begin{aligned} P(5) &= \binom{5}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0 \\ &= \frac{1}{1024} \\ &\approx 0.00098 \end{aligned}$$

(ii) Probability that only 3 cards are spades is $P(3)$.

$$\begin{aligned} P(3) &= \binom{5}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 \\ &= \frac{45}{512} \\ &\approx 0.08789 \end{aligned}$$

(iii) Probability that no card is spade is $P(0)$.

$$\begin{aligned} P(0) &= \binom{5}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5 \\ &= \frac{243}{1024} \\ &\approx 0.23730 \end{aligned}$$