Assignment:- 1

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11017

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Exercise 12.13.1.10 A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution. Let X and Y be the random variables denoting the number which comes up on black and red die respectively.

$$X \in \{1,2,3,4,5,6\} \tag{1}$$

$$Y \in \{1,2,3,4,5,6\} \tag{2}$$

where, for $\forall k \in \{1,2,3,4,5,6\}$

$$\Pr(X=k) = \Pr(Y=k) = \frac{1}{6} \tag{3}$$

Also, conditional probability of $X > r$ given $Y = k$ is:

$$\Pr(X > r | Y = k) = \frac{\Pr(X > r, Y = k)}{\Pr(Y = k)} \tag{4}$$

$\because$Rolling of black and red die is independent of each other,

$$\therefore \Pr(X = r, Y = k) = \Pr(X = r)\Pr(Y = k) \tag{5}$$

(a) here we need to find,

$$\Pr(X + Y > 9 | X = 5) = \frac{\Pr(X + Y > 9, X = 5)}{\Pr(X = 5)} \tag{6}$$

$$= \frac{\sum_{i=5}^{6} \Pr(X = 5, Y = i)}{\Pr(X = 5)} \tag{7}$$

$$= \frac{\sum_{i=5}^{6} \Pr(X = 5, Y = i)}{\Pr(X = 5)} \tag{8}$$

from (5)

$$\Pr(X + Y > 9 | X = 5) = \frac{\sum_{i=5}^{6} \Pr(X = 5)\Pr(Y = i)}{\Pr(X = 5)} \tag{9}$$

$$= \Pr(Y = 5) + \Pr(Y = 6) \tag{10}$$

$$= \frac{1}{6} + \frac{1}{6} \tag{11}$$

$$= \frac{2}{6} \tag{12}$$

$$= \frac{1}{3} \approx 0.33 \tag{13}$$

(b) here, we need to find,

$$\Pr(X + Y = 8 | Y < 4) = \frac{\Pr(X + Y = 8, Y < 4)}{\Pr(Y < 4)} \tag{14}$$

$$= \frac{\sum_{i=2}^{3} \Pr(X = 8 - i, Y = i)}{\sum_{i=1}^{3} \Pr(Y = i)} \tag{15}$$

from (5)

$$\Pr(X + Y = 8 | Y < 4) = \frac{\sum_{i=2}^{3} \Pr(X = 8 - i)\Pr(Y = i)}{\sum_{i=1}^{3} \Pr(Y = i)} \tag{16}$$

$$= \frac{\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)}{3 \cdot \left(\frac{1}{6}\right)} \tag{17}$$

$$= \frac{1}{9} \approx 0.11 \tag{18}$$