Assignment:- 1

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11017

Dikshant Khandelwal
30 April, 2023

Exercise 12.13.1.10 A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution. Let X and Y be the random variables denoting the number which comes up on black and red die respectively.

$$\Pr(X = n) = \Pr(Y = n) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \tag{1}$$

$$\Pr(X + Y = n) = \begin{cases} 0 & n \le 1 \\ \frac{n-1}{36} & 2 \le n \le 7 \\ \frac{13-n}{36} & 7 < n \le 12 \\ 0 & n > 12 \end{cases} \tag{2}$$

Let us define cumulative frequency distribution of some random variable A,

$$F_A(i) = \Pr(A \le i) \tag{3}$$

$$\therefore F_X(i) = F_Y(i) = \begin{cases} 0 & i < 1 \\ \frac{i}{6} & 0 < i \le 6 \\ 1 & i > 6 \end{cases} \tag{4}$$

Also, conditional probability of $X > r$ given $Y = k$ is:

$$\Pr(X > r | Y = k) = \frac{\Pr(X > r, Y = k)}{\Pr(Y = k)} \tag{5}$$

$\because$Rolling of black and red die is independent of each other,

$$\therefore \Pr(X = r, Y = k) = \Pr(X = r)\Pr(Y = k) \tag{6}$$

(a) Let us define,

$$F_{X+Y,X}(k, r) = \Pr(X + Y \le k, X = r) \tag{7}$$
$$F_{X+Y,X}(k, r) = F_Y(k - r)\Pr(X = r) \tag{8}$$

Using (**??**)

$$\Pr(X + Y > 9 | X = 5) \tag{9}$$
$$= \frac{\Pr(X + Y > 9, X = 5)}{\Pr(X = 5)} \tag{10}$$
$$= \frac{F_{X+Y,X}(12, 5) - F_{X+Y,X}(9, 5)}{\Pr(X = 5)} \tag{11}$$
$$= \frac{F_Y(7)\Pr(X = 5) - F_Y(4)\Pr(X = 5)}{\Pr(X = 5)} \tag{12}$$
$$= F_Y(7) - F_Y(4) \tag{13}$$
$$= 1 - \frac{4}{6} \tag{14}$$
$$= \frac{1}{3} \approx 0.33 \tag{15}$$

$$\therefore \Pr(X + Y > 9 | X = 5) = \frac{1}{3} \approx 0.33 \tag{16}$$

(b) Let us define,

$$F_{X+Y,Y}(k, r) = \Pr(X + Y = k, Y \le r) \tag{17}$$
$$F_{X+Y,Y}(k, r) = \sum_{i=1}^{r-1} \Pr(Y = i)\Pr(X = k - i) \tag{18}$$

Using (**??**)

$$
\begin{aligned}
\Pr(X+Y=8|Y<4) &= \frac{\Pr(X+Y=8, Y<4)}{\Pr(Y<4)} && (19)\\
&= \frac{F_{X+Y,Y}(8,3)}{F_Y(3)} && (20)\\
&= \frac{\left(\frac{2}{36}\right)}{\frac{3}{6}} && (21)\\
&= \frac{1}{9} \approx 0.11 && (22)
\end{aligned}
$$

$$
\therefore \Pr(X+Y=8|Y<5) = \frac{1}{9} \approx 0.11 \qquad (23)
$$