

Assignment:- 3

AI1110: Probability and Random Variables

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CS22BTECH11017

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Exercise 12.13.1.14 Given that 2 numbers appearing on throwing two dice are different. Find the probability of the event ‘ the sum of numbers on the dice is 4’ .

Solution. Let X and Y be the random variables denoting the number which comes up on Die1 and Die2 respectively.

$$\Pr(X + Y = n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \leq n \leq 7 \\ \frac{13-n}{36} & 7 < n \leq 12 \\ 0 & n > 12 \end{cases} \quad (14)$$

$$\therefore \Pr(A) = \Pr(X + Y = 4) = \frac{4-1}{36} = \frac{1}{12} \quad (15)$$

$$\Pr(X + Y = 4 | X \neq Y) = \frac{\Pr(X + Y = 4, X \neq Y)}{\Pr(X \neq Y)} \quad (1)$$

Probability of an event E , written as $\Pr(E)$

$$\Pr(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total Number of possible outcomes}} \quad (2)$$

$$\Pr(X \neq Y) = 1 - \Pr(X = Y) \quad (3)$$

$$= 1 - \frac{6}{36} \quad (4)$$

$$= \frac{5}{6} \quad (5)$$

$$\Pr(AB) = \Pr(X + Y = 4, X \neq Y) = \frac{1}{12} - \frac{1}{36} \quad (16)$$

$$= \frac{1}{18} \quad (17)$$

$$\Pr(X + Y = 4 | X \neq Y) = \frac{\left(\frac{1}{18}\right)}{\left(\frac{5}{6}\right)} \quad (18)$$

$$\therefore \Pr(X + Y = 4 | X \neq Y) = \frac{1}{15} \quad (19)$$

Let A be the event $X + Y = 4$ and B be the event $X \neq Y$

$$A = A(B + B') \quad (6)$$

$$= AB + AB' \quad (7)$$

$$\therefore \Pr(A) = \Pr(AB) + \Pr(AB') \quad (8)$$

$$\Pr(AB) = \Pr(A) - \Pr(AB') \quad (9)$$

$$X = Y \quad (10)$$

$$X + Y = 4 \quad (11)$$

$$X = Y = 2 \quad (12)$$

$$\therefore \Pr(AB') = \Pr(X + Y = 4, X = Y) = \frac{1}{36} \quad (13)$$