Assignment:- 1

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11017

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Exercise 12.13.1.10 A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution. Let X and Y be the random variables denoting the number which comes up on black and red die respectively.

$$\Pr(X=n)=\Pr(Y=n)=\begin{cases}\frac{1}{6} & 1\le n\le 6\\ 0 & otherwise\end{cases} \tag{1}$$

Let $Z=X+Y$,

$$\Pr(Z=n)=\begin{cases}0 & n\le 1\\ \frac{n-1}{36} & 2\le n\le 7\\ \frac{13-n}{36} & 7<n\le 12\\ 0 & n>12\end{cases} \tag{2}$$

Also, conditional probability of $X>r$ given $Y=k$ is:

$$\Pr(X>r|Y=k)=\frac{\Pr(X>r,Y=k)}{\Pr(Y=k)} \tag{3}$$

$\because$Rolling of black and red die is independent of each other,

$$\therefore \Pr(X=r,Y=k)=\Pr(X=r)\Pr(Y=k) \tag{4}$$

$$\Pr(Z=k,X=r)=\Pr(Y=k-r,X=r) \tag{5}$$

$$=\Pr(Y=k-r)\Pr(X=r) \tag{6}$$

$$\therefore \Pr(Z=k,X=r)=\begin{cases}\frac{1}{36} & 0<r\le 6, 0<k-r\le 6,\\ 0 & otherwise\end{cases} \tag{7}$$

WLOG, Z and Y follow,

$$\therefore \Pr(Z=k,Y=r)=\begin{cases}\frac{1}{36} & 0<r\le 6, 0<k-r\le 6\\ 0 & otherwise\end{cases} \tag{8}$$

(a) here we need to find,

$$\Pr(Z>9|X=5)=\frac{\Pr(Z>9,X=5)}{\Pr(X=5)} \tag{9}$$

$$=\frac{\sum_{i=10}^{12}\Pr(X=5,Z=i)}{\Pr(X=5)} \tag{10}$$

from (7)

$$\Pr(Z>9|X=5)=\frac{\frac{1}{36}+\frac{1}{36}+0}{\frac{1}{6}} \tag{11}$$

$$=\frac{2}{6} \tag{12}$$

$$=\frac{1}{3}\approx 0.33 \tag{13}$$

(b) here, we need to find,

$$\Pr(Z=8|Y<4)=\frac{\Pr(Z=8,Y<4)}{\Pr(Y<4)} \tag{14}$$

$$=\frac{\sum_{i=1}^{3}\Pr(Z=8,Y=i)}{\sum_{i=1}^{3}\Pr(Y=i)} \tag{15}$$

from (8)

$$\Pr(Z=8|Y<4)=\frac{0+\frac{1}{36}+\frac{1}{36}}{\frac{1}{6}+\frac{1}{6}+\frac{1}{6}} \tag{16}$$

$$=\frac{1}{9}\approx 0.11 \tag{17}$$