Assignment:- 1

AI1110: Probability and Random Variables

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CS22BTECH11017

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Exercise 12.13.1.10 A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution. Let X and Y be the random variables denoting the number which comes up on black and red die respectively.
Let us define cumulative frequency distribution of some random variable A,

$$F_A(i) = \Pr(A \le i) \tag{1}$$

$$\therefore F_X(i) = F_Y(i) = \begin{cases} 0 & i < 1 \\ \frac{i}{6} & 0 < i \le 6 \\ 1 & i > 6 \end{cases} \tag{2}$$

X and Y are independent random variables.

$$\Pr(X = k, Y = r) = \Pr(X = k)\Pr(Y = r) \tag{3}$$

$$\therefore \Pr(X = k, Y = r) = \frac{1}{36} \tag{4}$$

(a)

$$\Pr(X + Y > 9 | X = 5) = \frac{\Pr(X + Y > 9, X = 5)}{\Pr(X = 5)} \tag{5}$$

$$= \Pr(Y > 4) \tag{6}$$

$$= F_Y(6) - F_Y(4) \tag{7}$$

$$= 1 - \frac{4}{6} \tag{8}$$

$$= \frac{1}{3} \approx 0.33 \tag{9}$$

$$\therefore \Pr(X + Y > 9 | X = 5) = \frac{1}{3} \approx 0.33 \tag{10}$$

(b)

$$\Pr(X + Y = 8 | Y < 4) = \frac{\Pr(X + Y = 8, Y < 4)}{\Pr(Y < 4)} \tag{11}$$

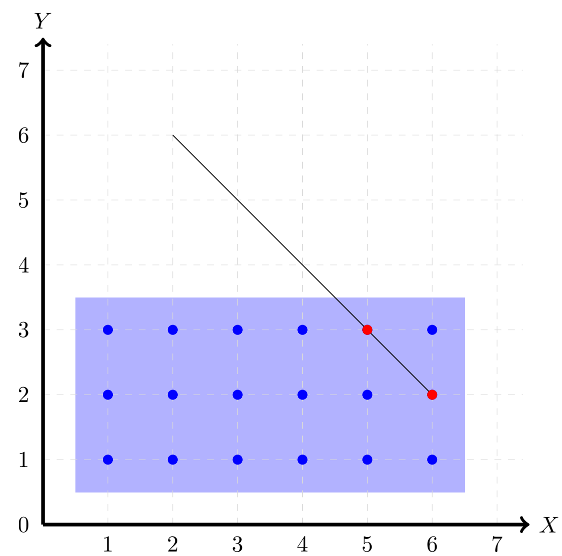

Fig. 1. $X + Y = 8 | Y < 4$

Probability of an event E, written as $\Pr(E)$

$$\Pr(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total Number of possible outcomes}} \tag{12}$$

Total number of (X, Y) such that $0 < X \le 6, 0 < Y \le 6$ is 36
In Fig(1), the blue region represent $Y < 4$ and the line is $X + Y = 8$, therefore red dots represent, $X + Y = 8, Y < 4$

$$\Pr(Y < 4) = \frac{\text{Number of } (X, Y) \text{ in blue region}}{36} \tag{13}$$

$$= \frac{18}{36} \tag{14}$$

from (11),

$$\Pr(X+Y=8, Y<4) = \frac{\text{Number of red dots }(X,Y)}{36} \tag{15}$$

$$= \frac{2}{36} \tag{16}$$

$$\therefore \Pr(X+Y=8 \mid Y<4) = \frac{\left(\frac{2}{36}\right)}{\left(\frac{18}{36}\right)} \tag{17}$$

$$= \frac{1}{9} \approx 0.11 \tag{18}$$