

Assignment:- 2

AI1110: Probability and Random Variables

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CS22BTECH11017

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11th May, 2023

Exercise 12.13.1.10 A black and a red dice are rolled. (b)

- (a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
- (b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution. Let X and Y be the random variables denoting the number which comes up on black and red die respectively.

Let us define cumulative frequency distribution of some random variable A ,

$$F_A(i) = \Pr(A \leq i) \quad (1)$$

$$\therefore F_X(i) = F_Y(i) = \begin{cases} 0 & i < 1 \\ \frac{i}{6} & 0 < i \leq 6 \\ 1 & i > 6 \end{cases} \quad (2)$$

X and Y are independent random variables.

$$\Pr(X = k, Y = r) = \Pr(X = k) \Pr(Y = r) \quad (3)$$

$$\therefore \Pr(X = k, Y = r) = \frac{1}{36} \quad (4)$$

(a)

$$\Pr(X + Y > 9 | X = 5) = \frac{\Pr(X + Y > 9, X = 5)}{\Pr(X = 5)} \quad (5)$$

$$= \Pr(Y > 4) \quad (6)$$

$$= F_Y(6) - F_Y(4) \quad (7)$$

$$= 1 - \frac{4}{6} \quad (8)$$

$$= \frac{1}{3} \approx 0.33 \quad (9)$$

$$\therefore \Pr(X + Y > 9 | X = 5) = \frac{1}{3} \approx 0.33 \quad (10)$$

$$\Pr(X + Y = 8 | Y < 4) = \frac{\Pr(X + Y = 8, Y < 4)}{\Pr(Y < 4)} \quad (11)$$

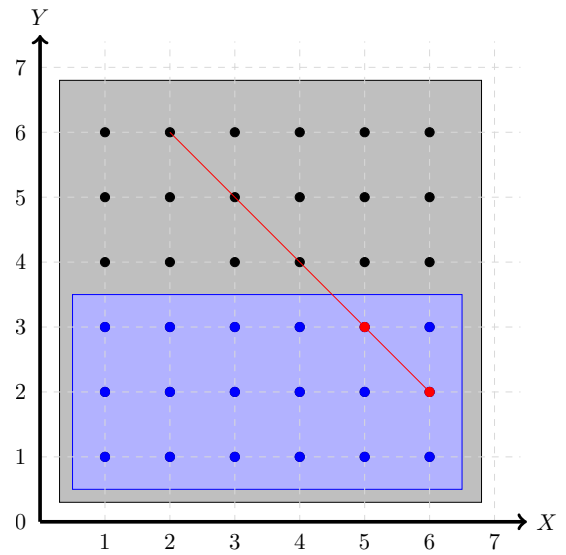


Fig. 1. $X + Y = 8 | Y < 4$

Probability of an event E , written as $\Pr(E)$

$$\Pr(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total Number of possible outcomes}} \quad (12)$$

$$\Pr(Y < 4) = \frac{\text{Number of } (X, Y) \text{ in blue region}}{\text{Number of } (X, Y) \text{ in gray region}} \quad (13)$$

$$= \frac{(3) \cdot (6)}{(6) \cdot (6)} \quad (14)$$

$$= \frac{1}{2} \quad (15)$$

$$\Pr(X + Y = 8, Y < 4) = \frac{\text{Number of red dots } (X, Y)}{\text{Number of } (X, Y) \text{ in gray region}} \quad (16)$$

$$= \frac{2}{(6) \cdot (6)} \quad (17)$$

$$= \frac{1}{18} \quad (18)$$

$$\therefore \Pr(X + Y = 8|Y < 4) = \frac{\left(\frac{1}{18}\right)}{\left(\frac{1}{2}\right)} \quad (19)$$

$$= \frac{1}{9} \approx 0.11 \quad (20)$$