Assignment:- 1

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11017

Dikshant Khandelwal

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Exercise 12.13.1.10 A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution. Let X and Y be the random variables denoting the number which comes up on black and red die respectively.

$$\Pr(X = n) = \Pr(Y = n) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \tag{1}$$

Let us define cumulative frequency distribution of some random variable A,

$$F_A(i) = \Pr(A \le i) \tag{2}$$

$$\therefore F_X(i) = F_Y(i) = \begin{cases} 0 & i < 1 \\ \frac{i}{6} & 0 < i \le 6 \\ 1 & i > 6 \end{cases} \tag{3}$$

$$M_X(z) = M_Y(z) = \frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6} \tag{4}$$

$\because$Rolling of black and red die is independent of each other,

$$\Pr(X = r, Y = k) = \Pr(X = r)\Pr(Y = k) \tag{5}$$

$$\therefore M_{X+Y}(z) = M_X(z)M_Y(z) \tag{6}$$

$$\Pr(X + Y = n) = \begin{cases} 0 & n \le 1 \\ \frac{n-1}{36} & 2 \le n \le 7 \\ \frac{13-n}{36} & 7 < n \le 12 \\ 0 & n > 12 \end{cases} \tag{7}$$

(a) We want sum of all the coefficient in $M_{X+Y}(z)$ such that $X + Y > 9$ when z has a exponent -5 in $M_X(Z)$ which implies $Y > 4$

$$\Pr(X + Y > 9|X = 5) \tag{8}$$
$$= \frac{\Pr(X + Y > 9, X = 5)}{\Pr(X = 5)} \tag{9}$$
$$= \Pr(Y > 4) \tag{10}$$
$$= F_Y(7) - F_Y(4) \tag{11}$$
$$= 1 - \frac{4}{6} \tag{12}$$
$$= \frac{1}{3} \approx 0.33 \tag{13}$$

$$\therefore \Pr(X + Y > 9|X = 5) = \frac{1}{3} \approx 0.33 \tag{14}$$

(b) We want the coefficient of z^{-8} in $M_{X+Y}(z)$ when $Y < 4$.

$$\Pr(X + Y = 8|Y < 4) = \frac{\Pr(X + Y = 8, Y < 4)}{\Pr(Y < 4)} \tag{15}$$
$$= \frac{\frac{2}{36}}{F_Y(3)} \tag{16}$$
$$= \frac{\left(\frac{2}{36}\right)}{\frac{3}{6}} \tag{17}$$
$$= \frac{1}{9} \approx 0.11 \tag{18}$$

$$\therefore \Pr(X + Y = 8|Y < 5) = \frac{1}{9} \approx 0.11 \tag{19}$$