# Assignment:- 1

## AI1110: Probability and Random Variables
## Indian Institute of Technology, Hyderabad

CS22BTECH11017

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**Exercise 12.13.1.10** A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

**Solution.** Let $X$ and $Y$ be the random variables denoting the number which comes up on black and red die respectively.

$$\Pr(X = n) = \Pr(Y = n) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \quad (1)$$

Let us define cumulative frequency distribution of some random variable A,

$$F_A(i) = \Pr(A \le i) \quad (2)$$

$$\therefore F_X(i) = F_Y(i) = \begin{cases} 0 & i < 1 \\ \frac{i}{6} & 0 < i \le 6 \\ 1 & i > 6 \end{cases} \quad (3)$$

$\because$Rolling of black and red die is independent of each other,

$$\therefore \Pr(X = r, Y = k) = \Pr(X = r)\Pr(Y = k) \quad (4)$$

We know that, for a random variable Z-transform is ,

$$M_X(z) = E\left[s^{-X}\right] = \sum_{i=-\infty}^{\infty} \Pr(X = i)\, s^{-X} \quad (5)$$

$$M_{X+Y}(z) = E\left[z^{-(X+Y)}\right] \quad (6)$$

$$= \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \Pr(X = i, Y = j)\, z^{-(X+Y)} \quad (7)$$

$$(8)$$

So,

$$M_{X+Y}(z) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \Pr(X = i)\Pr(Y = j)\, z^{-X} z^{-Y} \quad (9)$$

$$= \left(\sum_{i=-\infty}^{\infty} \Pr(X = i)\, z^{-X}\right)\left(\sum_{j=-\infty}^{\infty} \Pr(Y = j)\, z^{-Y}\right) \quad (10)$$

$$\therefore M_{X+Y}(z) = M_X(z)M_Y(z) \quad (11)$$

$$M_X(z) = M_Y(z) = \frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6} \quad (12)$$

$$M_{X+Y}(z) = \left(\frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6}\right)^2 \quad (13)$$

$$\Pr(X + Y = n) = \begin{cases} 0 & n \le 1 \\ \frac{n-1}{36} & 2 \le n \le 7 \\ \frac{13-n}{36} & 7 < n \le 12 \\ 0 & n > 12 \end{cases} \quad (14)$$

(a) We want sum of all the coefficient in $M_{X+Y}(z)$ such that $X + Y > 9$ when z has a exponent $-5$

in $M_X(Z)$ which implies $Y > 4$

$$\begin{aligned}
&\Pr(X+Y>9|X=5) && (15)\\
&= \frac{\Pr(X+Y>9, X=5)}{\Pr(X=5)} && (16)\\
&= \Pr(Y>4) && (17)\\
&= \frac{F_Y(7)\Pr(X=5) - F_Y(4)\Pr(X=5)}{\Pr(X=5)} && (18)\\
&= F_Y(7) - F_Y(4) && (19)\\
&= 1 - \frac{4}{6} && (20)\\
&= \frac{1}{3} \approx 0.33 && (21)
\end{aligned}$$

$$\therefore \Pr(X+Y>9|X=5) = \frac{1}{3} \approx 0.33 \quad (22)$$

(b) We want the coefficient of $z^{-8}$ in $M_{X+Y}(z)$ when $Y < 4$ .

$$\Pr(X+Y=8|Y<4) = \frac{\Pr(X+Y=8, Y<4)}{\Pr(Y<4)} \quad (23)$$

$$\begin{aligned}
&= \frac{\frac{2}{36}}{F_Y(3)} && (24)\\
&= \frac{\left(\frac{2}{36}\right)}{\frac{3}{6}} && (25)\\
&= \frac{1}{9} \approx 0.11 && (26)
\end{aligned}$$

$$\therefore \Pr(X+Y=8|Y<5) = \frac{1}{9} \approx 0.11 \quad (27)$$