

# Assignment:- 1

## AI1110: Probability and Random Variables

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CS22BTECH11017

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**Exercise 12.13.5.4** Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

- (i) all the five cards are spades?
- (ii) only 3 cards are spades?
- (iii) none is a spade?

**Solution.** A deck has 52 cards among which 13 are spades. Since we are replacing drawn cards, the probability of getting spade on any draw,  $p = \frac{13}{52} = \frac{1}{4}$ . This is a binomial distribution where getting a card of spades is considered success.

The probability of getting  $r$  successes in a binomial distribution having  $n$  independent Bernoulli trials and probability of success in each Bernoulli trial being  $p$  is

$$\Pr(r) = {}^nC_r p^r (1-p)^{n-r} \quad (1)$$

Here,  $p = \frac{1}{4}$  and  $n = 5$ .

- (i) Probability that all 5 cards are spades is  $\Pr(5)$ .

$$\Pr(5) = {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0 \quad (2)$$

$$= \frac{1}{1024} \approx 0.00098 \quad (3)$$

- (ii) Probability that only 3 cards are spades is  $\Pr(3)$ .

$$\Pr(3) = {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 \quad (4)$$

$$= \frac{45}{512} \approx 0.08789 \quad (5)$$

- (iii) Probability that no card is spade is  $\Pr(0)$ .

$$\Pr(0) = {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5 \quad (6)$$

$$= \frac{243}{1024} \approx 0.23730 \quad (7)$$