Assignment:- 1

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11017

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Exercise 12.13.1.10 A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution. Let X and Y be the random variables denoting the number which comes up on black and red die respectively.

$$X \in \{1,2,3,4,5,6\} \tag{1}$$
$$Y \in \{1,2,3,4,5,6\} \tag{2}$$

where, for $\forall k \in \{1,2,3,4,5,6\}$

$$\Pr(X=k) = \Pr(Y=k) = \frac{1}{6} \tag{3}$$

Also, conditional probability of $X > r$ given $Y = k$ is:

$$\Pr(X > r|Y=k) = \frac{\Pr(X>r, Y=k)}{\Pr(Y=k)} \tag{4}$$

$\because$Rolling of black and red die is independent of each other,

$$\therefore \Pr(X=r, Y=k) = \Pr(X=r)\Pr(Y=k) \tag{5}$$

(a) here we need to find,

$$\Pr(X+Y>9|X=5) = \frac{\Pr(X+Y>9, X=5)}{\Pr(X=5)} \tag{6}$$
$$= \frac{\sum_{i=5}^{6}\Pr(X=5, Y=i)}{\Pr(X=5)} \tag{7}$$
$$= \frac{\sum_{i=5}^{6}\Pr(X=5, Y=i)}{\Pr(X=5)} \tag{8}$$
$$\tag{9}$$

from (5)

$$\Pr(X+Y>9|X=5) = \frac{\sum_{i=5}^{6}\Pr(X=5)\Pr(Y=i)}{\Pr(X=5)} \tag{10}$$
$$= \Pr(Y=5) + \Pr(Y=6) \tag{11}$$
$$= \frac{1}{6} + \frac{1}{6} \tag{12}$$
$$= \frac{2}{6} \tag{13}$$
$$= \frac{1}{3} \approx 0.33 \tag{14}$$

(b) here, we need to find,

$$\Pr(X+Y=8|Y<4) = \frac{\Pr(X+Y=8, Y<4)}{\Pr(Y<4)} \tag{15}$$
$$= \frac{\sum_{i=2}^{3}\Pr(X=8-i, Y=i)}{\sum_{i=1}^{3}\Pr(Y=i)} \tag{16}$$

from (5)

$$\Pr(X+Y=8|Y<4) = \frac{\sum_{i=2}^{3}\Pr(X=8-i)\Pr(Y=i)}{\sum_{i=1}^{3}\Pr(Y=i)} \tag{17}$$

$$= \frac{\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)}{3\cdot\left(\frac{1}{6}\right)} \tag{18}$$

$$= \frac{1}{9} \approx 0.11 \tag{19}$$