MEC E 652
Autonomous Driving and Navigation

Winter 2025 – Assignment 2

Due on March 17th, 7:00 PM (MST)
Submission through the dropbox on eClass

Assignment 2

Q1. For the following single-track front-steered robot model, with linear tire force assumptions, front/rear tire cornering stiffness $C_{\alpha f}, C_{\alpha r}$, mass/inertia parameters m, I_z, front and rear axles to CG a, b, and control input $\delta(t)$:

$$\begin{Bmatrix} \dot{v_y} \\ \dot{r} \end{Bmatrix} = \begin{bmatrix} -\dfrac{C_{\alpha f} + C_{\alpha r}}{v_x m} & -\left(\dfrac{aC_{\alpha f} - bC_{\alpha r}}{u_x m} + v_x\right) \\ -\dfrac{aC_{\alpha f} - bC_{\alpha r}}{v_x I_z} & -\dfrac{a^2 C_{\alpha f} + b^2 C_{\alpha r}}{v_x I_z} \end{bmatrix} \begin{Bmatrix} v_y \\ r \end{Bmatrix} + \begin{Bmatrix} \dfrac{C_{\alpha f}}{m} \\ \dfrac{aC_{\alpha f}}{I_z} \end{Bmatrix} \delta(t) \quad \Longrightarrow \quad \dot{x}(t) = A\, x(t) + B\delta(t)$$

i. Study the observability of the system **<u>in discrete-time</u>** to estimate the lateral speed v_y as long as we have access to the longitudinal/lateral accelerations a_x, a_y and the yaw rate r measured (in the body frame $x - y$) by a 3-axis IMU installed at the robot CG. Note that the measured acceleration and yaw rate include Gaussian (zero-mean) noises.

ii. If the system is observable, develop a linear Kalman state observer to estimate the lateral speed v_y and derive the error covariance, state prediction, and state update equations for this system.

iii. If the measured signals (i.e., accelerations and yaw rate by the IMU at CG) are corrupted by the bias, which variant of the Kalman observer could be used? Elaborate on reasons and assumptions.

Q2. For the following kinematic model for autonomous driving, study the controllability of the system in discrete-time, i.e., capability of the control input vector (front steering δ_f and acceleration a) to move the state variables from any initial state to any final state in a finite time. Note that controllability could be interpreted as a dual of observability, but involves the input and state matrices B and A.

<u>Hint:</u> Make any simplifying assumption to form a linear representation of the kinematic model to be able to check controllability conditions of linear time-invariant (LTI) systems.

$$
\begin{aligned}
\dot{x} &= v\cos(\psi + \beta) \\
\dot{y} &= v\sin(\psi + \beta) \\
\dot{\psi} &= \frac{v}{l_r}\sin(\beta) \\
\dot{v} &= a \\
\beta &= \tan^{-1}\left(\frac{l_r}{l_f + l_r}\tan(\delta_f)\right)
\end{aligned}
$$

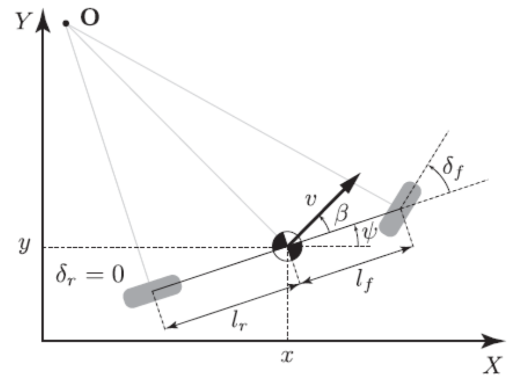

Fig. 1: Kinematic model for the controllability analysis in Q2 (images are from [1])

[1] J. Kong et al. “Kinematic and Dynamic Vehicle Models for Autonomous Driving Control Design,” 2015 IEEE IV symposium, COEX, Seoul, Korea, pp. 1094-99, 2015.