MEC E 652
Autonomous Driving and Navigation

Winter 2025 – Assignment 1

Due on Feb. 14th, 7:00 PM (MST)
Submission through the Dropbox on eClass

Assignment 1

Q1. For the front-steered mobile robot shown in Fig. 1, the configuration is described by $q = [x, y, \theta, \phi]$, in which the position of the point at the rear axle is denoted by x and y, the front steering angle is shown by ϕ, and the heading angle is θ (i.e., the yaw rate is $\dot{\theta}$). Assume there is rolling without slipping, thus, the sideways velocity of the rear wheels is zero.

i. Prove that the rolling without slipping constraints lead to the following set of constraints:

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0$$
$$\dot{x}\sin(\phi+\theta) - \dot{y}\cos(\phi+\theta) - \dot{\theta}\,d\cos\phi = 0$$

ii. Obtain the allowable velocity vectors as a combination of the basis vectors to derive $\dot{q} = q_1(q)u_1 + \cdots + g_m u_m$ where each velocity vector $\dot{q}$ satisfies $w_i(q)\dot{q} = 0$, and elaborate on control inputs and physical representation of the motion for each g_j controlled by $u_1, \ldots, u_m$

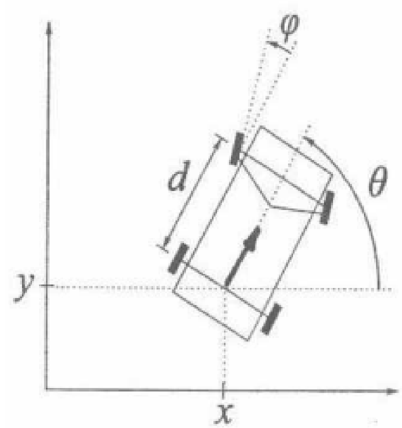

Fig. 1: Kinematic representation of a front-steered mobile robot (having rolling without slippage)

Q2. For an autonomous vehicle platform with the geometrical parameters of mass: 1780 kg; front axles to CG: 1.32 m; rear axles to CG: 1.46 m; moment of inertia I_z about the vertical axis: 4400 $\mathrm{kg.m}^2$; front cornering stiffness $C_{\alpha f}$: 70,500 N/rad; rear cornering stiffness $C_{\alpha r}$: 70,500 N/rad on a dry surface; and coordinates shown in Fig. 2,

i) Simulate conventional vehicle handling responses by using a bicycle model for longitudinal speeds of 65 and 135 kp/h (note: you need to change the speed to m/s for the stability and response analysis). For both of these maneuvers, set the front steering input angle to $\delta = 0.04\ \mathrm{rad}$. Compare lateral velocity (in the body frame), yaw rate, lateral acceleration (in the body frame), vehicle path (in the World frame), and tire slip angle responses for the maneuvers (i.e., through plots with time in the horizontal axes).

ii) Discuss on the stability of the system and pole locations, and calculate under-steer coefficient. Comments on the results and discuss validity of the bicycle model for the maneuvers considering bicycle model linearity assumptions.

iii) Redo all components requested in items i) and ii) above with a bilinear tire force model with the same slope for the linear part (i.e., $C_{\alpha f} = C_{\alpha} r$ = 70,500 N/rad), and saturation after $\alpha_s = 7$ deg for the nominal normal/vertical load.

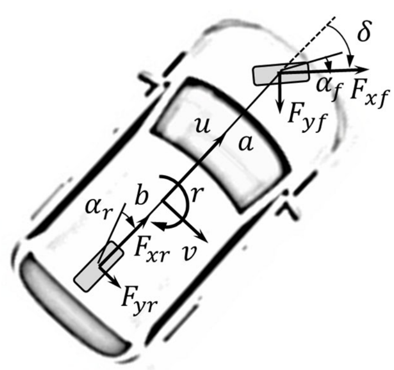

Fig. 2: Vehicle bicycle model used for Q2